$$\begin{array}{rl} 17,35 & \text{Breadth.} \\ 30,5 & \text{Length.} \\ \hline 8675 & \\ 52050 & \\ \hline 529,175 & \text{Feet.} \end{array}$$

$$\frac{529,175}{9}\Big(\ 58,797 \text{ Yards.}$$

If the Demand were for Inches. But if in any of these *Examples*, the Demand had been to know the Inches, then forasmuch as Inches are of lesser Denomination than the Feet given in the Proposition, the Feet found should have been multiplied by the Inches in 1 square Foot, that is (12 x 12 or) 144, Or the Sides given multiplied by 12, and then multiplied one into the other, would have produced the same Effect.

For the Side unknown. In the second Case, when the *Area* and Side is given, to find the other Side, divide the *Area* by the given Side. But oftentimes the *Area* is given implicitly, and to be understood in the Denomination demanded.

Q. Of Length to make a Foot of Board. *Example* 1. A Board is 6 Inches broad: how much Length thereof shall make a square Foot.

Answer. *Answ.* 24 Inches, or two Feet: For either 144 the Inches in 1 square Foot divided by 6, or 1 Foot by $\frac{1}{2}$ in Fractions, effecteth the Desire; and the Analogy in all such Questions is reciprocal, and resolved by the *Indirect Rule of Three*, as in Plain Proportions before.

As *Breadth.* 12 . *Length.* 12 :: *Breadth.* 6 . *Length.* 24

6) 144 (24 Inches.

Or as 1 Breadth . 1 Length :: $\frac{1}{2}$ Breadth . 2 Length.

$$\frac{1}{2}\Big)\frac{1}{1}\Big(\frac{2}{1} \text{ Feet.}$$

Q. Of Length to make a Square of Tiling. *Example* 2. A Roof $16\frac{1}{4}$ Feet broad: how much Length thereof shall make a Square?

Answer. *Answ.* $6\frac{2}{13}$ Feet?

As *Breadth.* 10 . *Length.* 10 :: *Breadth.* $16\frac{1}{4}$. *Length.* $6\frac{2}{13}$

$$\frac{65}{4}\Big)\frac{100}{1}\Big(\frac{400}{65}\Big(6\tfrac{2}{13} \text{ Feet.}$$

Q. Of Length to make a Yard of Pavement. *Example* 3. A Pavement $16\frac{1}{2}$ Feet broad: how much Length thereof shall make a Square Yard?

Answer. *Answ.* $\frac{6}{11}$ of a Yard.

As *Breadth.* 3 . *Length.* 3 :: *Breadth.* $16\frac{1}{2}$. *Length.* $\frac{6}{11}$

9

$$\frac{33}{2}\Big)\frac{9}{1}\Big(\frac{6}{11} \text{ Yard.}$$

Duplicate Ratio's. Among like Plains in a Duplicate *Ratio*, that is, as the Squares of their Homologal Sides: If 3 Numbers be given, in which as the Square of the First is to the Square of the Second, so the Third to a Number sought. Then as the first Square to the third Number; so is the second Square to the Number sought.

Q. Of the Number of paving Tiles. *Example* 1. Two like Rectangled *Area's* or Plains, the Greater in Length 40 Feet, the Lesser 24, each paved with paving Tiles, the Greater hath 1200 Tiles: how many hath the Lesser?

Answer. *Answ.* 432: Because 1600, the Square of 40 to 576 the Square of 24, is as 1200 to 432, that is as 25 to 9.

(Q. 40) (Q. 24.)
As 16,00 . 12,00 :: 576 . 432 Tiles.

$$\begin{array}{r} 576 \\ 12 \\ \hline 1152 \\ 576 \\ \hline 16)6912(432 \end{array}$$

Example 2. How many Acres of Wood-land, measured with a Perch of 18 Feet, are there in 73 Acres of Land measured with a Perch of $16\frac{1}{2}$ Feet? Q. *Of Acres Woodland Measure.*

Answ. 61,34, and somewhat over: For 18 and $16\frac{1}{2}$ reduced to their least Terms, are as 12 to 11: Wherefore, Answer.

(Q. 12) (Q. 11)
As 144 . 73 :: 121 . 61,34 Acres.

$$\begin{array}{r} 121 \\ 73 \\ \hline 363 \\ 847 \\ \hline 8833 \end{array}$$

$$8833,00 \,(61,34$$

Concerning the Values of Quadrangles in exchange or otherwise, the Analogy lies between the two *Area's*, Direct if the Price be the second Number, but Reciprocal if the Quantity. For the Value.

Example 1. A Merchant hath 40 Yards of Cloth of $1\frac{1}{4}$ Yard broad, and would exchange for another Piece of $\frac{3}{4}$ Broad: how many Yards of this ought he to have for the other? Q. *Of Cloth of one Breadth for another.*

Answ. $66\frac{2}{3}$ Yards: For the 40 multiplied by $1\frac{1}{4}$, being the Content of 1 Piece, shall be divided by $\frac{3}{4}$, as in the *Indirect Rule of Three.* Answer.

($40 \times 1\frac{1}{4}$)
As 50 . 1 :: $\frac{3}{4}$. $66\frac{2}{3}$ Yards.

$$\frac{3}{4}\Big)\frac{50}{1}\Big(\frac{200}{3}\Big(66\frac{2}{3}$$

Example 2. How many Rods of Land, 4 Rods broad, shall I have for a Piece of Land that is 3 Rods broad and 12 Rods long? Q. *Of a Piece of Land for another.*

Answ. 9 Rods in length. Answer.

(3×12)
As 36 . 1 :: 4 . 9 Rods.

Example 3. A Gentleman bespeaks a Piece of Wainscot of 36 Feet 3 Inches long, and 8 Feet 4 Inches broad; and agreeth to pay for it by the Yard square, at 10*s.* every Square: what doth this Piece of Work come to? Q. *Of the Price of a Piece of Wainscot.*

Answ. 16*l.* : 15*s.* : $7\frac{7}{9}$*d.*: For the Length multiplied into the Breadth, is $302\frac{1}{12}$ square Feet: And 9 square Feet making 1 square Yard, the Analogy is; As 9 to 10*s.* so is $302\frac{1}{12}$, to 16*l.* : 15*s.* : $7\frac{7}{9}$*d.*: Or reducing the square Feet into square Yards, it is, As 1 . 10*s.* :: $33\frac{61}{108}$. 16*l.* : 15*s.* : $7\frac{7}{9}$*d.* Answer.

Length.

Length. *Breadth.*
$36\frac{1}{4} \times 8\frac{1}{3}$
Or,
$\frac{145}{4} \times \frac{25}{3}$ $\Big\} = \frac{3625}{12}$

(6
38(1
Or $\frac{3625}{12 \times 9}$ (*Square Yards.* $33\frac{61}{108}$
108

As 9 . 10 s. :: $\frac{3625}{12}$. 16 l. : 15 s. : $7\frac{7}{9}$ d.

(7
$\frac{9}{1}$) $\frac{36250}{12}$ ($\frac{3625(0}{108}$ (33| 5 s.
l. $16:15\frac{3}{5}\frac{5}{4}$

As 1 . 10 s. :: $33\frac{61}{108}$. 16 l. : $15\frac{3}{5}\frac{5}{4}$ s.

Q. Of a Piece of Heben Wood, the Price.

Example 4. If a Foot Square of *Heben* Wood be worth 7 *s.* 3 *d.* what will that Piece be worth that is $9\frac{1}{2}$ Feet long, and $4\frac{1}{3}$ Feet broad, but of equal Thickneſs with the other?

Anſwer.

Anſw. 14 *l.* 18 *s.* $5\frac{1}{2}$ *d.* For $9\frac{1}{2}$ the Length, multiplied into $4\frac{1}{3}$ the Breadth, produce $41\frac{1}{6}$ ſquare Feet; the Reſt is reſolved by the *Rule of Three Direct.*

Length. *Breadth.*
$9\frac{1}{2} \times 4\frac{1}{3}$
Or,
$\frac{19}{2} \times \frac{13}{3}$ $\Big\} = \frac{247}{6}$

Sq.Foot. *s.* *Sq.Feet.* *l.* *s.* *d.*
As 1 . $7\frac{1}{4}$ :: $\frac{247}{6}$. $14 : 18 : 5\frac{1}{2}$
2(1
230(1
$\frac{7163}{24}$ ($29{,}8\frac{1}{2}\frac{1}{4}$ s. $\times 12 = 13(2$ (1 $\frac{}{24}$ ($5\frac{1}{2}$ d.
l. $14:18:5\frac{1}{2}$

Concerning the Forms of Quadrangles.

About the Alteration of the Forms of *Quadrangles*, beſides what may be underſtood in ſome of the Premiſſes, more may be ſeen in ſome Military Propoſitions, wherein ſome of the other Propoſitions are alſo mixed.

Military Propoſitions.

Military Propoſitions principally concern the Forming of Battels.

Battels, how conſidered.

Battels are conſidered in reſpect to the Number of Men, or Form of the Ground.

Square Battel of Men.

Square Battel of Men, is that which hath an equal Number of Men in Rank and File, or (as ſome phraſe it) *Front* and *Flank*.

Square Battel of Ground.

Square Battel of Ground, is that which hath the Rank as long as the File, though the Men in Rank be more than in File.

Of the firſt four Sorts.

In reſpect of Men, there is a fourfold Variety, *viz.* a Square Battel, a Double Battel, a Battel of the Grand Front, which is called a *Quadruple Battel*, and a Battel of any Proportion of the Number in Rank to the Number in File.

1. Square Battel of Men.

1. For a Square Battel of Men, extract the Square Root out of the whole Number of Men, the ſame ſhall be the Number of Souldiers to be ſet in Rank, and likewiſe in File.

Q. Of two Brigades, how many the Front, &c.

Example 1. A Captain-General hath under his Command two Brigades, one of *Poles* conſiſting of 7225, and the other of *Switzers* conſiſting of 6084 Men; and upon occaſion of Service would form each of them into Square *Battalia*; and again, all of them into one Square Battel: how many Men ſhall be in the Front, and accordingly in the Flank of every of them?

Anſwer.

Anſw. Of the *Poles* 85, of the *Switzers* 78. And in Square Battel of the Total 115; and there will be 84 Men ſpare.

8	11		12'84
7225 (85	6084 (78	7225	13309 (115
64	49	6084	1
80	112	13309	21
25	64		1125

Q. Of an Army marching, what in Rank and File.

Example 2. An Army marching in a narrow Paſſage, were 20 in Rank, and 1620 in File; and when they come to pitch the Field, they would embody into a ſquare Form: how many muſt there be in Rank and File?

Anſw.

Answ. 180: For multiplying 1620 by 20, the whole Number of the Men is found to be 32400; which is a Square Number, and hath 180 for the Root. Answer.

```
                 26
  1620          32400( 180
    20          1
  -----         16
  32400          64
```

2. For a double Battel of Men, extract the Square Root of half the Number of Men, and the same doubled shall be the Number of Souldiers to be set in a Rank. 2. *Double Battel.*

Example. 1458 shall give 54, that is double of 27, the Root of 729, the half of 1458. *Example.*

3. For a Quadruple Battel, extract the Square Root of a Quarter of the Number of Men, and the same quadrupled, shall be the Number of Souldiers to be set in a Rank. 3. *Quadruple Battel.*

Example. 1024 shall give 64, that is Quadruple of 16, the Root of 256, the Quarter of 1024. *Example.*

4. For a Battel required of any other Form, that is, if a *Ratio* be given according to which the Number of Men in Rank shall be to the Number of Men in File; Multiply the 2 Terms of the *Ratio* given: Then as the Product is to the Square of the Term which is for the Rank; or as the Term which is for the File, is to the Term which is for the Rank; so is the whole Number of Souldiers, to the Square of the Number of Men to be placed in a Rank. 4. *Battel of any other Form.* Analogies.

For in *Species* FR . Rq :: F . R.

Example. 1944 Souldiers are to be marshalled, so that the Number of the Rank be to the Number of the File, as 8 to 3; that is, for 8 in Rank, 3 in File: what shall then be the Number of Men in the Rank? *Example.*

Answ. 72: For the Terms of 8 multiplied into 3 is 24, and the Square of 8 is 64: Therefore as 3 to 8, or as 24 to 64; so is 1944 to 5184, which is the Square of 72. Answer.

```
      File.  Rank.  Number.   Q. Rank.
  As   3   .  8   ::  1944   .  5184
                         8
                     3)15552( 5184( 72 Rank.
                               49
                               284
```

And for Proof thereof, If there be 72 in Rank, which is 9 times 8, then there must be 27 in File, which is 9 times 3: And 72 multiplied into 27, produce 1944 the Number given. *Proof thereof.*

In respect of the Form of Ground, the Battel is either a Square form of Ground, or longer one way than the other. For the Distance or Order of Souldiers marshalled in Array, may be distinguished either into *Open-Order*, or *Order*. *Battels of the second Sort.*

Open-Order, (as *Barriffe* in his *Military Discipline* tells us) is when the very Centers of their Places are distant 6 Feet asunder, both in Rank and File. Open-Order, *what.*

Order, is when the Centers of their Places are distant 3 Feet, but some will have it 3 Feet in Rank and 6 Feet in File, which last *Order* and whatsoever *Order* else there is, in which the Distance of the Ranks one from another is greater than the Distance of the Files, causeth that a Square of Men maketh not a Square of Ground, but the Ground is longer on the File than on the Rank. So, Order, *what.*

Distance, in Rank . R. in File . F.   Number, in Rank . R. in File . F.

1. If then it be a Square Battel of Ground, the Centers of the Distances being 3 Feet in Rank and 6 in File, because 3 is the Half of 6, the *Ratio* of the Distances is as 1 to 2. and seeing the Number in Rank, to the Number in File is reciprocal to the Distances, the *Ratio* of the Number of Men in Rank, to the Number of Men in File, shall be as 2 to 1. and the Rule may be as in the 4*th* sort of Battels abovementioned: As the Term of the File to the Term of the Ranks; *Square Battel of Ground.* Analogies.

So is the whole Number of Souldiers, to the Square of the true Number of the File.

RF . Fq :: R . F :: F . R.

Example. *Example.* 1352 Souldiers are to be ſet in a Square of Ground, that their Diſtances may be 3½ Feet in a Rank, and 7 Feet in a File: what ſhall the File be?

Anſwer. *Anſw.* The *Ratio* of the Rank to the File ſhall be reciprocally, as 7 to 3½, that is, as 2 to 1. Therefore,

```
F  .  R  ::   N  .  Rq.
1  .  2  ::  1352 . 2704
               2
           1)2704(52 File.
```

Proof thereof. So the File being 52, the Rank muſt be *half ſo much, which is* 26, and $52 \times 26 = 1352$, a Proof of the Truth thereof.

Battel when the Ground is a long Square. 2. If a Battel, wherein the Diſtance in Rank is unequal to that in File, that is, be longer one way than the other, according to any *Ratio* given, there is to be conſidered a double *Ratio*, one reciprocal in reſpect of the Diſtances, the other according to the Form of the Ground: wherefore to find the *Ratio* of Men in Rank to the Men in File, multiply the 2 Terms of the Rank for the Rank, and the 2 Terms of the File for the File; and then the Rule ſhall be as in the *4th* ſort of Battels above. Analogies.

Example. *Example.* 10290 Souldiers are to be ſet in *Battalia*, ſo that they may ſtand only 3 Feet aſunder in Rank, and 7 Feet in File, and the Length of the Ground for the Rank to the Length of the Ground for the File, ſhall have the *Ratio* of 5 to 2: what number of Men ſhall be in the Front?

Anſwer. *Anſw.* 245. For the *Ratio* of the Rank to the File, is as 7 to 3, in reſpect of the Diſtances, but in reſpect of the Ground, the *Ratio* of the Rank to the File, is as 5 to 2. So the like Terms multiplied, make 35 and 6. Then as 6 . 35 :: 10290. 60025. whoſe Square Root is 245 the Number of Men to be ſet in Rank, briefly expreſſed in Species thus;

Ff . Rr :: N . Rq   or, Rr. Ff :: N . Fq.

Proof thereof. And for Proof of the Truth thereof, ſeeing for every time 35 in the Rank (that is 7×5) there muſt be 6 in the File (that is 3×2) there being then 7 times 35 in 245, there muſt be 7 times 6, that is 42, in the File, and $42 \times 245 = 10290$ the whole Number of Souldiers.

How to enlarge or diminiſh a Plot of Ground. Hence ariſe Propoſitions, touching enlarging of a Plot of Ground for a Camp, Building, &c. or the contrary.

Example 1. *Example* 1. If 1000 Souldiers may be lodged in a Square of 300 Feet; how many muſt be the ſide of a Square which will ſerve to lodg 5000?

Anſwer. *Anſw.* Almoſt 671, for the Analogy is thus.

```
     N.S       Q:G       N.S    Q:G
  As 1000 . 300 x 300 :: 5000 . 450000

              90000     (1
               5000     96(1
  1000)450000,000(450 0(00(670 +
                  36
                   889
```

Example 2. *Example* 2. An Architect projecting a Building, at firſt layeth out a Plot of Ground of 58 Feet ſquare every way; but afterwards enlarging his Intentions, findeth it neceſſary to take in double the Ground: what muſt then the Side of that Square be that is double to the former?

Anſwer. *Anſw.* 82 Feet and a ſmall matter over: For ſeeing the Ground is to be doubled, the Square of 58 is to be doubled, which is 6728, and the greateſt ſquare Root therein 82, as before.

$$\text{Root } 58 \times 58 = \overset{\textit{Square.}}{3364} \times 2 = \overset{\textit{Doubled.}}{6728} \left(82 \text{ Side.} \right.$$

$$\begin{array}{l} 4 \\ 64 \\ 324 \end{array}$$

But if the Propoſition had been to double the Side of the Ground, then the Square muſt have been the Square of 116 the Double of 58, which is four times as much as the former.

$$\text{Root } 58 \times 2 = 116 \times 116 = \overset{\textit{Square.}}{13456} \textit{ Area}. \quad \frac{13456}{3364} \left(4 \right.$$

Proof of doubled Proportions.

In all theſe Operations, ſo far as concerns the Extraction of Roots, or *Rule of Three*, have the Proof of their Work already ſpoken to in other Chapters; the reſt proper to this Chapter, hath the Proof in the Anſwers to the Propoſitions particularly illuſtrated.

CHAP. XVI. *Proportions tripled, &c.*

Tripled Proportions.

AFTER the Proportions about Plain Figures, come thoſe uſed about Bodies, or Solids.

Proportions of Bodies large.

Proportions about Bodies taking in the *Ratio* of the ſeveral Parts, are of themſelves enough to fill a large Treatiſe, eſpecially if the word Body be largely taken.

Of Man's Body. *Meaſures taken from thence.* *Proportions of Noah's Ark.*

Albertus Durens hath wrote a whole Book of the Meaſures of Man's Body, and *Pythagoras* calls Man *the Meaſure of all Things*, becauſe he is the moſt Perfect among all ſublunary Bodies; and according to the Maxim of Philoſophers, that which is moſt Perfect, and the firſt in Rank, meaſureth all the reſt. And we know that the Meaſures of an Inch, Foot, Cubit, Pace, *&c.* have taken their Names and Quantities from Humane Bodies: Yea, ſuch is the admirable Symmetry and Concordancy in the Parts of Man's Body, that ſome well-proportioned Works have been faſhioned and compoſed after his Proportion. As *Noah*'s Ark was in Length 300 Cubits, in Breadth 50, and in Height 30; ſo that the Length doth contain the Breadth 6 times, and ten times the Depth: and ſuch Proportion will be found in the Body of a Man being meaſured.

Of Man's Body.

Obſervations of Man's Body.

1. The Length of a Man well made (which is commonly called his Height) is equal to the Diſtance, from the one end of his longeſt Finger to the other, when the Arms are extended as far as they may.
2. The Breadth of a Man (or the Space which is from one ſide to another) makes the ſixth part of all the Body, taken in Length or Height.
3. The Thickneſs of the Body (taken from the Belly to the Back) is the tenth Part of the whole Body, ſome ſay the ninth.
4. The Length of the Face, is equal to the Length of the Hand, taken from the Wriſt to the extremity of the longeſt Finger.
5. The Height of the Brow, the length of the Noſe, the ſpace between the Noſe and the Chin, the Length of the Ears, and the greatneſs of the Thumb, are perfectly equal the one to the other.
6. If a Man have his Feet and Hands extended or ſtretched forth in Form of St. *Andrew*'s Croſs, placing one Foot of a Pair of Compaſſes upon his Navel, there may be deſcribed a Circle which will paſs by the ends of his Fingers and Toes; and drawing Lines by the Terms of them, there may be inſcribed a Square within a Circle.

What Bodies are here intended.

But theſe and ſuch like are ſet aſide, the intent of this Chapter being to call over thoſe Proportions, converſant about inanimate Bodies or Geometrical Figures that have Solidity; and among them, ſuch as are moſt uſeful in meaſuring of Timber, Stone and other Solids, Queſtions of Gunnery, Pyrotechnic, Gaug-

ing

ing of Vessels, and other Mathematical Propositions; most of which depend upon the Knowledg of the Cube and Globe, or Sphere.

Of Globes and Cubes. Globes have their capacities one to another, as the Cubes of their Diameters; for by *Euclid* 18 *Prop.* 12 Book, they bear triple that Proportion their Diameters do. And Cubes by *Euclid*, 19 *Prop.* 8 Book, bear triple Proportion in comparison of their Sides. *Triplicate Ratio's, what.* Hence like *Solids*, are said to be in a Triplicate *Ratio*, when they are as the Cubes of their Homologal Sides; and so not unfitly may this Chapter, concerned in the Proportions of such Bodies, bear the Title of *Tripled Proportions*.

This agreement between Cubes and Globes, occasions the Resolutions to several Propositions concerning them, to agree also in the Method of Resolution.

Propositions relating to these and other Bodies, commonly concern either their Solidity, Gravity, Value, or Form, or a Mixture of one with the other; some more particularly to one Body, and others to another.

Globe or Sphere.

Particularly of the Globe *or* Sphere.

Principal Parts. Because the Product of the *Axis* into the *Periphery* is the *Superficies*, and that again by $\frac{1}{6}$ of the *Axis* is the *Solidity*: Therefore,

If the *Data* be the { *Axis*, *Superficies*, *Axis*, *Solidity*, } and the *Quesita* be the { *Superficies*, *Axis*, *Solidity*, *Axis*,

According to *Archimedes* and *Oughtred* before mentioned,

Analogies therein.

The Analogies *are:*

As 7, to 22, or 1, to 3,1416	So is the Square of the *Axis*, to the *Superficies*.
Contrary, as 22, to 7, or 1, to 0,31831	So is the *Superficies*, to the Square of the *Axis*.
As 21, to 11, or 1, to 0,5236	So is the Cube of the *Axis*, to the *Solidity*.
Contrary, as 11, to 21, or 1, to 1,90986	So is the *Solidity*, to the Cube of the *Axis*.

Propositions.

Q. Of the Superficies and Solidity. A Sphere whose *Axis* is 14; what is the Superficies thereof? and what the *Solidity* by *Archimedes*?

Answer. *Answ.* 616 Superficies, and $1437\frac{1}{3}$ *Solidity*.

As 7 . 22 :: 196 (Q. 14) . 616 *Superficies*.

As 21 . 11 :: 2744 (C. 14) . $1437\frac{1}{3}$ *Solidity*.

Q. Of the Axis. A *Sphere*, whose *Superficies* is 616: what is the *Axis* by *Archimedes*?

Answer. *Answ.* 14 *Axis*.

As 22 . 7 :: 616 . 196 (Q. 14) *Axis*.

Q. Of the Axis. A *Sphere*, whose *Solidity* is $1437\frac{1}{3}$: what is the *Axis* by *Archimedes*?

Answer. *Answ.* 14 *Axis*.

As 11 . 21 :: $1437\frac{1}{3}$. 2744 (C. 14) *Axis*.

Cube.

Particularly of the Cube.

Principal Parts. If the *Data* be the { Side or Root, Solidity } and the *Quesita* be the { Solidity. Side or Root:

For

For the Solidity.
For the first, multiply the side cubically, and you have the Solidity.

For the Root.
For the second, extract the Cube Root of the Solidity, as before in Figural Numbers.

If another Denomination be sought.
But if the Proposition require the Number sought, in another Denomination to that given: then, as in the Chapter last before about Squares and long Squares, so here in Cubes and long Cubes, *&c.* divide for a Greater, and multiply for a lesser Denomination.

Q. Of the Feet in a Piece of Timber.
Example 1. A piece of Timber is 18 Inches Square: how many solid Feet are therein?

Answer.
Answ. $3\frac{3}{8}$ Feet: for seeing Feet are an higher Denomination than Inches, the Cube of 18, is divided by the solid Inches in one Foot, that is, 1728 (or $12 \times 12 \times 12$): And if 18 be turned into $1\frac{1}{2}$ Feet, the Triple *Ratio* will effect as much.

$$\begin{array}{l} \text{Side } 18 \times 18 \times 18 = \dfrac{\overset{(648}{5832}}{1728}\Big(\, 3\tfrac{3}{8} \text{ Feet} \\ \text{Solid Inches in 1 Foot} \end{array} \qquad \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{\overset{(3}{27}}{8}\Big(\, 3\tfrac{3}{8}$$

Q. Of the Inches in a Cube.
Example 2. A Cubical Body is $1\frac{1}{2}$ Foot Square every way: how many solid Inches are therein?

Answer.
Answ. 5832 Inches: For Inches being lesser than Feet, the $3\frac{3}{8}$ Feet, Solid Content, are multiplied by 1728, or $1\frac{1}{2}$ Foot is reduced into Inches, and multiplied cubically.

$$\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{27}{8}\Big(\, 3\tfrac{3}{8} \text{ Feet.}$$

$$\begin{array}{r} 1728 \\ 3\frac{3}{8} \\ \hline 5184 \\ 648 \\ 18 \times 18 \times 18 = \underline{5832} \text{ Inches.} \end{array}$$

Q. Of Inches in the Side of a Rock.
Example 3. A Rock exactly cubical, containeth 1728 solid Feet: how many Inches does one Side contain?

Answer.
Answ. 144 Inches: The Cube Root of 1728 Feet extracted, is 12 Feet; but Inches being of lesser Denomination, 12 is multiplied by 12, the Inches in a Foot.

$$\begin{array}{l} 1728\Big(\, 12 \times 12 = 144 \text{ Inches.} \\ 16 \\ \;\;128 \end{array}$$

Q. Of Feet in the Side of a Mount.
Example 4. A Mount of Cubick Form containeth 2985984 solid Inches: how many Feet doth one Side contain?

Answer.
Answ. 12 Feet: The Cube Root of 2985984 is 144, to be divided by 12, because Feet desired is the higher Denomination.

$$\begin{array}{l} 1241 \\ 2985984\Big(\, \dfrac{144}{12}\Big(\, 12 \text{ Feet.} \\ 1 \\ 1744 \\ \;241984 \end{array}$$

Cubical Bodies.
Those Bodies that are cubical, yet not exact Cubes, order as Cubes.

Q. Of Feet in a Tree.
Example 1. A Tree hewed Square, 2 Feet broad, $1\frac{1}{2}$ Foot thick, and 3 Yards long: how many solid Feet are therein?

Answer.
Answ. 27 Feet: For $2 \times 1\frac{1}{2}$ produces 3 Feet for the *Area* of the Plain; which multiplied into the Length 9 Feet, every Yard being 3, makes the Product 27 as before for the Solidity. And in like manner the Solidity of other Bodies, not regular Cubes, is to be found.

$$\begin{array}{lr} \text{Feet } 2 \times 1\frac{1}{2} = & 3 \\ \text{Yards } 3 \times 3 = & \underline{9} \\ & 27 \text{ Feet solid.} \end{array}$$

Q. Of Feet in a Plank.

Example 2. A Plank 20 Inches broad, $2\frac{1}{2}$ Inches thick, and 3 Yards long: how many solid Feet are therein?

Answer.

Answ. $3\frac{1}{8}$ Feet: For 50 the *Area* of the Plain, made by the Breadth and Thickness, multiplied into the Length, which is 108 Inches, gives the Solidity in Inches 5400; this divided by 1728 the solid Inches in a Foot, gives in the Quotient $3\frac{1}{8}$ as before.

Inches $20 \times 2\frac{1}{2} = 50$
Yards $3 \times 36 = 108$
Solid $\underline{5400}$ Inches.

(216
~~5400~~
~~1728~~ ($3\frac{1}{8}$ Feet.

What Part of such Bodies, &c.

In these and such other Bodies, if it be desired to know what Portion of Length there must be to any Breadth and Thickness given to make a Foot cubical, or such other solid Content: then by the *Area* made of the Breadth and Thickness, divide the Cubick Foot, or other Solid Content; for the Analogy is reciprocal, as in the *Indirect Rule of Three.*

Q. Of Length for a Foot.

Example 1. A Piece of Timber is 8 Inches broad, and 9 Inches thick: how much in Length shall make a Foot thereof?

Answer.

Answ. 24 Inches, or 2 Feet: For the *Area* of 8×9, is 72; which dividing 1728, the solid Inches in 1 Foot, the Quotient is 24: Or by the *Indirect Rule of Three*, As 144 to 12; so 8×9 to 24.

~~28~~
~~1728~~
$8 \times 9 =$ ~~72~~ (24 Inches. Or as $\underbrace{\overset{(12\times12)}{144} \;.\; 12}$ :: $\overset{(8\times9)}{72}$. 24 Inches.

72)1728(24

Q. Of the Length for a Yard of a Wall.

Example 2. A Bricklayer hath undertaken to build a Brick-Wall of 3 Feet thick and 4 Feet high, to be paid for his Work by the Yard cubick; and would know how much Length shall make a cubical Yard of that Wall?

Answer.

Answ. $2\frac{1}{4}$ Feet: For 27 the Yard cubick divided by 12, the Product of the Height and Thickness, gives $2\frac{1}{4}$ in the Quotient.

(3
$3 \times 3 \times 3 =$ ~~27~~
$3 \times 4 =$ ~~12~~ ($2\frac{1}{4}$ Feet: Or as $\underbrace{\overset{(3\times3)}{9} \;.\; 3}$ :: $\overset{(3\times4)}{12}$. $2\frac{1}{4}$ Feet.

12)27($2\frac{1}{4}$

For the Weight, Value & Form.

Jointly, of the Gravity, Value *and* Form.

Analogy between the Weight of Bodies of the same Kind.

The *Gravity* or Weight of Bodies of the same Kind, is according to the Cubes of their Diameters or Axis in Spherical Bodies, and of their Sides in cubical Bodies in direct Proportion.

Q. Of the Weight of a Bullet.

Example 1. Suppose a Bullet 8 Inches Diameter weigh 30 lb: what shall a Bullet of the same Kind weigh that is but 4 Inches Diameter?

Answer.

Answ. $3\frac{3}{4}$ lb: Here the Cube of 8 to 30, is as the Cube of 4 to $3\frac{3}{4}$: Or in less Terms, the Cube of 2 to 30, is as the Cube of 1 to $3\frac{3}{4}$: For 8 to 4, is as 2 to 1.

$\frac{8}{4} = \frac{2}{1}$ As $\overset{(C.8)}{512}$. $\overset{lb}{30}$ :: $\overset{(C.4)}{64}$. $\overset{lb}{3\frac{3}{4}}$ Or, As $\overset{(C.2)}{8}$. $\overset{lb}{30}$ :: $\overset{(C.1)}{1}$. $\overset{lb}{3\frac{3}{4}}$

64
512)1920($3\frac{3}{4}$

(6
~~30~~
8 ($3\frac{3}{4}$

Q. Of the Weight of a Bullet.

Example 2. If a Gun of $5\frac{1}{2}$ Inches Diameter in the Mouth, shoot a Bullet of $20\frac{51}{64}$ lb: what shall that Bullet weigh that serveth for a Gun of 8 Inches Diameter?

Answer.

Answ. 64 lb: Here the Cubes may be reduced into Halfs; that is, 8 into 16, and $5\frac{1}{2}$ into 11: Or else the *Analogy* set as in Fractions;

As

```
   (C.11)  lb      (C.16)  lb
As 1331 . 20 51/64 :: 4096 . 64
                      20 51/64
                     --------
                      81920
                       3264
               1331)85184(64
```

Or, As $\frac{1331}{8}$. $\frac{1331}{64}$ :: 512 . 64 (C.$5\frac{1}{2}$) lb (C.8) lb

And by cancelling the like Numerators 1331, after the manner of Fractions, it ſhall be,

As $\frac{1}{8}$. $\frac{1}{64}$ :: 512 . 64

Example 3. Let a Cube of Metal of 4 Inches ſquare, weigh 8 lb: what ſhall a Cube of the ſame Metal weigh that is 5 Inches ſquare? *Q. Of the Weight of a Cube.*

Anſw. $15\frac{5}{8}$ lb: For the Cube of 4 to 8, is as the Cube of 5 to $15\frac{5}{8}$. Anſwer.

```
   (C.4)   lb   (C.5)   lb
As  64  .  8  ::  125  . 15 5/8
                    8
             64)1000(15 5/8
```

Example 4. When a Cube of Metal, that is $3\frac{1}{2}$ Inches every way, weigheth 64 Ounces *Troy* Weight: what ſhall a Cube of the ſame Metal weigh that is $5\frac{1}{4}$ Inches? *Q. Of the Weight of a Cube.*

Anſw. 216 Ounces *Troy*: For the Cube of 14 the Quarters in $3\frac{1}{2}$, is to 64, as the Cube of 21 the Quarters in $5\frac{1}{4}$ is to 216. Anſwer.

```
   (C.14)  ℥    (C.21)  ℥
As 2744 . 64 :: 9261 . 216      164
                 64             4396
              ------          592704(216
              37044            2744
              55566             2744
              ------             2744
              592704
```

Contrariwiſe; If the Gravity of one Body be given, to find the Side of another Cubical, or the Axis of a Spherical Body: Then as the Roots of the given Numbers one to the other; ſo ſhall the Side or Axis given be, to that required: Or as the given Numbers one to another; ſo ſhall the Cube of the Side or Axis given be, to that required. *Analogy to find the Side or Axis.*

Example 1. If a Cube of 64 Ounces *Troy*, have the Side $3\frac{1}{2}$ Inches: what ſhall the Side of that Cube be that weigheth 216 Ounces *Troy*? *Q. Of the Side of a Cube.*

Anſw. $5\frac{1}{4}$ Inches: The Cube Roots of 64 and 216, which are 4 and 6, ſet in leſs Terms are 2 and 3: Therefore as 2 to $3\frac{1}{2}$; ſo is 3 to $5\frac{1}{4}$. Anſwer.

```
Cubes       .  64 . 216          Inches.    Inches.
Roots       .   4 .   6     As 2 . 3 1/2 :: 3 . 5 1/4
Leaſt Terms .   2 .   3         2/1 ) 21/2 ( 21/4
```

Example 2. In caſe a Bullet of 4 Inches in the Axis weigh $3\frac{3}{4}$ lb *Averdupois*: what ſhall be the Axis of a Bullet of the ſame Kind that weigheth 30 lb? *Q. Of the Axis of a Bullet.*

Anſw. 8 Inches: Where the firſt and third Numbers not being cubical, 4 is cubed; and after *Multiplication* and *Diviſion*, the Root of the Quotient is extracted. Anſwer.

```
           (C.4)
As 3 3/4 .  64  ::  30  .  8 Inches.
            30
    15/4 ) 1920/1 ( 7680/15 ( 512 ( 8
```

Among like Solids in a Triplicate *Ratio*, that is as the Cubes of their Homologal Sides: If 3 Numbers be given, in which as the Cube of the Firſt is to the Cube of the Second; ſo the Third to a Number ſought: Then as the firſt Cube to the third Number; ſo is the ſecond Cube to the Number ſought. *Triplicate Ratio's.*

Example

Q. Of Powder to charge a Gun.

Example 1. If $\frac{43}{100}$ ℔ of Gunpowder suffice to charge a Gun, whose Concave Diameter is $1\frac{1}{2}$ Inch: how many Pounds of the same Powder will suffice to charge a Gun whose Concave Diameter is 7 Inches?

Answer.

Answ. $43,7\frac{1}{135}$ ℔: For as the Cube of $1\frac{1}{2}$, or 1,5 to the Cube of 7; so is 0,43 to 43,7 and $\frac{1}{135}$ of a Prime.

$$\begin{array}{lllllll} & (C.1,5) & & & (C.7) & & \\ \text{As} & 3,375 & \cdot\ 0,43 & :: & 343 & \cdot\ 43,7\frac{1}{135} & \text{Gunpowder.} \end{array}$$

```
            343              (2
           ----           22 365(5   ℔
            129          147,490 0( 43,7 1/135
           172             3,375
           129              3,375
         ------              3,375
          147,49
```

Q. Of Powder to charge a Gun.

Example 2. Suppose 43,7 ℔ of Gunpowder are sufficient to charge a Gun, whose Diameter in the Concave is 7 Inches: And there is another Sort of Gunpowder more strong, that is to the former as 5 to 2: how much of this will suffice to charge a Gun of 4 Inches Diameter?

Answer.

Answ. 3,26 ℔ and somewhat more: As appears, first, by the Proportions of the Powder, seeking how much of that strong Powder is enough to charge a Gun of 7 Inches Diameter; and then, by a second Work, with the Proportions of the Diameters.

```
         ℔              ℔            (C.7)   ℔      (C.4.)  ℔
As  5 . 43,7  ::  2 .  17,48      As  343 . 17,48 :: 64 . 3,26 54/343
         2                                  64
       ----     32     ℔                 -------        21(5
       87,4   87,40(  17,48               69 92        89 1(4   ℔
               5                         1048 8      1118,72( 3,26 54/343
                                        --------       343
                                         1118,72
```

Proportions of Magnitudes and Gravities of Bodies, a curious Enquiry.

The Proportions as well of the Magnitudes, as of the Gravities of Bodies one to another, is a nice Inquisition; and among Authors some difference is found, according as the Experiments of the one have been more or less curious than the other.

What by Van Etten.

Henry van Etten, in his *Mathematical Recreations*, acquaints us, That a Quantity of Water to an equal Quantity of Metal, is in Proportion as,

Water.	*Tin.*	*Iron.*	*Copper.*	*Silver.*	*Lead.*	*Gold.*
10 .	75 .	81 .	91 .	104 .	$116\frac{1}{2}$.	$187\frac{1}{2}$.

What by Alsted.

Herewith agreeth the Learned *Alsted*, save that *Iron* he makes to be but $80\frac{1}{2}$, and adds, Oil 9, Honey 15, and Quicksilver 150.

So as a Magnitude of Gold, weighing $187\frac{1}{2}$ ℔. shall have an equal magnitude of Silver weigh but 104 ℔. and the like of the rest. But for that there is not only difference in the Weight of Fresh Water and Salt, but between Fresh Waters themselves, some being more Mineral than others, according to the Tincture they receive by the Mines, along which they glide in the Bowels of the Earth; therefore some accompt this but an uncertain Basis, and as unstable as the Water it self.

What by Mr. Oughtred, according to Ghetald.

Mr. *Oughtred* (often already mentioned) in his *Circles of Proportion*, pag. 67, 68, &c. tells us, That pieces of Metal, if they be of equal Magnitude, have their Weights in direct Proportion; but if of equal Weight, they have their Magnitudes in Proportion reciprocal. And inserts the Proportions of their Weights, according to the Experiments of *Marinus Ghetaldus*, in his Tractate, called *Archimedes Promotus*, thus:

☉ Gold

☉	Gold	3990	Wherefore	☉ . ☿ :: 7 . 5 :: 3990 . 2850.
☿	Quickfilver	2850		☉ . ♄ :: 38 . 23 :: 3990 . 2415.
♄	Lead	2415		☉ . ☽ :: 57 . 31 :: 3990 . 2170.
☽	Silver	2170		☉ . ♀ :: 19 . 9 :: 3990 . 1890.
♀	Brafs	1890		☉ . ♂ :: 19 . 8 :: 3990 . 1680.
♂	Iron	1680		☉ . ♃ :: 95 . 37 :: 3990 . 1554.
♃	Tin	1554		

Therefore if 4 Pieces of Metals, whereof the Third is of the fame kind with the firft, and the fourth of the fame kind with the fecond, are Proportional; their Gravities alfo are Proportional.

And again, if there be 4 Pieces of Metals, whereof the Third is of the fame kind with the Firft, and the Fourth of the fame kind with the Second, and the Firft and Second of equal Greatnefs, and the Third and Fourth of equal Weight; the Gravities of the Firft and Second fhall be reciprocal to the Magnitudes of the Third and Fourth.

Weight of a Cylinder of Tin of an Inch long, &c.

The aforefaid *Ghetald* (ufing the antient Roman Foot, which by his Accompt feems very little lefs than ours) found a Cylinder of Tin of 1 Inch thick and long, to weigh 1824 Grains, whereof $\frac{2}{3}$ is 1216 for the Weight of a Sphere of that Thicknefs; becaufe every Sphere is $\frac{2}{3}$ of a Cylinder that hath the Height and Diameter of the Bafe, the fame with the *Axis* of the *Sphere*.

Weight of a Sphere.

(C. 10) (C. 12)
Then as 1000 . 1216 :: 1728 . 2101,248. the Weight of a Sphere whofe Diameter is $\frac{1}{10}$ of a Foot; or fet according to the manner of other Proportions, the Queftion will ftand thus: If a Sphere whofe Diameter is $\frac{1}{12}$ of a Foot weigh 1216 Grains, what fhall that Sphere weigh, whofe Diameter is $\frac{1}{10}$ of a Foot? which in reafon becaufe biggeft muft weigh moft; and fo the Cube of 10 being lefs than the Cube of 12, will be Divifor as before.

Weight of a Cube.

And by the fame Accompt, a cubed Inch of Tin weigheth 2322,3887324; and a cubed tenth Part of a Foot 4013,0877296.

How thereby to find the Weight of a Sphere of another Axis,

Accordingly to find the Weight of a Sphere of Tin of any other *Axis*, multiply the Cube of the *Axis* given, by 1216, if it be in Inch-meafure, or by 2101,248, if it be by Decimal Parts of a Foot; and the Product will be the Weight of that Sphere.

And the Weight of a Sphere of another Metal.

But to find the Weight of a Sphere of any other Metal at any Diameter, affigned either in Inch-meafure, or Decimal Parts of a Foot: feek the Weight of a Sphere of Tin at that Diameter given; and then as the proportional Number of Tin is to the Weight of the Sphere of Tin; fo is the Proportional Number of the other Metal, to the Weight of the Sphere propofed.

Example in Iron for the Weight.

Example. Suppofe a Sphere of Iron have the Diameter 3 Inches: what fhall the Weight be?

Anfw. 35494,054 Grains, and fome fmall Overplus.

(C. 1) (C. 3)
As 1 . 1216 :: 27 . 32832. Grains in the Sphere of Tin.
♃ ♂
As 1554 . 32832 :: 1680 . 35494,054. Grains in the Sphere of Iron.

To find the Diameter or Axis of a Sphere of other Metal.

And to find the Diameter of a Sphere of any Metal in Inch-meafure, or Decimal Parts of a Foot by the Weight, feek the Cube of the Diameter of a Sphere of Tin of that Weight: and then as the Proportional Number of the other Metal, is to the Cube of the Diameter found; fo is the Proportional Number of Tin, to the Cube of the Diameter of the Sphere propofed.

Example in Iron for the Axis.

Example. A Sphere of Iron weigheth 35494,054 Grains: what is the Diameter thereof?

Anfw. 3 Inches.

Cube Cube
As 1216 . 1 :: 35494,054 . 29,189189144, &c.
♂ ♃
As 1680 . 29,18918914 :: 1554 . 27 Root 3.

Of the Value of Bodies.

The Value of all Bodies, is accompted according to their Solidity or Gravity; so as the Solidity or Gravity gotten, the Value is to be had by the *Rule of Three Direct.*

Q. Of the Worth of an Iron Bullet.

Example 1. At 15 *l.* the Tun, what is an Iron Bullet worth of 3 Inches Diameter?

Answer.

Answ. 0,618967. *s.* or 7 Pence Farthing and somewhat above: For by the Work above, the Weight of such a Bullet is found to be 35494,054 Grains; and then if one Tun cost 15 *l.* or 1 Hundred 15 *s.* which is all as one, because 20 Hundred are 1 Tun, the Weight of the Bullet, shall give the Sum aforesaid.

	Grains in 1 C.	*s.*	*Grains in the Bullet.*	*s.*
As	860160 .	15 ::	35494,054 .	0,618967, &c.

Q. Of the Worth of a Piece of Timber.

Example 2. A Piece of Timber is 2 Feet broad, 3 Feet thick, and 6 Feet long: what doth it come to at 20 *s.* a Tun?

Answer.

Answ. 18 *s.* For the Solidity found as before to be 36 solid Feet; and 40 Feet being a Tun, it shall be worth 18 *s.*

$2 \times 3 \times 6 = 36$ As 40 . 20 *s.* :: 36 . 18 *s.*

Of the Form of Bodies, 2 Things.

Touching the Form of Bodies, Propositions of two sorts are usual.

1. To increase or diminish the Body, and yet keep the same Form: Or,
2. To alter the Form, and yet keep the same Solidity or Gravity.

1. To increase or lessen, &c.

To increase a Body Spherical or Cubical, Double or Triple, &c. as is desired, the Solidity; and to diminish the same, do the contrary.

Q. Of the Axis of a Globe, and Side of a Cube, doubled.

Example 1. A Globe whose *Axis* is 14, and a Cube of the same side, are both to be doubled: what shall the *Axis* of the One, and the Side of the other so doubled be?

Answer.

Answ. The desired *Axis* and Side shall be between 17 and 18, because the Solidity of the Globe, whose *Axis* is 14, being according to *Archimedes* $1437\frac{1}{3}$, doubled is $2874\frac{2}{3}$; and this Solidity shall have for the *Axis* the Cube Root of 5488, which is the Double of 2744 the Cube of 14.

Cube.
Root $14 \times 14 \times 14 = 2744$
Doubled 5488

(5
29(7
431(5
5488(17 $\underline{575}$
1
21
147
343

Periphery.
Axis $14 \times 44 = 616$ Superficies.
$\frac{1}{6}$ of the Axis $2\frac{1}{3}$
1232
$205\frac{1}{3}$
$1437\frac{1}{3}$ Solidity.
Doubled $2874\frac{2}{3}$

As 11 . 21 :: $2874\frac{2}{3}$. 5488 (C. 17 $\underline{575}$)
21
2874
5748
14
11) 60368 (5488

Q. Of the Axis of a Globe, and Side of a Cube halfed.

Example 2. A Globe and *Cube*, whose Axis and Side is 14, are both to be made but half so much: what then shall the *Axis* of the one, and the Side of the other be?

Answer.

Answ. 11 and a small matter over: For so half 2744 the Cube of 14, and half $1437\frac{1}{3}$ the solid Sphere of 14, according to *Archimedes*, will both give.

Root

Root 14. Cube 2744
Half 1372

(41
1372(11 41
1331

Axis 14 . Solidity $1437\frac{1}{3}$
Half $718\frac{2}{3}$

As 11 . 21 :: $718\frac{2}{3}$. 1372 (C. 11 41
21
718
1436
14
11)15092(1372.

How to increase or lessen the Burden of Ships.

And thus the Book called, *The Treasure of Travellers*, teacheth in the Building of Ships, how to increase or diminish their Burden, by increasing or diminishing accordingly their Length of the Keel, Breadth at the Main-Beam, and Depth in the Hold: and also to fit their Cables, Ropes, Masts, *&c.* though as to the Tackle and Rigging of Ships, Nautical Experience gives the best Directions.

Example.

So if a Ship of 60 Tuns, have the Length of the Keel 32 Feet within the Post; and another were to be built of 120 Tuns, or of 30 Tuns: Then the Cube of 32, which 32768 doubled for the Greater, and halfed for the Lesser, and the Roots accordingly taken, the Keel of the Greater will be somewhat above 40 Feet, and of the Lesser 25 Feet and better.

Keel 32. Cube 32768, Doubled 65536, Halfed 16384.

1
65536(40 1536
64

8|759
16384(25 759
8
7625

What the doubling or halfing of the Side of Cubical Bodies effects.

In all Cubical Bodies, the doubling of one Side doubles the whole Body, and halfing of one Side accordingly lessens the Body by half: So as in Bodies that are not exact Cubes, if the Increase or Decrease be only desired; then their Solidity is to be increased or decreased accordingly: But if their Form be to be kept, then the best way is to increase or decrease one Side.

Example in the Double.

As if a Piece of Timber 2 Feet broad, three Feet thick, and 6 Feet long, be doubled; then the Solidity 36 doubled shall be 72 equal to a Piece of 4 Feet broad, 3 Feet thick, and 6 Feet long; or 2 Feet broad, 6 Feet thick, and 6 Feet long; or 2 Feet broad, 3 Feet thick, and 12 Feet long.

Breadth.		*Thickness.*		*Length.*		*Solidity.*
2	×	3	×	6	=	36
					Doubled	72

$$\left.\begin{matrix} 4 \times 3 \times 6 \\ 2 \times 6 \times 6 \\ 2 \times 3 \times 12 \end{matrix}\right\} = 72$$

Example in the Half.

Likewise if the same Piece of Timber were to be halfed, Then as the Half of 36 is 18; so shall be the Product of half one of the Sides multiplied into the rest.

Breadth.		*Thickness.*		*Length.*		*Solidity.*
2	×	3	×	6	=	36
					Half	18

$$\left.\begin{matrix} 1 \times 3 \times 6 \\ 2 \times 1\frac{1}{2} \times 6 \\ 2 \times 3 \times 3 \end{matrix}\right\} = 18$$

2. How to alter the Form, and keep the Solidity or Gravity.

To alter the Form, and keep the Solidity or Gravity, is most usual in irregular Bodies, to reduce them to Cubes or Spheres; which when their Solidity or Gravity is had thereby, the Axis of the one, or Side of the other, is to be had as in the *Examples* before.

And not only Irregulars, but others also, as Cones, Cylinders, Pyramids, Prisms, *&c.* may be thus reduced without any Difficulty, when their Solidity or Gravity is found; but in getting thereof they differ one from another, as is well known by the Rudiments of *Geometry*, to which their Measure properly belongs: Yet it may not be unprofitable here to see of some of them, some of their *Analogies*.

Of

Cone.

Of a Right Cone.

Becauſe a Sphere is equal to two *Cones* that have their Height, and the Diameter of their Baſe the ſame with the Axis of the Sphere: The *Analogies*, by *Archimedes*, common to the *Cone*, are:

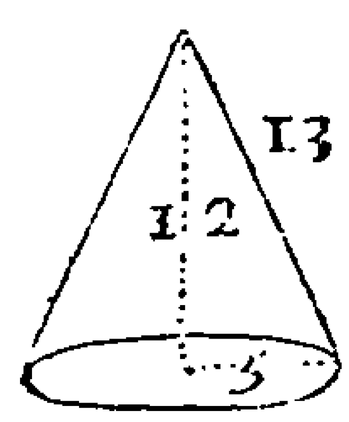
Analogies thereof. As 7 to 22; ſo is half the Diameter multiplied in the Side, to the Conical Superficies.

As 14 to 11; ſo is the Square of the Diameter multiplied in $\frac{1}{3}$ of the Axis, to the Solidity.

To the Conical Superficies, if the *Area* of the Baſe be added, the Total ſhall be the Superficies of the whole *Cone*.

Example. Wherefore if a right *Cone* have the Diameter at the Baſe 10, and the Height or Axis 12; then ſhall the Side be 13, the Conical Superficies $204\frac{2}{7}$, the Total Superficies $282\frac{6}{7}$, and the Solidity $314\frac{2}{7}$.

The Meaſure of a Cone.



$5 \times 5 = 25$
$12 \times 12 = 144$ √
169 (13 Side.

As $7 . 22 :: 10 . 31\frac{3}{7}$ *Periphery.*

$5 \times 15\frac{5}{7} = 78\frac{4}{7}$ *Area* of the Baſe.

(5×13)
As $7.22::65.204\frac{2}{7}$ Conical Superficies.
$78\frac{4}{7}$ *Area* of the Baſe added.
$282\frac{6}{7}$ Total Superficies.

(Q. 10×4)
As $14 . 11 :: 400 . 314\frac{2}{7}$ Solidity.
11
14) 4400 ($314\frac{2}{7}$

Variety. So if the Side and half the Periphery be multiplied, the Product is the Conical Superficies.

And if $\frac{1}{3}$ of the *Area* at the Baſe be multiplied into the Height, or $\frac{1}{3}$ of the Height into the *Area*, the Solidity is had.

$13 \times 15\frac{5}{7} = 204\frac{2}{7}$ Conical Superficies.

$26\frac{4}{21} \times 12$ or $78\frac{4}{7} \times 4$ } $= 314\frac{2}{7}$ Solidity.

Cylinder.

Of a Right Cylinder.

Becauſe $\frac{2}{3}$ of a *Cylinder* is equal to a Sphere, that hath the Height and Diameter of the Baſe the ſame with the Axis of a Sphere: The Common *Analogies* by *Archimedes* in the *Cylinder* are;

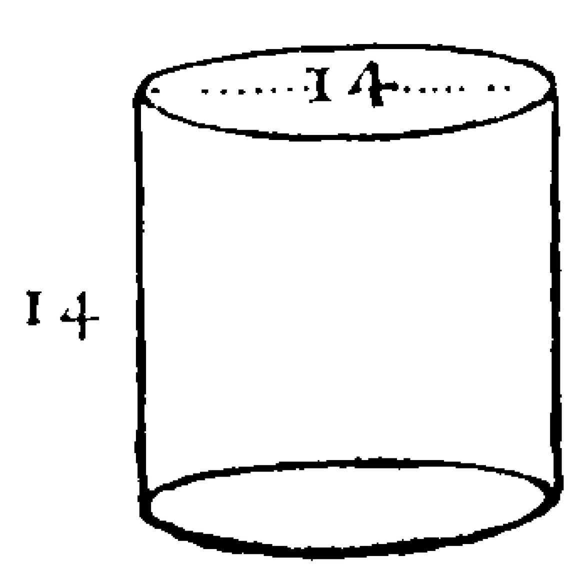
Analogies thereof. As 7, to 22; ſo is the Diameter multiplied in the Axis, to the Superficies-Cylindrical: to which add the *Area* of both the Baſes for the Total Superficies.

As 14, to 11; ſo is the Square of the Diameter multiplied in the Side to the Solidity.

Or as 88, to 7; ſo is the Square of the *Periphery* multiplied in the Side to the Solidity.

Example. Therefore if a Right *Cylinder* have the Diameter 14, and the Height or Axis as much; then ſhall the *Area* of each Baſe be 154, the Cylindrical Superficies 616, the Total Superficies 924, and the Solidity 2156.

The Meaſure of a Cylinder.



D
As $7 . 22 :: 14 . 44$ *Periphery.*
Half $7 \times 22 = 154$ *Area* of the Baſe.
Doubled 308 Baſes.

D L
As $7 . 22 :: 14 \times 14 . 616$ Cylindrical Superficies.
308 Baſes added.
924 Total Superficies.

(Q. 14 D) L
As $14 . 11 :: 196 \times 14.$
(Q. 44 P) L } 2156 Solidity.
Or as $88 . 7 :: 1936 \times 14.$

Variety. So alſo if the *Periphery* be multiplied into the Height or Axis; the Product is the Cylindrical Superficies.

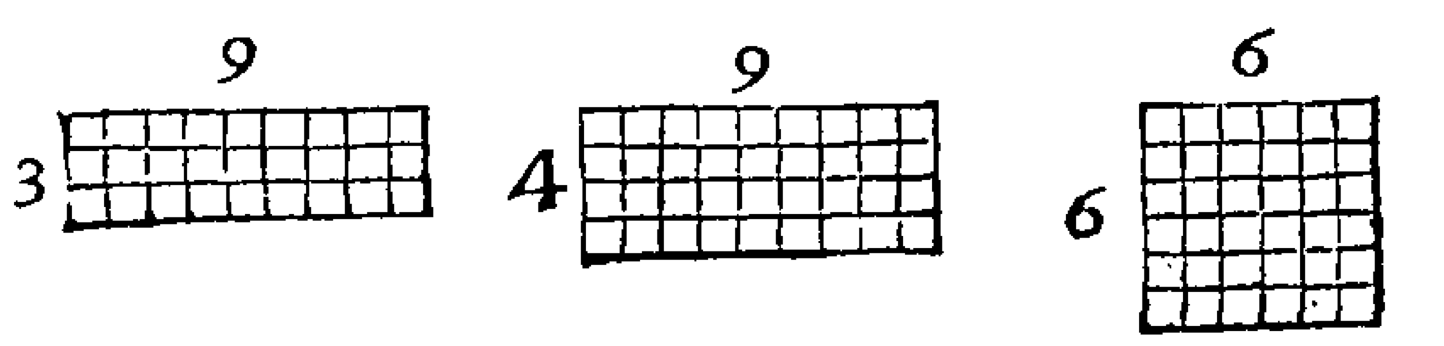
And if the *Area* of one Baſe be multiplied into the Axis, the Solidity is found.

$44 \times 14 = 616$ Cylindrical-Superficies.
$154 \times 14 = 2156$ Solidity.

Hereby

Hereby appeareth, 1. That it is not ſafe to judg the Value of Bodies by their Surface or Perimeter, but by their Solidity or Gravity, as was ſaid before: For Plain Figures, called *Ipſoperimeters*, and alſo Bodies of Equal Surface, may be vaſtly different in their *Area's* and Solid Contents; as thoſe of different Sides and Diameters may have their *Area's* and Solidities equal. *How to value Ipſoperimeters and Bodies of equal Surfaces.*

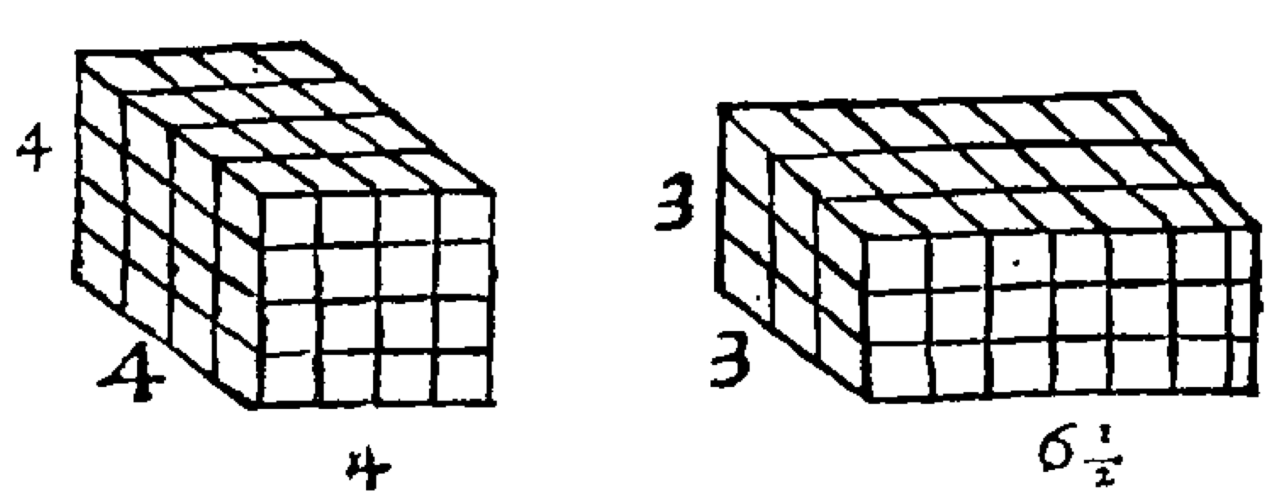
For let there be among plain Rectangle Figures, one whoſe Sides are 3 and 9, another 4 and 9, and a Square 6 and 6; it is evident that the Sides of the Firſt are Ipſoperimetral with the Sides of the Square, that is, both 24: Yet the *Area* of the one is but 27, a Quarter leſs than 36 the Square *Area*. And though the *Perimeter* of the Second be 26, when the *Perimeter* of the Square is but 24; yet the *Area* of the ſecond Figure is 36, equal to the *Area* of the Square. *Examples in Plains.*



$6 + 6 \times 2 = 24$
$6 \times 6 = 36$

Perimeter 24	*Perimeter* 26	*Perimeter* 24
Area 27	*Area* 36	*Area* 36

So in Solids, if one Body whoſe Length is $6\frac{1}{2}$ Feet, Breadth and Thickneſs alike 3 Feet, be examined and compared with the Cube of 4 Feet; their Surfaces will be equal, but their Solidities as different as $58\frac{1}{2}$ from 64. *Examples in Solids.*



Cube.

Superficies $16 \times 6 = 96$.
Solidity $4 \times 4 \times 4 = 64$.

Long Cube.

Solidity $3 \times 3 \times 6\frac{1}{2} = 58\frac{1}{2}$

Superficies $\left\{ \begin{matrix} 3 \times 3 \times 2 \\ 6\frac{1}{2} \times 3 \times 4 \end{matrix} \right\} = 96$.

Again, let a Cube whoſe ſide is 14, be compared with a Sphere of the ſame Diameter, and a Cone and Cylinder of equal Diameter and Height; both the Superficies and Solidities will be thus different.

		Superficies	Solidities
14	Side or Root of the Cube,	1176	2744
	Diameter or Axis of the Sphere,	616	$1437\frac{1}{3}$
	Height and Diameter of the Cone,	$498\frac{6}{31}$	$718\frac{2}{3}$
	Height and Diameter of the Cylinder,	924	2156

Wherefore it is evident, if *Sempronius* borrow of *Caius* a Sack of Corn 4 Feet high, and 2 Feet over, and pay him again with 2 Sacks of Corn, each 4 Feet high and 1 Foot over; yet hath *Caius* been paid but one Quarter of his Corn. *Caius paid too little.*

As $7 . 22 :: 2 . 6\frac{2}{7}$
$1 \times 3\frac{1}{7} = 3\frac{1}{7}$
4
Lent $12\frac{4}{7}$

As $7 . 22 :: 1 . 3\frac{1}{7}$
$\frac{1}{2} \times 1\frac{4}{7} = \frac{11}{14} \times 4 = \frac{44}{14}(3\frac{1}{7}$ Paid.
Paid $3\frac{1}{7}$ due $9\frac{3}{7}$.

2*ly*. It is evident, that if a Body partake of different Forms, the true Meaſure of that Body muſt be mixt accordingly; as in the Gaging of Veſſels will be further clear. *How to meaſure Bodies of mixt Forms.*

A Wine or Beer Veſſel is in form of a *Spheriode*, partly like a Sphere, and partly *Oval* or *Cylindrical*: Wherefore meaſure the two Diameters, *viz.* that at the Head, and that at the Bung, and alſo the Length within their Veſſel, either in Inches or in Decimal Parts of a Foot, and by the Diameters find out the Circles. Then, *Spheroide, what. To gage Veſſels.*

Add together 2 third Parts of the greater Circle, and 1 third Part of the leſſer Circle, and multiply the Total by the Length; and this ſhall produce the Content or Solidity.

To find $\frac{1}{3}$ and $\frac{2}{3}$ of any Circle, the Analogies by the Common Way of *Archimedes*, and alſo by Mr. *Oughtred*, from *Van Ceulen*, are thus: *Analogies.*

For $\frac{2}{3}$ As 21, to 11. Or as 1, to 0,5236 } So is the Square of the Diameter, to the $\frac{2}{3}$ of the Circle.

For $\frac{1}{3}$ As 42, to 11. Or as 1, to 0,2618 } So is the Square of the Diameter, to the $\frac{1}{3}$ of the Circle.

Example.

Example. A Veſſel whoſe Diameter at the Bung is 32 Inches, and at the Head 18 Inches, is in Length 40 Inches: what Content ſhall this Veſſel be of?

Anſw. By the common Proportions $24849\frac{11}{21}$ Solid Inches, and by the other 24839,584 Solid Inches; as by the following Operations of both appears.

Common Way.

Solidity by Archimedes.

As 7 . 22 :: 32 . $100\frac{4}{7}$ *Periphery.*

$16 \times 50\frac{2}{7} = 804\frac{4}{7}$ *Area* at the Bung. $\frac{2}{3}$ is $536\frac{8}{21}$

As 7 . 22 :: 18 . $56\frac{4}{7}$ *Periphery.*

$9 \times 28\frac{2}{7} = 254\frac{4}{7}$ *Area* at the Head. $\frac{1}{3}$ is $84\frac{6}{7}$

Total	$621\frac{5}{21}$
Length	40
Solidity	$24849\frac{11}{21}$

(Q.32.)
As 21 . 11 :: 1024 . $536\frac{8}{21}$ }
Or, (Q.18) } as above.
As 42 . 11 :: 324 . $84\frac{6}{7}$ }

Other Way.

Solidity by Van Ceulen.

As 1 . 3,1416 :: 32 . 100,5312 *Periphery.*

$16 \times 50,2656 = 804,2496$ *Area* at the Bung. $\frac{2}{3}$ is 536,1664

As 1 . 3,1416 :: 18 . 56,5488 *Periphery.*

$9 \times 28,2744 = 254,4696$ *Area* at the Head. $\frac{1}{3}$ is 84,8232

Total	620,9896
Length	40
Solidity	24839,5840

(Q.32)
As 1 . 0,5236 :: 1024 . 536,1664 }
Or, (Q.18) } as above.
As 1 . 0,2618 :: 324 . 84,8232 }

Content how otherwiſe to be found.

The Content of a Veſſel in Cubick Inches is alſo to be had thus; to 2 Squares of the Diameter at the Bung, add the Square of the Diameter at the Head; and dividing the Total into 3 Parts, ſay, As 452 to 355; ſo is $\frac{1}{3}$ of the ſaid Total multiplied by the Length, to the Content in Cubick Inches.

And ſo $18 \times 18 = 324$ And $32 \times 32 = 1024$
Double of 1024 is 2048

$3)2372(790\frac{2}{3} \times 40 = 31626\frac{2}{3}$

And as 452 . 355 :: $31626\frac{2}{3}$. 24839, *&c.* as before.

To turn the Solidity into Concave Meaſure. Example.

Then having the Cubick Inches, or Cubick tenth Parts of a Foot in one Gallon of Wine or Beer, the Content of the Veſſel in Gallons may be found.

For there being in 1 Foot 1000 Cubick 10*th* Parts, that is $10 \times 10 \times 10$; And 1728 Cubick Inches, that is $12 \times 12 \times 12$: And as Mr. *Oughtred* and others write, 231 Cubick Inches in a Wine Gallon, and $272\frac{1}{4}$ in a Beer Gallon; and the *Ratio* between them in leſſer Terms, as 14 to $16\frac{1}{2}$: If Cubick Inches be given, the *Analogy* is,

(C.12) (C.10)
As 1728 . 272,25 :: 1000 . 157,5521 — Cubick 10*th* Parts in a Beer Gallon.
As 1728 . 231 :: 1000 . 133,6805 + Cubick 10*th* Parts in a Wine Gallon.

But if Cubick 10*th* Parts be given, the *Analogy* is contrary.

(C.10) (C.12)
As 1000 . 157,5521 — :: 1728 . 272,25. Cubick Inches in a Beer Gallon.
As 1000 . 133,6805 + :: 1728 . 231. Cubick Inches in a Wine Gallon.

Wherefore if the Solidity of the Veſſel aforeſaid were deſired in Cubick 10*th* Parts: Then, by the

Common Way.

As 1728 . 24849,5238 :: 1000 . 14380,5114,&c.

Other Way.

As 1728 . 24839,584 :: 1000 . 14374,7592,&c.

And if the Content of the ſame Veſſel were deſired in Gallons Wine-meaſure or Beer-meaſure; then accordingly diviſion is to be made by 231 for Wine, if Cubick Inches, or by 133,6805, if Cubick 10*th* Parts be given: And by 272¼ or 272,25 for Beer, if Cubick Inches be given; or by 157,5521, if Cubick 10*th* Parts: So will be found in the ſaid Veſſel by the

Common Way { Beer Gallons 91, 27 / Wine Gallons 107, 57 } and ſomewhat above.

Other Way { Beer Gallons 91, 23 / Wine Gallons 107, 53 } and ſomewhat above.

By one of the Officers of Exciſe, I have heard that their Gage, or Rule by which they meaſure the Brewers Tuns and Veſſels, is computed after the Rate of 282 Cubick Inches to a Gallon of Beer; which if ſo, ſeems to be for allowance of Lees, &c.

3*ly*. Hence alſo is apparent, That by the help of Figural Proportions, not only Bodies of mixed Forms may be meaſured, but alſo one and the ſame Body, whoſe Solidity or Gravity is mixed, may be meaſured, and in higher Proportions than tripled. *How to meaſure a mixt Body in Higher Proportions.*

Example 1. Suppoſe a Merchant have a Piece of Wine of 128 Gallons, and draw out thereof 16 Gallons, and fill it up again with Water: And again draw out 16 Gallons, and fill it again with Water, and do the like the third and fourth Time: How much Wine and Water were then in the Veſſel? Q. *Of Wine mixt with Water four times.*

Anſw. 75$\frac{1}{32}$ Gallons of Wine, and the reſidue Water. Anſwer.

Here after the Numbers 128 and 16 are ſet in their leaſt Terms, that is, 8 and 1, the Leſſer is taken from the Greater, ſo it is 8 and 7; and the like will be if 16 be taken from 128, and the Remain 112 abbreviated with 128, both which are to be Figurate to the 4*th* Quantity, becauſe the Mixture was fourfold, (for always the Figuration muſt be according to the Mixture) and then the Analogy is,

As the greater Figural Number to the whole Quantity: So is the leſſer Figural Number to the Quantity deſired.

	(QQ. 8)			(QQ. 7)		
As	4096	. 128	::	2401 .	75$\frac{1}{32}$	Wine.
				Complement.	52$\frac{31}{32}$	Water.
					128	Gallons.

Example 2. If a Goldſmith have an Ingot of Silver 12 Penny Weights Fine, weighing 8 Marks, and cut off a Mark thereof, and melt with the reſidue a Mark of Copper; and from this mingled Maſs cut off another Mark, and put thereto again a Mark of Copper, and do the like the third Time: The Queſtion is, how much a Mark as it is mix'd will hold? Q. *Of Silver mixt with Copper.*

Anſw. 8$\frac{5}{128}$ Penny Weights fine. Anſwer.

For here the Proportion between the Cubes of 8 and 7, becauſe the Mixture is but triple, guide the Analogy.

Whole	8	Marks.
Cut off	1	Mark.
Difference	7	

(C. 8) (C. 7)
As 512 . 12 :: 343 . 8$\frac{5}{128}$ Fine.

Thus ſuch Mixtures are more eaſily reſolved than by *Alligation*, as in that Chapter was before noted: and hereby alſo the Value of ſuch Mixtures are as eaſily diſcovered. *Theſe Mixtures better reſolved here than by Alligation; alſo their Values*

Example

Q. Of the Worth of mixed Wine and Water.

Example 1. A Merchant had 128 Gallons of Wine in a Cask, worth 5 *s.* the Gallon, and draweth out thereof 16 Gallons, and filleth up the Cask again with Water; and then draweth out 16 Gallons more, and filleth up again the Cask with Water: what will a Gallon of this mixture be worth?

Answer.

Answ. $3\frac{53}{64}$ *s.* for the Analogy is, as the whole Quantity Figurate, according to the Number of Mixtures, is to the highest Price; so is the Difference between the whole Quantity and the first Draught so figurate as the other, to the Price of the Mixture.

Whole. First Draught. Difference.

128 —— 16 $=$ 112 abbreviated $\frac{112}{128} = \frac{7}{8}$ Figurate twice $\frac{49}{64}$.

Therefore as 64 . 5 *s.* :: 49 . $3\frac{53}{64}$ *s.*

Q. Of the Worth of Wine mixt with other Wine.

Example 2. If a Merchant have 128 Gallons of Wine, worth 5 *s.* a Gallon, and draw out thereof 16 Gallons, and fill up the Vessel again with Wine of 4 *s.* the Gallon, and afterwards draw out 16 Gallons, and fill up the Cask again with the Wine of 4 *s.* the Gallon, and so do the third time: what shall a Gallon of this mingled Wine be worth?

Answer.

Answ. $4\frac{343}{512}$ *s.* After 16, the first Draught, is taken from 128 the whole Quantity, and the Remain abbreviated therewith to their least Terms, and each of them figurate according to the Mixture, because the Wine put in was of Value as well as that drawn out, the *Analogy* here may be,

As the greater Figural Number, to 1; so is that greater Figural Number multiplied by the lowest Price given, and added to the lesser Figural Number, to the Price desired.

128 —— 16 $=$ 112 Abbreviated $\frac{112}{128} = \frac{7}{8}$ Figurate thrice $\frac{343}{512}$.
And $512 \times 4 + 343 = 2391$
Therefore 512 . 1 :: 2391 . $4\frac{343}{512}$ Price desired.

Proof of this Work.

And the Truth hereof may appear, if, according to the Work of *Alligation* before, it be examined: for thereby being found upon the third Mixture in the Vessel $85\frac{3}{4}$ of the highest Price, and $42\frac{1}{4}$ of the lowest Price, and each of them multiplied respectively by their several Prices, and the Total of the Products divided by the whole Quantity so mixed, will bring forth in the Quotient the mean Price as before.

Prices.

Gallons $\left\{ \begin{array}{l} 42\frac{1}{4} \times 4 = 169 \\ 85\frac{3}{4} \times 5 = 428\frac{3}{4} \end{array} \right.$

And as 128 . 1 :: $597\frac{3}{4}$. $4\frac{343}{512}$.

$\frac{128}{1}\Big)\frac{2391}{4}\Big(\frac{2391}{512}\Big(4\frac{343}{512}$ *s.*

Proof of tripled Proportions.

The Operations of this Chapter being composed of *Figurals* and *Plain Proportions*, have their Proofs accordingly spoken to in other Places. And by reversing the Questions and Varieties of Work, the Truth of the Conclusions will sufficiently appear without further Assay or Example.

Partis secundæ Libri quarti

FINIS.

The Third PART of the Fourth BOOK.

CHAP. I.

Of PROPORTIONS Continued.

SImple *Disjunct Proportions* have at large, with their *Comparative Elements*, been unravelled in the foregoing Part; therefore this will concern it self with *Continual Proportions*. *Continual Proportions their Comparative Elements.*

Continual Proportions, as before in the first Chapter of this Fourth Book, were observed to be both *Arithmetical* and *Geometrical*, that having the Difference or Excess, and this the *Ratio* between every two Numbers or Terms equal. As, *Arithmetical and Geometrical, how different.*

Examples of both.

Arithmetical	1 .	2 .	3 .	4 .	5 .	6.	&c.	*Difference*	1.
	1 .	3 .	5 .	7 .	9 .	11.	&c.	*Difference*	2.
	2 .	4 .	6 .	8 .	10 .	12.	&c.	*Difference*	2.
Geometrical	1 .	2 .	4 .	8 .	16 .	32.	&c.	*Ratio*	2.
	1 .	3 .	9 .	27 .	81 .	343.	&c.	*Ratio*	3.
	2 .	8 .	32 .	128 .	512 .	2048.	&c.	*Ratio*	4.

Both these are called *Progression*, because the Numbers go forward from the first Term, and have their Progress in an orderly Series, increasing by an equal and continued *Difference* in the one, and *Ratio* in the other. *Both called Progression, and why.*

They both agree also in their placing or setting down, either the first Term to be set to the left Hand, and the rest in order to the Right, as above: Or else the first Term at top, and the other underneath in order, as in *Addition of Integers*. *Wherein they agree. How placed.*

Yet further, the first Term in both is called the *Antecedent*, and all the rest in reference thereto *Consequents*: And moreover the first and last Terms in both are the *Extreams*, and all the other *Means*. *Antecedent and Consequent. Extreams and Means.*

As they herein agree among themselves, so in some things they agree with *Disjunct Proportions*; that is to say, *Arithmetical Continued*, with *Arithmetical Disjunct*; and *Geometrical Continued*, with *Geometrical Disjunct*. *Agreement with Disjunct Proportions.*

1. Numbers in *Arithmetical Proportion Continued*, or *Disjunct*, have the *Aggregate* of the *Extreams*, equal to the *Aggregate* of the *Means*. *1. Total of the Extreams and Means equal.*

Example.

Arith. Proportion Continued.

3 . 6 . 9 . 12 . 15

Aggre-18-gate.

Arith. Proportion Disjunct.

4 . 7 : 5 . 8

Aggre-12-gate.

For in the *Disjunct*, as 4+8, so 7+5 are 12. And in the *Continued*, as 3+15 are 18, so also 6+12; and so likewise 9, the odd Mean added to himself, makes also 18.

2. Numbers in *Geometrical Proportion Continued*, or *Disjunct*, have the *Product* of the *Extreams* equal to the *Product* of the *Means*. *2. Product of the Extreams and Means equal.*

Example

Geom. Proportion Continued.

2 . 4 . 8 . 16 . 32

Pro-64-duct.

Geom. Proportion Disjunct.

5 . 15 :: 6 . 18

Pro-90-duct.

For $\left.\begin{array}{l} 2 \times 32 \\ 4 \times 16 \end{array}\right\} = 64$ (For 2 × 32 and 4 × 16) Also $\left.\begin{array}{l} 5 \times 18 \\ 15 \times 6 \end{array}\right\} = 90$

And so 8 the odd Mean multiplied by himself, produceth 64.

3. Four Proportionals added to or taken from 4 others, the Consequence.

3. If 4 Numbers in *Arithmetical Proportion Continued* or *Disjunct*, be added to, or substracted from 4 other alike Proportional, the Totals and Remains will respectively be like Proportionals; and in the Totals the Excess will also be added, but in the Remains diminished.

Example.

	Arith. Proportion continued.		*Arith. Proportion disjunct.*	
	1 . 4 . 7 . 10	Excess 3	3 . 5 : 7 . 9	Excess 2
	4 . 8 . 12 . 16	Excess 4	2 . 4 : 5 . 7	Excess 2
Totals	5 . 12 . 19 . 26	7	5 . 9 : 12 . 16	4
Remains	3 . 4 . 5 . 6	1	1 . 1 : 2 . 2	0

4. Four Proportionals multiplied or divided by 4 others, the Consequence.

4. If 4 Numbers in *Geometrical Proportion continued*, or *disjunct*, be multiplied or divided by 4 other Numbers respectively proportional, the Products and Quotients shall be accordingly proportional; and the *Ratio* in the Products will likewise be multiplied, but in the Quotients divided.

Example.

	Geom. Proportion continued.		*Geom. Proportion disjunct.*	
	1 . 2 . 4 . 8	Ratio 2	7 . 28 :: 9 . 36	Ratio 4
	3 . 6 . 12 . 24	Ratio 2	8 . 32 :: 9 . 36	Ratio 4
Products	3 . 12 . 48 . 192	4	56 . 896 :: 81 . 1296	16
Remains	3 . 3 . 3 . 3	1	$1\frac{1}{7}$. $1\frac{1}{7}$ :: 1 . 1	1

Computation of Continual Proportions, and their Issues.

Wherein further both Sorts of *Continued Proportions* agree, or disagree with themselves, or *Disjunct Proportions*, may be observed in uncovering their several *Comparative Elements.* Those *Arithmetical* in the next Chapter, and those *Geometrical* in the Fifth following, between which is placed the Computation of what issues from the First, and afterwards the Proceeds of the Second in the Order following, *viz.*

Continued Proportions
- *Progression Arithmetical*, Chap.2.
  - *Transposition*, Chap.3.
  - *Technologie*, Chap.4.
- *Progression Geometrical*, Chap.5.
  - *Transmutation*, Chap.6.
  - *Anatocism*, Chap.7.

CHAP. II. *Progression Arithmetical.*

Arithmetical Progression. New Proportionals how gotten. Common Way of proceeding.

BEcause in every *Arithmetical Progression*, the Antecedent substracted from the Consequent, shews the Excess or Difference; therefore to beget new Proportionals, the Excess is added to the Antecedent successively for the new Consequents. So as the Common Way, and indeed the most of what old Authors have left concerning this Sort of *Progression*, is, to set down orderly all the Increase in their due Places, and then add them together as in common Addition of *Integers.*

Q. Of Sattin sold encreasing the Price of every Yard.

For Example-sake: Suppose a Merchant selleth 15 Yards of Sattin, to be paid for the first Yard 4 *s.* for the second Yard 6 *s.* and so for every Yard an orderly Increase of 2 *s.* And it were demanded, what the 15 Yards of Sattin did amount to?

Answer.

Answ. By setting down orderly all the Terms, and adding them together, into one Total; the Sum is found 270 *s.*; or 13 *l.* : 10 *s.* after the Common Way: Thus,

Terms

Terms or Places.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15

First Term, 4. 6. 8. 10. 12. 14. 16. 18. 20. 22. 24. 26. 28. 30. 32 Last Term.

Excess or Difference 2. Total or Sum 270 *s.* or 13 *l.* 10 *s.*

Or thus;

Yards of Sattin	1	4 *s.*	First or least Term.
	2	6	
	3	8	
	4	10	
	5	12	
	6	14	
	7	16	Excess or Difference 2.
	8	18	
	9	20	
	10	22	
	11	24	
	12	26	
	13	28	
	14	30	
Terms of Places	15	32	Last or greatest Term.
		270	Total or Sum.

In this Example, the first Term, and as it were the Root, is 4, the second is made of the first and one Difference, the third of the first and 2 Differences, and generally every Term is made of the first and the Sum of the Differences, the Number of which is less by 1 than the Number of Terms, and so the thirteenth Term shall be 4 and twelve Differences, that is, $4 + 24$, seeing in this Example the Difference is 2. *How the Terms are made.*

The Number of Differences is noted by $T - 1$.
The Sum of the Differences by $TX - X = \omega - \alpha$.

But for a more orderly Proceeding in the Computation of this first sort of Progression, two things in general are to be noted. *What to be noted.*

1. That there are 5 Principals in every *Arithmetical Progression.*
2. That any 3 of the 5 being given, the other 2 may be found.

The 5 Principals are. *What the five Principals are.*

1. The first or least Term of any *Progression*, which in the Example above is 4, this is noted in Species with the Greek α, being the first small Letter of their Alphabet called *Alpha*. *1. The first Term and his Species.*

2. The last or greatest Term noted in Species with the Greek ω, being the last small Letter of their Alphabet called *Omega*, and in the Example above is 32. *2. The last Term and his Species.*

3. The Number of Terms or Places in the whole *Progression*, this in the Instance above is 15, and in Species noted commonly with the Capital Roman T. *3. The Number of Terms and Species thereof.*

4. The common Difference or Excess (*sometime called Increase*), whose Note in Species is X, another Capital Letter of the Roman Alphabet, in the Example above is 2. *4. The Excess or Difference, and his Species.*

5. The Total or Sum of all the Terms in the *Progression*, which above in the Example is 270; for this the Common Note in Species is Z, the last Capital Roman Letter in their Alphabet. *5. The Total or Sum, and his Species.*

The Invention of any 2 of the 5 by 3 given, constitutes 20 Propositions, set down in Species by Mr. *Oughtred*, *p.* 78, 79, and 80, *of his Clavis*, that is 4 Propositions for every of the 5 Principals: So as every Question may be varied 20 several ways, though the thing sought be one of those 5. *How by any 3 of these the other may be found.*

1st. To find the first Term (or α) of any *Arithmetical Progression.* *To find the first Term.*

1. If the last Term, the Number of Terms, and the Excess be given, that is, $\omega . T . X .$ Then $\omega + X - TX = \alpha$. *1. Data. ω. T. X.*

That is, from the Sum of the second Principal added to the Fourth, substract the Product of the third Principal, multiplied by the Fourth. *Rule.*

As in the former Instance 32. 15. & 2. to find 4. *Example.*

$$\begin{array}{lll} 32\ (\omega) & 15\ (T) & 34 \text{ Total.} \\ \underline{2\ (X)} & \underline{2\ (X)} & \underline{30} \text{ Product.} \\ 34\ (\omega+X) - & 30\ (TX) = & 4\ (\alpha) \end{array}$$

2. Data. ω.T.Z. 2. If the last Term, the Number of Terms, and the Sum be given, that is, ω . T . Z . Then $\frac{2Z}{T} - \omega = \alpha$.

Rule. That is, divide the double of the fifth Principal by the Third, and from the Quotient take the Second.

Example. As in the former Instance 32 . 15 . and 270 . to find 4.

$$\begin{array}{lll} 270\ (Z) & & \\ \underline{2} & & 540\,(36 \\ 540\ (2Z) & (T)\ 15 & \underline{32}\ (\omega) \\ & & 4\ (\alpha) \end{array}$$

3. Data. ω.X.Z. 3. If the last Term, the Excess and the Sum be given, that is, ω . X . Z.

Then $\frac{1}{2}X \pm \sqrt{:\omega q + \omega X + \frac{1}{4}Xq - 2ZX} = \alpha$.

As α shall happen to be $\left\{\begin{array}{l}\text{greater}\\\text{lesser}\end{array}\right\}$ than $\frac{1}{2}$X

Rule. That is, add together the Square of the second Principal, with the Product of the second multiplied into the Fourth, and the fourth Part of the Square of the Fourth; and from the Total, take the Product of the fourth and fifth doubled: to the Square Root of the Remain, add half the Fourth, except the first Term (or Number sought) be less than half the Fourth, and then this Root is to be taken from half the Fourth.

Example. As in the former Instance 32 . 2. and 270 . to find 4.

$$\begin{array}{llll} 32\ (\omega) & & 2\ (X) & 270\ (Z) \\ \underline{32} & & \underline{2} & \underline{2\ (X)} \\ 64 & 32\ (\omega) & 4\ (Xq) & 540\ (ZX) \\ \underline{96} & \underline{2\ (X)} & & \underline{2} \\ 1024\ (\omega q) + & 64\ (\omega X) + & 1\ (\tfrac{1}{4}Xq) = 1089 - & 1080\ (2ZX) = 9\,(\sqrt{3} \\ & & & \qquad\qquad \underline{1\ (\tfrac{1}{2}X)} \\ & & & \qquad\qquad 4\ (\alpha) \end{array}$$

The half of X is here added, because 4 the First Term is greater than 2 the Excess.

4. Data. T.X.Z. 4. If the Number of Terms, the Excess, and the Sum be given, that is, T . X . Z . Then $\frac{2Z}{2T} - \frac{TX}{2} + \frac{X}{2} = \alpha$.

Rule. That is, divide the double of the fifth Principal, by the double of the Third; to the Quotient add half the Fourth, and from the Total, substract half the Product of the Third multiplied into the Fourth.

Example. As in the former Instance 15 . 2 . and 270 . to find 4.

$$\begin{array}{lllll} 270\ (Z) & 15\ (T) & & & 15\ (T) \\ \underline{2} & \underline{2} & 540\,(18 & & \underline{2\ (X)} \\ 540\ (2Z) & 30\ (2T) & 30 & 1\ (\tfrac{1}{2}X) & 30\ (TX) \\ & & & 19 - & 15 = 4\ (\alpha) \end{array}$$

To find the last Term. 2dly. To find the last Term (or ω) of any *Arithmetical Progression.*

1. Data. α.T.X. 1. If the first Term, the Number of Terms, and the Excess be given, that is, α . T . X . Then $\alpha + TX - X = \omega$. or $T - 1 \times X + \alpha = \omega$.

Rule. That is, to the Product of the third Principal multiplied by the Fourth, add the first and substract the Fourth; or multiply the Number of Terms lacking 1, by the fourth Principal, and to the Product add the First.

As in the former Instance 4 . 15 . and 2 . to find 32. Example.

$$\begin{array}{ll} 15\ (T) \\ \underline{\ 2\ (X)} \\ 30\ (TX) \\ \underline{\ 4\ (\alpha)} \\ 34\ (TX+\alpha) \\ \underline{\ 2\ (X)} \\ 32\ (\omega) \end{array} \qquad \begin{array}{l} 15-1=14\ (T-1) \\ \underline{\ 2\ (X)} \\ 28\ (T-1\times X) \\ \underline{\ 4\ (\alpha)} \\ 32\ (\omega) \end{array}$$

2. If the first Term, the Number of Terms, and the Sum be given, that is, $\alpha . T . Z .$ 2. Data. α.T.Z.

$$\text{Then } \frac{2Z - T\alpha}{T} = \omega.$$

That is, double the fifth Principal, and take out thereof the Product of the third multiplied by the First; then divide the Remainder by the Third. Rule.

As in the former Instance 4 . 15 . and 270 . to find 32. Example.

$$\begin{array}{ll} 270\ (Z) & 15\ (T) \\ \underline{\ 2\ } & \underline{\ 4\ (\alpha)} \\ 540\ (2Z) - & 60\ (T\alpha) = 480 \Big(32\ (\omega) \\ & \qquad (T)\ 15 \end{array}$$

3. If the first Term, the Excess, and the Sum be given, that is, $\alpha . X . Z .$ Then $\sqrt{:\alpha q - \alpha X + \frac{1}{4}Xq + 2ZX:} - \frac{1}{2}X = \omega.$ 3. Data. α.X.Z.

That is, Square the first Principal, double the Product of the fifth multiplied by the Fourth, and add them together with the fourth Part of the Square of the Fourth: From the Total, take the Product of the first multiplied by the Fourth, and from the Square Root of the Remain take half the Fourth. Rule.

As in the former Instance 4 . 2 . & 270 . to find 32. Example.

$$\begin{array}{lllll} & & XX & ZX \\ & & & 270\times 2 \\ 4\ (\alpha) & 4\ (\alpha) & 2\times 2 & 540\ (ZX) \\ \underline{\ 4\ } & \underline{\ 2\ (X)} & \underline{\ 4\ (Xq)} & \underline{\ 2\ } \\ 16\ (\alpha q) & -8\ (\alpha X) + & 1\ (\tfrac{1}{4}X) + & 1080\ (2ZX) = 1089 \Big(33 \\ & & & \qquad \underline{\ 1\ (\tfrac{1}{2}X)} \\ & & & \qquad 32\ (\omega) \end{array}$$

4. If the Number of Terms, the Excess, and the Sum be given, that is, T . X . Z . 4. Data. T.X.Z.

$$\text{Then } \frac{2Z}{2T} + \frac{TX}{2} - \frac{X}{2} = \omega.$$

That is, divide the fifth Principal doubled, by the double of the Third; to the Quotient add half the Product of the Third multiplied by the Fourth, and from the Total take half the Fourth. Rule.

As in the former Instance 15 . 2 . and 270 . to find 32. Example.

$$\begin{array}{llll} 270\ (Z) & 15\ (T) & 15\ (T) \\ \underline{\ 2\ } & \underline{\ 2\ } & \underline{\ 2\ (X)} & \frac{540}{30}\Big(18 \\ 540\ (2Z) & 30\ (2T) & 30\ (TX) & \qquad \underline{\ 15\ } \\ & & 15\ (2) & \qquad 33 \\ & & & \qquad \underline{\ 1\ (\tfrac{1}{2}X)} \\ & & & \qquad 32\ (\omega) \end{array}$$

3dly. To find the Number of Terms (or T) of any *Arithmetical Progression.* *To find the Number of Terms.*

1. If the first Term, the last Term, and the Excess be given; that is, $\alpha . \omega . X .$ 1. Data. α.ω.X.

$$\text{Then } \frac{\omega - \alpha}{X} + 1 = T.$$

 That

Rule. That is, substract the first Principal from the Second, divide the Remain by the Fourth, and to the Quotient add an Unit.

Example. As in the former Instance 4 . 32 . & 2 . to find 15 .

$$\overset{\omega}{32} - \overset{\alpha}{4} = \frac{28}{\underset{(X)}{2}}\Big(14 + 1 = 15\,(T)$$

2.Data. α.ω.Z. 2. If the first Term, the last Term, and the Sum be given ; that is, α . ω . Z.

$$\text{Then } \frac{2\ Z}{\omega+\alpha} = T.$$

Rule. That is, divide the Double of the fifth Principal, by the First added to the Second.

Example. As in the former Instance 4 . 32 . & 270 . to find 15.

$$\overset{\alpha}{4} + \overset{\omega}{32} = 36\Big)\overset{2\ Z}{540}\Big(15\ (T)$$

3.Data. α.X.Z. 3. If the first Term, the Excess, and the Sum be given ; that is, α . X . Z.

$$\text{Then } \surd : \frac{\alpha q - \alpha X + \frac{1}{4}Xq + 2ZX :}{Xq} - \alpha + \tfrac{1}{2}X = T.$$

Rule. That is, square the first Principal, double the Product of the fifth multiplied by the Fourth, and add them together with the fourth Part of the Square of the Fourth ; From the Total take the Product of the First multiplied by the Fourth, divide this Remainer by the Square of the Fourth ; and from the Square Root of the Quotient take the First, and add half the Fourth.

Example. As in the former Instance 4 . 2 . & 270 . to find 15 .

$$\begin{array}{cccc} & & X\ X & Z\ \ X \\ 4\ (\alpha) & 4\ (\alpha) & 2\times 2 & 270\times 2 \\ 4 & 2\ (X) & 4\ (Xq) & 540\ (Z\,X) \\ & & & 2 \end{array}$$

$$16\,(\alpha q) - 8\,(\alpha X) + 1\,(\tfrac{1}{4}Xq) + 1080\,(2ZX) = \frac{1089}{\underset{(Xq)}{4}}\Big(\overset{\surd}{\frac{33}{2}}$$

$$\frac{33 - \overset{\alpha}{4} + \overset{\frac{1}{2}X}{1}}{2} = 15\,(T)$$

4.Data. ω.X.Z. 4. If the last Term, the Excess, and the Sum be given ; that is, ω . X . Z.

$$\text{Then } \frac{\omega + \frac{1}{2}X -}{X} + \surd : \frac{\omega q + \omega X + \frac{1}{4}Xq - 2ZX}{Xq} = T.$$

As α shall happen to be $\left\{\begin{array}{l}\text{greater}\\ \text{lesser}\end{array}\right\}$ than ½ X.

Rule. That is, substract the Product of the fourth and fifth Principals doubled, from the Product of the Second and Fourth, added to the Square of the Second and Fourth Part of the Square of the Fourth ; divide the Remainer by the Square of the Fourth, and take the Square Root of the Quotient from the Quotient of the Second added to half the Fourth, and divided by the Fourth ; except the first Term be less than half the Fourth, for then this Root is to be added.

Example. As in the former Instance, 32 . 2 . & 270 . to find 15.

$$\begin{array}{ccccc} 32\ (\omega) & 32\ (\omega) & & X\ X & Z\ \ X \\ 1\ (\frac{1}{2}X) & 32 & & 2\times 2 & 270\times 2 \\ & 64 & 32\ (\omega) & & 540 \\ 33\ (\omega+\frac{1}{2}X) & 96 & 2\ (X) & 4\ (Xq) & 2 \\ 2\ (X) & & & & \end{array}$$

$$\frac{1024\,(\omega q) + 64\,(\omega X) + 1\,(\tfrac{1}{4}Xq) - 1080\,(2ZX)}{4\,(Xq)} = \frac{9}{4}\Big(\overset{\surd}{\frac{3}{2}}$$

$$\frac{33}{2} - \frac{3}{2} = 15\,(T)$$

The

The Root $\frac{1}{2}$ is substracted here, because 4, the First Term, is greater than 2 the Excess.

4*thly*. To find the Excess (or X) of any Arithmetical Progression. *To find the Excess.*

1. If the first Term, the last Term, and the Number of Terms be given; that is, $\alpha . \omega . T$. 1. Data. $\alpha . \omega . T$.

$$\text{Then } \frac{\omega - \alpha}{T - 1} = X.$$

That is, Take the first Principal from the Second, and divide the Remainer by the Third made less by an Unit. Rule.

As in the former Instance, 4 . 32 . & 15 . to find 2. Example.

$$\begin{array}{r|r}
15\ (T) & 32\ (\omega) \\
1 & 4\ (\alpha) \\
\hline
14 & 28\ \big(2\ (X)
\end{array}$$

2. If the first Term, the last Term, and the Sum be given; that is, $\alpha . \omega . Z$. 2. Data. $\alpha . \omega . Z$.

$$\text{Then } \frac{\omega q - \alpha q}{2Z - \omega - \alpha} = X.$$

That is, Substract the Square of the first Principal from the Square of the Second, and divide the Remainer by double the Fifth, lacking the First and Second. Rule.

As in the former Instance, 4 . 32 . & 270 . to find 2. Example.

$$\begin{array}{r}
32\ (\omega) \\
32 \\
\hline
64 \\
96 \\
\hline
1024\ (\omega q) \\
4 \times 4 = 16\ (\alpha q) \\
\hline
1008 \\
504
\end{array}
\quad
\begin{array}{r}
270 \\
2 \\
\hline
540\,(2Z)
\end{array}
-
\begin{array}{r}
32+4 \\
\hline
36\ (\omega+\alpha)
\end{array}
= \frac{1008}{504}\Big(2\ (X)$$

3. If the first Term, the Number of Terms, and the Sum be given; that is, $\alpha . T . Z$. 3. Data. $\alpha . T . Z$.

$$\text{Then } \frac{2Z - 2T\alpha}{Tq - T} = X.$$

That is, Take the Product of double the first Principal multiplied by the Third from the Fifth doubled, and divide the Remain by the Remain of the Third substracted from his Square. Rule.

As in the former Instance, 4 . 15 . & 270 . to find 2. Example.

$$\begin{array}{r}
270\ (Z) \\
2 \\
\hline
540 \\
540 \\
\hline
\end{array}
\qquad
\begin{array}{r}
15(T) \times 4(\alpha) \\
\hline
60 \\
2 \\
\hline
120\ (2T\alpha)
\end{array}
= 420$$

$$T \quad T$$

$$15 \times 15 = 225(Tq) - 15(T) = 210 \qquad \frac{420}{210}\Big(2\ (X)$$

4. If the last Term, the Number of Terms, and the Sum be given, that is, $\omega . T . Z$. 4. Data. $\omega . T . Z$.

$$\text{Then } \frac{2\,T\omega - 2Z}{Tq - T} = X.$$

That is, double the Product of the third Principal multiplied by the Second, and substract therefrom the double of the Fifth: divide the Remain by the Remain of the Third substracted from his Square. Rule.

As in the former Instance 32 . 15 . and 270 . to find 2. Example.

32

$$\begin{array}{l} 32\ (\omega) \\ \underline{15\ (T)} \\ 160 \\ \underline{32} \\ 480 \qquad 270\ (Z) \\ \underline{2} \qquad\quad \underline{2} \\ 960\ (2T\omega) - 540\ (2Z) = 420 \\ \qquad T\ T \\ 15 \times 15 = 225\ (Tq) - 15\ (T) = 210 \end{array} \Big(2\ (X)$$

To find the Sum. 5*thly*. To find the Sum (or Z) of any *Arithmetical Progression*.

1.Data. α.ω.T. 1. If the first Term, the last Term, and the Number of Terms be given, that is, α . ω . T.

$$\text{Then } T\omega + T\alpha = 2Z \quad \text{or} \quad \frac{T\omega + T\alpha}{2} = Z$$

Rule. That is, the Product of the third Principal multiplied by the second, and added to the Product of the Third multiplied by the First, shall be equal to twice the Sum: or if the two Products be halfed, or if half the first and second be multiplied by the Third, or the whole first and second by half the Third, the Sum will be had.

Example. As in the former Instance, 4. 32. and 15. to find 270.

$$\begin{array}{l} 15\ (T) \\ \underline{32\ (\omega)} \\ 30 \qquad 15\ (T) \\ \underline{45} \qquad\ \underline{4\ (\alpha)} \\ 480\ (T\omega) + 60\ (T\alpha) = \frac{540}{2} \Big(270\ (Z) \end{array} \qquad \begin{array}{l} \ \ \omega \quad \alpha \qquad \tfrac{1}{2}T \\ 32 + 4 = 36 \times 7\tfrac{1}{2} = 270\ (Z) \\ \qquad\quad\ \underline{18} \\ \qquad\quad\ \underline{15} \\ \qquad\quad\ 90 \\ \qquad\quad\ \underline{18} \\ \qquad\quad\ 270\ (Z) \end{array}$$

2.Data. α.ω.X. 2. If the first Term, the last Term, and the Excess be given, that is, α . ω . X.

$$\text{Then } \frac{\omega q - \alpha q}{X} + \omega + \alpha = 2Z.$$

Rule. That is, substract the Square of the first Principal from the Square of the Second, divide the Remain by the Fourth; to the Quotient add the First and Second, and take half the Total.

Example. As in the former Instance, 4 . 32 . and 2 . to find 270.

$$\begin{array}{l} 32\ (\omega) \\ \underline{32} \\ 64 \qquad 4\ (\alpha) \\ \underline{96} \qquad\ \underline{4} \\ 1024\ (\omega q) - 16\ (\alpha q) = \frac{1008}{(X)\ \ 2} \Big(504 + \overset{\omega}{32} + \overset{\alpha}{4} = \frac{540}{2} \Big(270\ (Z) \end{array}$$

3.Data. α. T.X. 3. If the first Term, the Number of Terms, and the Excess be given; that is, α . T . X.

$$\text{Then } TX - X + 2\alpha \text{ in } T = 2Z.$$

Rule. That is, multiply the third and fourth Principals; from the Product take the Fourth, to the Remain add double the First, multiply the Total by the Third, and half the Product.

Example. As in the former Instance 4. 15. and 2. to find 270.

$$\begin{array}{l} 15\ (T) \\ \underline{2\ (X)} \\ 30\ (TX) - \overset{X}{2} = 28 + \overset{2\alpha}{8} = 36 \times \overset{T}{15} = \frac{540}{2} \Big(270\ (Z) \end{array}$$

4.Data.ω.T.X. 4. If the last Term, the Number of Terms, and the Excess be given, that is, ω . T . X.

$$\text{Then } 2\omega + X - TX \text{ in } T = 2Z.$$

That

That is, from the doubled Sum of the second Principal added to the Fourth, take the Product of the third multiplied by the Fourth; multiply the Remain by the third and half the Product. *Rule.*

As in the former Instance 32 . 15 . and 2 . to find 270. *Example.*

$$\begin{array}{l} 32\ (\omega) \\ \underline{\ \ 2} \\ 64 \\ \underline{\ \ 2\ (X)} \\ \underline{66}\ (2\omega+X) - \underline{30}\ (TX) = 36 \times 15 = \frac{540}{2} \Big(270\ (Z) \end{array} \qquad \begin{array}{l} 15\ (T) \\ \underline{\ \ 2\ (X)} \\ 30 \end{array}$$

Touching these Principals and Propositions, two things more come under farther observance. *What further to be noted.*

First, That by every three of the 5 Principals given, both the other 2 wanting are to be found. *1. Both the two unknown found by three given.*

As *Data.*	*Quesita.*			*Propositions.*		
α . ω . T	Z & X	1	of the	5 . & 1	of the	4
α . ω . X	T & Z	1		3 . & 2		5
α . ω . Z	T & X	2		3 . & 2		4
α . T . X	ω & Z	1		2 . & 3		5
α . T . Z	ω & X	2		2 . & 3		4
α . X . Z	ω & T	3		2 . & 3		3
ω . T . X	α & Z	1		1 . & 4		5
ω . T . Z	α & X	2		1 . & 4		4
ω . X . Z	α & T	3		1 . & 4		3
T . X . Z	α & ω	4		1 . & 4		2

Secondly, That all Questions duly propounded in *Arithmetical Progression*, give 3 of the 5 Principals, and require sometime one, sometime both the other, and sometime one or other of the middle Terms; sometime the Increase is inverted, and sometime one Question includes another, so as 5 Cases will compleat all needful to this sort of *Progression*. *2. The Data and Quesita in every Question. Five Cases.*

Case 1. When but one of the 5 is sought, the Proposition under which the Resolution falls, is to be used, as by the Examples above is largely to be seen; alteration only to be made for Fractions and Decimals, where occasion requires, according to their Nature and Use. *1. If but one sought, then as before. Alteration for Fractions and Decimals, &c.*

Case 2. When 2 of the Principals are required in the Question, after one is found, the other is to be sought; and because the Propositions get 2 of the Principals, and some of the Propositions are more easy than others, sometimes it happens that the Principal found in the Work of the First, will more easily procure the other sought, and so may be used as if one of the *Data*. *2. If 2 be sought, after one found the easiest to be chosen.*

Example 1. A Grocer selleth 80 lb. of Spice conditionally, to be paid for the first Pound 2 *d.* for the second 5 *d.* and so increasing by 3: The Question is, what was paid for the last Pound, and for the whole 80 lb? *Q. Of Spice sold, what paid for the last Pound, and for the Whole.*

Answ. For the last Pound 239 Pence, and for the whole 9640, or 40 *l.* 3 *s.* 4 *d.* *Answer.*

In this Question are,

The *Data* α . T . X
2 . 80 . 3 *Resolution.*
The *Quesita* ω (2) Principal . 1 of the 2 Propositions.
Z (5) Principal . 3 of the 5 Propositions.

$$\text{Wherefore}\quad \begin{array}{l} 80\ (T) \\ \underline{\ \ 3\ (X)} \\ 240 \\ \underline{\ \ \ 2\ (\alpha)} \\ \underline{242}\ (TX+\alpha) \\ \underline{\ \ \ 3\ (X)} \\ \underline{239}\ (\omega) \end{array} \qquad \text{And}\quad \begin{array}{l} 80\ (T) \\ \underline{\ \ 3\ (X)} \\ 240 \\ \underline{\ \ \ 3\ (X)} \\ 237\ (TX-X) \\ \underline{\ \ \ 4\ (2\alpha)} \\ \underline{241} \times 80 = \frac{19280}{2} \Big(9640 (Z) \end{array}$$

Q. *Of how many Men a Sum was received, and what each paid.*

Example 2. If a Man receive 84 *l.* of certain Men, by an orderly Increase, remembring only the Paiment of the first Man to be 9 *l.* and the last 33 *l.* and would know of how many Men he received the said 84 *l.* and what each Man paid one more than another: what shall the Answer be?

Answer. *Answ.* Of 4 Men, and each Man paid 8 *l.* more than the other.

Here are *Data* . α . ω . Z — 9 . 33 . 84; *Quesita* T (3) Principal . 2, X (4) Principal . 2 of the 3, 4 Propositions. *Resolution.*

Wherefore

$$\overset{\alpha}{9}+\overset{\omega}{33}=42)\ \overset{2Z}{168}\ (4\ (T)$$

And

$$33\ (\omega)\times 33 = 99,\ 99 \qquad 1089\ (\omega q)\quad 9\times 9 = 81\ (\alpha q)$$

$$84\ (Z)\times 2 = 168\ (2Z) - \underset{42}{\overset{33+9}{}}\ (\omega+\alpha) = \frac{1008}{126}\Big(\ 8\ (X)$$

Q. *Of Gain in the first and last Months.*

Example 3. Suppose a Man gain every Month in the Year 30 *s.* more than he did the first Month, and at 12 Months End found the Whole to amount to 280 *l.* what were his Gains the first and last Months of the 12?

Answer. *Answ.* The first Month 15 *l.* : 1 *s.* : 8 *d.* : And the last Month 31 *l.* : 11 *s.* : 8 *d.*

Here are *Data* . T . X . Z . — 12 . 30 . 5600 the Shillings in 280 *l.*; *Quesita* α (1) Principal . 4, ω (2) Principal . 4 of the 1, 2 Propositions. *Resolution.*

Wherefore

$$5600\ (Z)\times 2 = 11200\ (2Z)\qquad 12\ (T)\times 2 = 24\ (2T)\qquad \frac{11200}{24}\Big(\ 466\tfrac{2}{3}\quad 15\ (\tfrac{1}{2}X)\qquad 12\ (T)\times 30\ (X) = 360\ (TX)$$

$$481\tfrac{2}{3} - 180 = 301\tfrac{2}{3}\ (\alpha)\ s.$$

And

$$5600\ (Z)\times 2 = 11200\ (2Z)\qquad 12\ (T)\times 2 = 24\ (2T)\qquad 12\ (T)\times 30\ (X) = 360\ (TX)\quad \tfrac{360}{2}=180\qquad \frac{11200}{24}\Big(\ 466\tfrac{2}{3}$$

$$466\tfrac{2}{3}+180 = 646\tfrac{2}{3} - 15\ (\tfrac{1}{2}X) = 631\tfrac{2}{3}\ (\omega)\ s.$$

Q. *Of the Number and Ages of Children.*

Example 4. A Man had divers Children; the Youngest 6 Years old, and the Eldest 40, and every one elder than the other by 2 Years: how many Children had he, and what was the Sum of all their Ages?

Answer. *Answ.* He had 18 Children, and their Ages together were 414 Years.

Here are *Data* . α . ω . X — 6 . 40 . 2; *Quesita* T (3) Principal . 1, Z (5) Principal . 2 of the 3, 5 Propositions. *Resolution.*

Wherefore

$$\overset{\omega}{40} - \overset{\alpha}{6} = \frac{34}{\underset{(X)}{2}}\Big(\ 17+1 = 18\ (T)$$

And

$$40\ (\omega)\times 40 = 1600\ (\omega q) - 6\ (\alpha)\times 6 = 36\ (\alpha q) = \frac{1564}{\underset{(X)}{2}}\Big(\ 782+\overset{\omega}{40}+\overset{\alpha}{6} = \frac{828}{2}\Big(\ 414\ (Z)$$

In all these Examples the Demands being double, after the first is found, he may be taken with two other of the *Data* for finding the second Demand: As in the last Example after T, the Number of Terms was found to be 18; Resolution of the

the second Demand Z might have been found by α . T . X . or α . ω . T . or ω . T . X . as well as by α . ω . X . the first *Data*. And if by α . ω . T : that is the first Proposition of the 5*th* Principal, the Work had been shorter than that above; and the like is to be understood of others.

Data.
α . ω . T   ω   α
6 . 40 . 18   40 + 6 = 46   T   *Quesita.*
($\frac{1}{2}$) 23 × 18 = 414 (Z)

Case 3. When together with a Question in *Progression* is involved, explicitly or implicitly, another Question, whose Resolution belongs to some other Element of Numbers, Operation is to be made accordingly. *3. If one Question be included in another.*

Example. If 100 Eggs be placed, every one a Yard distant from other in length, and the first a Yard distant from a Basket: whether one might gather up the Eggs one after another, still returning to the Basket to put them in, before one can run 4 Miles. *Q. Of gathering up 100 Eggs.*

Here are given in this Question the first Principal α, which is 2 Yards, (*viz.* a Yard from the Basket to the first Egg, and as much back again to the Basket) the third Principal T, that is, the 100 Eggs, and the fourth Principal X or 2 Yards, (*viz.* a Yard forth and a Yard back): And Z the fifth Principal is desired, that is, the Number of Yards he that doth gather up the Eggs runneth in all forward and backward. And then because another Question is included, that is, whether this Number of Yards will reach in Length 4 Miles or not? the Number found is to be compared with the Yards in 4 Miles, and allowing 1760 Yards to an English Mile, Z or 10100 Yards found to be run in gathering up the Eggs, is seen to amount to 5 Miles and almost 3 quarters of a Mile more. *Answer.*

Data   100 (T)
α . T . X.   2 (X)   *Quesita* *Yards. Mile. Yards. Miles.*
2 . 100 . 2   200   As 1760 . 1 :: 10100 . $5\frac{61}{88}$.
2 (X)
198   *Miles.*
4 (2α)   4 × 1760 = 7040.
202 × 100 = $\frac{20200}{2}$ (*Yards.* 10100 (Z)

Case 4. When the Increase is inverted, that is, turned into Decrease, and the Question propounded with every Term succeeding the First less than the First; then accompt the last Term the First, and the First the Last, till the Work of the Proposition be ended. *4. If the Increase be inverted.*

Example. A Scout-Master-General being commanded to discover the Quarters of an Enemy, returned this Accompt; If moving (saith he) from the Place where we now are the first Day 30 Furlongs, the second Day 28, the third Day 26, and so every Day lessening 2 Furlongs, in the 15*th* Day we shall come to the Enemy; but they would meet in 9 days: how far shall they march in a Day one day with another, to overtake the Enemy in their Quarters in 9 days? *Q. Of marching to overtake an Enemy.*

Here the first Work being to find out the Distance of the two Armies, or Z the 5*th* Principal, to the finding whereof is given α, the first day's march, 30 Furlongs; T the 15 days, and X the Difference of their March, which is 2 decreasing; so is the increase of the *Progression* inverted, wherefore α the 30 shall be ω the 15*th* Term, and that Term 2 (implicit in the Question) shall be instead of α. Then either by α . ω . T . or α . T . X . or ω . T . X . may Z be found, which will be 240 Furlongs; and then the other Question will be resolved by dividing 240 by 9, and their March thereby found to be $26\frac{2}{3}$ Furlongs in a Day. *Answer.*

Data.
ω . T . X.   30 (ω)
30 . 15 . 2   2   *Quesita.*
60   15 (T)
2 (X)   2 (X)   T   Z
62 (2ω+X) — 30 (TX) = 32 × 15 = $\frac{480}{2}$ ($\frac{240}{9}$ ($26\frac{2}{3}$ Furlongs.

Case 5.

5. If a middle Term be sought. *Case* 5. When any middle Term (that is, intermediate between the first and last Terms) is demanded, the same is to be found according to the *Data*. For seeing the Number of such middle Term may be represented by T, the whole Number of Terms in a *Progression*, the Sum of such Term may be represented by ω the last Term of the *Progression*: so as with little alteration all the Propositions before, for finding the second Principal ω, may serve to find the middle Term desired.

Examples. As if α and X be given, then by the first Proposition of the second Principal, take an Unite from the Term desired, and multiply the Remain by the Excess, and to the Product add the first Term.

And so in the foregoing Instance, if 4 and 2 be given to find the 12*th* Term of that *Progression*, that is, 26: An Unite taken from 12, and the Remain 11 multiplied by 2, is 22, to which 4 added, the Total is 26 desired.

But if Z be one of the *Data*, let it be understood to be the Sum of the *Progression* to that middle Term desired, and not the Total Sum of the whole *Progression*; and then Operation may be made therewith, as in the other 3 Propositions of the second Principal.

And if ω, or the last Term of the whole *Progression* be given, for one of the *Data*; then invert the Terms in the *Progression*, and multiplying 1 less than the Term desired by the Excess, subtract the Product from ω.

So the 12*th* Term of the former Instance shall be the 4*th* Term, from which 1 taken, and the rest 3 multiplied by 2, the Excess shall be 6, which taken from 32, the Remain will be 26 as before.

Terms	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	
	4.	6.	8.	10.	12.	14.	16.	18.	20.	22.	24.	26.	28.	30.	32.	
	15.	14.	13.	12.	11.	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.	inverted.

Proof of Arithmetical Progression. The Proof of the Operations in this sort of *Progression*, is by placing every Increase in its due Seat or Term, and by common Addition of Integers to collect the Numbers into one Total, as in the first Example of this Chapter.

CHAP. III. *Transposition.*

Transposition what, and the Sorts. THE first-born of *Arithmetical Progression* is *Transposition*, which is an orderly disposing of some Parts of a Number, so as there may be an equal Difference between the Parts so placed, or a disposing of a Number so that the Parts desired may be taken, and the other left.

The first Sort what it doth resemble. How wrought. The former sort resembles *Division*, in which though the Dividend be taken a thousand Times by the Divisor, yet the Divisor continues intire. This *Transposition* is to be done with due observation of the Parts: for if the Parts placed be some Aliquot Part of a Number, and the Places be equal to the Divisor, that Aliquot Part is the desired Number; but if 2 or more different Parts be taken, then half the Number of the Divisor of one sort, and half the Number of the Divisor of the other sort of Parts, must be taken.

Examples. Examples of both follow.

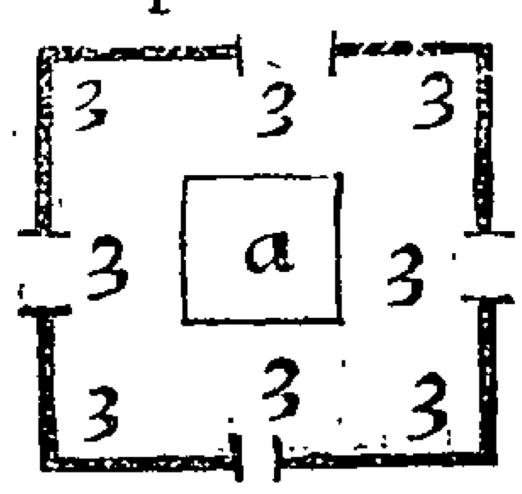
Q. Of Souldiers, how disposed, that a like Number may face the Side of a Fort, when some are entertained, and others discharged. A certain Passage of Square Form had 4 Gates opposite one to the other, that is, in the middle of each side one; and there were appointed 9 Men to defend each Front thereof, some at the Gates, and the other at each Corner or Angle: so each Angle served to assist two Faces of the Square if need required. Now this Square Passage being thus mann'd with 9 at each Side, it happened that 4 Souldiers coming by, desired of the Governor to be entertained into Service, who told them he could not admit of more than 9 upon each Side of the Square: to whom one of the Souldiers being skilled in the Art of Numbers replied, If he would take them into Pay, they would place themselves amongst the rest, and yet keep still the Order of 9 for each Face of the Square; to which the Governor agreed, and they were admitted. But afterwards liking not their Service, they indeavoured to remove themselves and also draw away each Man his Comrade, yet would leave 9 to defend each Side of the Passage: and how may this be?

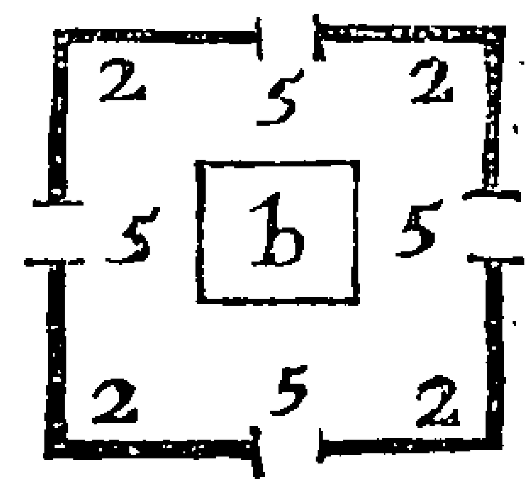
Answ. *Answ.* The Square having 4 Gates and 4 Angles, that is, 8 in all, 3 of each Side including the Angles, this 8 multiplying 3, produceth 24 for the whole Number,

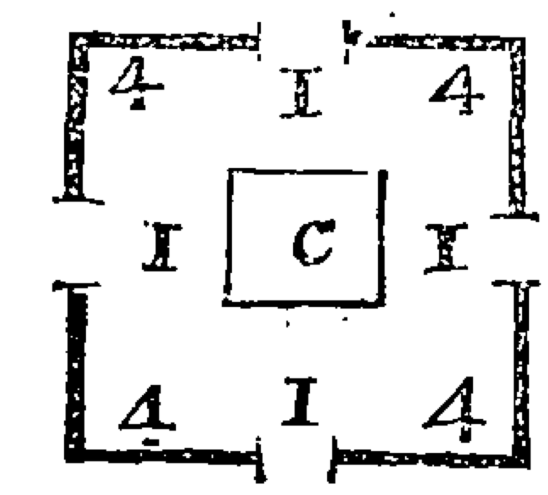
Number, whose Aliquot Part desired is 3; and so many were placed at each Gate and Angle before the 4 Souldiers were admitted, as at *a*.

Then each of the 4 placing themselves at the Gates, removed from every Angle to every Gate one Man, as at *b*. where the whole Number being 28, that is, 4 times 7, and 4 being half the Number of 8 the Divisor, must be now taken, because 8 will not evenly divide 28; and so it must be, that 7 shall be the Number belonging both to a Gate and an Angle, that is, 5 to one and 2 to the other, and 4 times 5 and 4 times 2 make up the 28.

But then 8 being taken away, and 20 Souldiers only left, which 20 will not be evenly divided by 8, but by 4 the half thereof; and therefore 5 the Quotient shall be the Number belonging both to an Angle and a Gate, that is, 4 to one, and 1 to the other, as at *c*. So as upon their departure, 2 were removed from the Gates, and placed at the Angles.



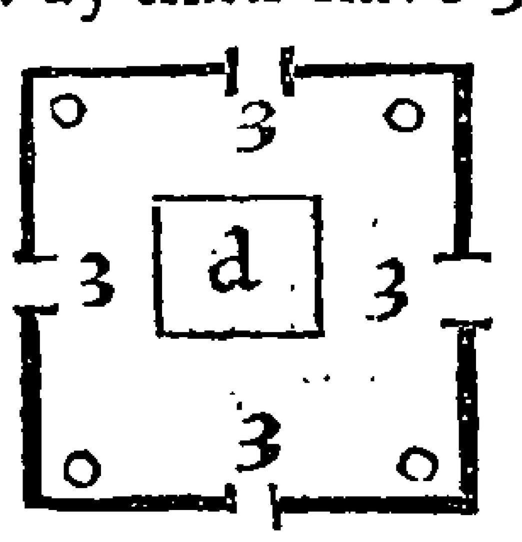


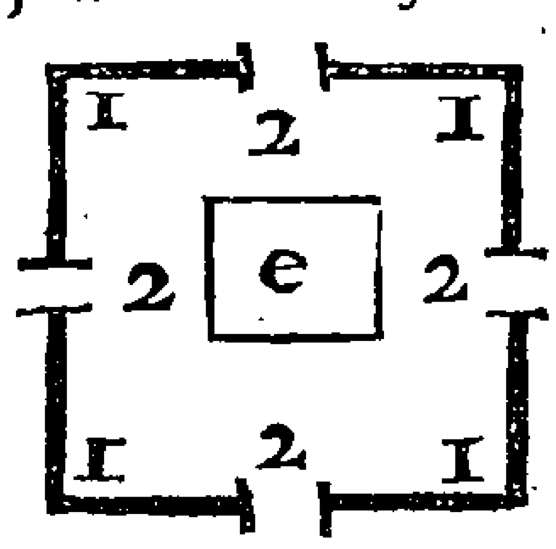


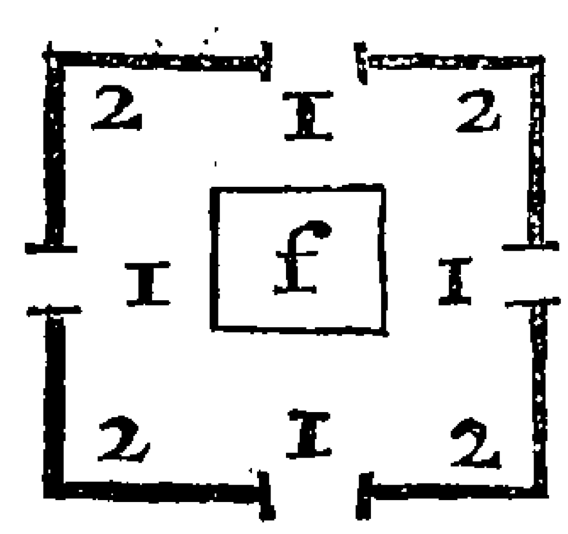
And the like is to be observed on the contrary, when the Parts of one and the same Number are to be so transposed, that their Fronts shall be more or less.

Example, where with a like Number the Side may have more or less.

As the Number 12 may be so parted and placed about a Square, that the Fronts every Way shall have 3, 4, or 5, as at *d. e. f.*







The other Sort of Transposition *used in Lottery.*

The other sort of *Transposition* to dispose a Number so that the Parts desired may be chosen, is used in Lottery, where sometime the 4, 5, 7, 20, &c. Person or Thing is respected to be secured, and this is performed thus: Take as many Units or Ciphers as there be Persons or Things on which the Lot is to pass, and dispose them on a Paper in a Row; then set a Prick on the respected Place as the Lot is to fall upon, whether the 3, 4, 5, &c. and so count along on the remaining Places, still pricking the alloted Place till the Complement of the Lottery be marked out; and then you shall see how to place the Persons or Things, that the Lot may fall where desired.

Q. Of Malefactors condemned, which to a Lot to suffer.

Example 1. If 24 Persons have done a Villany, and 6 of them more guilty than the rest, and being apprehended and found Guilty at Trial, the Judg determined by a Lot, that every 7*th* Man should die, till the Number of 6 were executed: how then should the Men be disposed, that so counting by 7, the Lot of Escape should always fall upon one of the 6 that were most notoriously guilty?

Answer.

Answ. By setting down 24 Ciphers, and telling along by 7 and 7, the 6 Men designed to die are seen to be set in the 4, 7, 12, 14, 20 and 21 Places.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10.11.12.13.14.15.16.17.18.19.20.21.22.23.24.
o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o . o.

1 2 3

4 5 6

Q. Of Christians and Turks, how to save the Christians.

Example 2. A Ship at Sea wanting Provision, having 15 Christians and as many Turks, and to preserve all from famishing, resolve to die to feed each other, yet so as by Lot, and that counting by 9 and 9 the ninth Person should die: Now the Master being a Christian, and desirous to save the Christians, disposed them so as the ninth Person still fell out to be a Turk: how may this be done?

Answer.

Answ. 4 Christians 5 Turks, 2 Christians 1 Turk, and so along according to the Vowels in *Populeam virgam mater Regina ferebat*, counting *a* 1, *e* 2, *i* 3, *o* 4, *u* 5. Or by the former way;

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10.11.12.13.14.15.16.17.18.19.20.21.22.23.24.25.26.27.28.29.30.
14 . 4 . 7 . 12 . 1 10 5 2 . 8 13.15.11 6 . 3 9
6 Y

Trans-

Proof of Transposition.

Transposition of both sorts, carries along with the Demonstration above such Evidence of the Truth of the Work, that nothing can be added for Proof more convincing.

CHAP. IV. *Technology.*

Technology, how called. What it doth import.

THE latter Issue of *Arithmetical Progression* called *Technology*, goes commonly under the Names of Sports and Pastimes ; but the Learned *Alsted* entitles it *Technology*, which imports as much as an Artificial discourse or discovery of Numbers or other things concealed.

How defined by Alsted, and the Sorts.

Geysius, and *Alsted* from him, defines *Technology* to be a *Progressional Arithmetical Division*, and that either by *Progressional Arithmetical Divisors* ; Or 2*ly* of *Progressional Dividends Arithmetically* : Or 3*ly*, into *Progressional Quotients Arithmetically*.

In the first Sort six things.

§. 1. In Division by *Progressional Arithmetical Divisors*, the Work is to find such Numbers to be divided as will leave the Remains of the Division, either first Equal ; or 2*ly*, Progressional ; or 3*ly*, Equal till the last Division, and then nothing ; or 4*ly*, Progressional till the Last, and then nothing ; or 5*ly*, Distant from the Divisor above an Unite ; or 6*ly*, Disorderly.

In all which there are many Numbers of such Properties, so as the Enquiry is sometimes for the least Number of that Property, and sometime for any such Number at random.

1. To find Remains equal.

1*st*. To find Numbers whose Remains shall be equal continually, multiply all the Divisors one into another, and to the last Product add the common Remain.

The least Number of such Property.

And if among the Divisors there be none of them compound one of another, then this Total will be the least Number of that Property, otherwise not. Therefore to get such least Number, omit the compound Divisors, and double the Product of the rest being multiplied one into another, if the compound Divisors be 4 or 6, or 4 and 6, and add thereto the common Remain.

But if the Divisors be 8, 9, 10, being 8 and 9 are doubly compound, besides the doubling multiply by 6, and so proceed accordingly for other Compounds : And having gotten the least Number, add him, lacking the Remain, to himself successively, and other like Numbers will be produced.

Q. Of a Number whose Remain 1, Divisors 2, 3, 4, 5, 6, 7. Answer.

Example. One trying whether a Number was Prime or Compound, found that by Division with 2 there was 1 remaining, and so likewise dividing by 3, 4, 5, 6 and 7, there was still left 1 for the Remain : what was that Number ?

Answ. If the Number quesited be intended the least of that sort, it shall be 421 ; but if any Number of that Property, add to 421 continually 420, and other like Numbers will be produced.

Here the Divisors 2, 3, 4, 5, 6, 7, multiplied one into another, produce 5040, to which 1 the common Remain added, the Total is 5041 : but this is not the least Number of that Property, because among the Divisors 4 is compound of 2, and 6 of 3, wherefore both 4 and 6 being omitted, the Product of the rest is 420, to which 1 the Remain added, makes 421 for the least Number of that Property.

$$\overset{6}{2\times 3}\times\overset{24}{4}\times\overset{120}{5}\times\overset{720}{6}\times 7 = 5040$$

1 Remain added.

5041

$$\frac{\overset{1}{5041}}{2}\Big(2520 \quad \frac{\overset{22}{5041}}{3}\Big(1680 \quad \frac{\overset{12}{5041}}{4}\Big(1260 \quad \frac{5041}{5}\Big(1008 \quad \frac{\overset{2}{5041}}{6}\Big(840 \quad \frac{\overset{1}{5041}}{7}\Big(720$$

$$\overset{6}{2\times 3}\times\overset{30}{5}\times 7 = 210$$

2

420 Doubled.

1 Remain added.

421 The least Number of that Property.

$$\frac{421}{2}\Big(210 \quad \frac{421}{3}\Big(140 \quad \frac{421}{4}\Big(105 \quad \frac{421}{5}\Big(84 \quad \frac{421}{6}\Big(70 \quad \frac{421}{7}\Big(60$$

If

If the Divisors proposed be 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10 . 11 . then the least Number which divided by them will still leave 1 remaining, is found to be 47721. For the Uncompound or Prime Numbers, 2 . 3 . 5 . 7 . 11, multiplied together, make 2310, which doubled for 4 and 6 makes 4620; and then for 8, 9 and 10, the other 3 Compounds multiplied by 6, that is 2 × 3, produceth 27720; to which 1 the common Remain added, is 27721: And by Addition of 27720 thereto, other-like Numbers will be produced. *Example in more Divisors.*

```
      6   30  210
2 × 3 × 5 × 7 × 11  ═══  2310    1   2
                            2    4 . 6
                         ─────────────
                         4620    1   2   3
                            6 . 8 . 9 . 10
                         ─────────────
                         27720
                             1 Remain added.
                         ─────
                         27721. Least Number.
```

2ly. To find Numbers whose Remains shall be Arithmetically Progressional: Multiply as before the Divisors, if they be all Prime, one into another; and from the last Product substract the Difference between the Divisor and Remainer given, and this Remain shall be the least Number of that Property; to which if the Product be added, other-like Numbers will be produced. *2. To find Remains Progressional.*

And if some of the Divisors be compound one of another, then as before omit them; and from the Product of the rest doubled, or otherwise multiplied, according to the Number of Compounds as before, take the Difference: For otherwise if all the Divisors be multiplied, the Number will be higher than the least. *The least Number of such Property.*

Example. Suppose one desire to know what Number that is, which being severally divided by 2, 3, 4, 5, 6, 7, the respective Remains will be 1,2,3,4,5,6. *Q. Of a Number whose Remains are 1, 2, 3, &c. Divisors 2, 3, 4, &c.*

Answ. The least Number of that Property will be found to be 419; to which 420 added successively, other Numbers will be produced. *Answer.*

Here 4 and 6 among the Divisors, being compound of 2 and 3 as before, they are omitted; and from 420 the double Product of the rest, 1 is substracted, so is 419 obtained.

```
    6   30
2 × 3 × 5 × 7 = 210
                  2
              ─────
                420 Doubled.
                  1 Difference substracted.
              ─────
                419 The least Number of that Property.

    (1          12(2          (3           (4          5(5          6(6
419(          419(          419(          419(        419(         419(
─── 209       ─── 139       ─── 104       ─── 83      ─── 69       ─── 59
 2             3             4             5           6            7

  6   24  120  720
2 × 3 × 4 × 5 × 6 × 7 = 5040
                           1 Difference substracted.
                        ─────
                         5039

  11(1         222(2          12(3           (4           25(5          16(6
5039(         5039(         5039(         5039(         5039(          5039(
──── 2519     ──── 1679     ──── 1259     ──── 1007     ──── 839       ──── 719
 2              3             4             5             6              7
```

If the Divisors proposed were 2, 3, 4, 5, 6, 7, 8, 9, 10, 11; then multiplying 2, 3, 5, 7, 11, and doubling the Product, and multiplying the Double by 6 as before; and from this Product 27720 taking 1, the least Number of that Property is found to be 27719: And if 27720 be added, other-like Numbers are produced. *Example in more Divisors.*

$$\overset{}{2} \times \overset{6}{3} \times \overset{30}{5} \times \overset{210}{7} \times 11 = 2310$$

```
                    1   2
              2 . 4 . 6
          4620    1   2    3
              6 . 8 . 9 . 10
         27720
             1  Difference substracted.
         27719  Least Number.
```

3. To find Remains equal till the last.

3*ly*. To find Numbers whose Remains shall be equal till the last Division, and then o left; multiply all the Divisors except the last one into another, and to the Product add the common Remain.

The least Number of such Property.

But if among the Divisors more be compound one of another than 4, the least of that Nature, this Number will not be the least of that Property; but the Product of the Prime Divisors doubled as before, for 4 and 6, and for the next 3 multiplied by 6, and so according to the Number of Compounds; and then dividing thereby the Number above-gotten, will leave the least Number remaining.

Q. Of Eggs broken.

Example. A Maid carrying Eggs to Market, met with an unruly Fellow who broke them; and afterward by determination of the Justice, was enforced to pay for them; and thereupon the Maid being demanded, how many Eggs she had? answered, That counting them by 2 and 2, there remained 1; and so likewise by 3 and 3, and 4 and 4, and 5 and 5, and 6 and 6; but when she counted them by 7, there rested o: How many Eggs had she at least?

Answer.

Answ. 301: For the Product of all the Divisors, except 7, is 720; to which 1 added is 721, a Number of like Property, but not the least of that Property: Wherefore because there were 2 compound Numbers, if the double Product of all the Prime Divisors, which is 420, be substracted from 721, the Remain will be 301. And by addition of 420 to 301, other-like Numbers will be produced.

$$2 \times \overset{6}{3} \times \overset{24}{4} \times \overset{120}{5} \times 6 = 720 \qquad 2 \times \overset{6}{3} \times \overset{30}{5} \times 7 = 210 \times 2 = 420$$

```
          1  Remain.
        721 — 420 = 301  Least Number desired.
```

$$\frac{301}{2}\Big(150 \quad \frac{301}{3}\Big(100 \quad \frac{301}{4}\Big(75 \quad \frac{301}{5}\Big(60 \quad \frac{301}{6}\Big(50 \quad \frac{301}{7}\Big(43$$

Example in more Divisors.

And if the least Number were desired, which being divided by 2, 3, 4, 5, 6, 7, 8, 9 and 10, would still leave 1 remaining, but would be evenly divided by 11; then 3628801, the Product of all the first 9 Divisors with the common Remain, divided by 27720, the Product of the Prime Divisors doubled and multiplied by 6, the Remain will be 25201, the least Number desired of that Property.

$$2 \times \overset{6}{3} \times \overset{24}{4} \times \overset{120}{5} \times \overset{720}{6} \times \overset{5040}{7} \times \overset{40320}{8} \times \overset{362880}{9} \times 10 = 3628800$$

```
                                     1  Remain.
                               3628801
```

$$2 \times \overset{6}{3} \times \overset{30}{5} \times \overset{210}{7} \times \overset{2310}{11} \times \overset{4620}{2} \times 6 = 27720$$

```
     (25
    856|2
  3628 8|01 (130
  2772 2 20
    2777
     27
```

4. To find Remains Progressional till the last.

The least Number of such Property.

4*ly*. To find Numbers whose Remains shall be *Arithmetically Progressional* till the last Division, and then o left; because all Numbers that will be evenly divided by a greater Compound Number, will be evenly divided by the Lesser whereof he is so compound, it follows therefore properly that the last Divisor given ought to be a Prime Number, and then the Rule may be thus: For Divisors to 4 multiply the Prime 2 into himself and abate 1, so shall 3 be divided by 2 and leave 1, by 3 and leave

leave o; for Divisors to 5, multiply the Prime Numbers 2 and 3 into themselves, and square the Product, from which substract 1; so shall 35 be divided by 2 and leave 1, by 3 and leave 2, by 4 and leave 3, by 5 and leave o. But for Divisors higher than 5, multiply all the Divisors except the two last one into another, and from the Product take the Difference as aforesaid, and this for 7 gets the least Number of that Property; and to get the least Number of such Property for higher Divisors, multiply only the Prime Numbers, and for 4 and 6 double the Product for 8, 9, 10, the next 3 Compounds; multiply the last Product by 6, and so for 13 the like; and this dividing the former Number shall leave the least Number remaining.

Example. There are Numbers which being divided by 2, 3, 4, 5, 6, 7, and 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, will leave their Remains orderly in *Arithmetical Progression* till the last Division, upon which o will remain: what are the least Numbers of those Properties? *Q. Of Numbers divided by 2, 3, 4, &c. whose Remains 2, 3, 4; &c. at last o.*

Answ. For 7 the least Number is 119, which is the Product of the first 4 Divisors lacking 1, and by continued Addition of 420 thereto, other like Numbers are produced: but for 11, the least Number is 2519, and by Addition of 27720 thereto, other like Numbers are produced. *Answer.*

6 24
2 × 3 × 4 × 5 = 120
1
119

119/2 (59 rem. 1 119/3 (39 rem. 2 119/4 (29 rem. 3 119/5 (23 rem. 4 119/6 (19 rem. 5 119/7 (17 rem. 0

6 30 210
2 × 3 × 5 × 7 × 2 = 420 Number to be added.

6 24 120 720 5040 40320
2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 = 362880
1
362879

6 30 210 2310 4620
2 × 3 × 5 × 7 × 11 × 2 × 6 = 27720 Number to be added.

(25
856|1
3628 7|9 (13
27722 0
277

And if the least Number were desired, which would evenly be divided by 13; but by all the other Numbers from 1, would leave the Remains in *Arithmetical Progression*; this accordingly will be found to be 277199: to which if 360360 be continually added, other-like Numbers will be produced.

6 24 120 720 5040 40320 362880 3628800
2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 = 39916800
1
39916799

6 30 210 2310 30030 60060
2 × 3 × 5 × 7 × 11 × 13 × 2 × 6 = 360360 Divisor.

(277
3880|1
39916 7|99 (110
36036 6 60
3603 3
360

277199/2 (138599 rem. 1 277199/3 (92399 rem. 2 277199/4 (69299 rem. 3 277199/5 (55439 rem. 4
277199/6 (46199 rem. 5 277199/7 (39599 rem. 6 277199/8 (34649 rem. 7 277199/9 (30799 rem. 8
277199/10 (27719 rem. 9 277199/11 (25199 rem. 10 277199/12 (23099 rem. 11 277199/13 (21323 rem. 0

5*ly*. To find Numbers that leave the Remains Progressional throughout, or Progressional till the last Division, and then o; but distant from the Divisor above 1, as sometimes 2, 3, 4, *&c.* *5. To find Remains Progressional, &c. but distant above 1 from the Divisor.*

6 Z Proceed

Progressional throughout. Proceed for the Remains Progressional throughout, as in the second *Case* before, only in getting the least Numbers after the Prime Numbers are multiplied, their Product is to be multiplied by the Prime Numbers omitted from the Unit to the first Divisor multiplied one into another; and for higher Divisors than 7 by 6 besides. And for ease it may be observed, that the least Numbers to the half of the greatest Divisor, is but to abate the Difference between the Divisor and Remainer from the least Number that hath the Difference but an Unit. Other Necessaries may be discerned by the *Examples* following.

Examples to 7.

Examples to 7.

Remains. / Divisors.	Products		Difference.		Numbers desired.
1.2.3.4.5 / 3.4.5.6.7	12 60 360 / 3×4×5×6×7 = 2520 / 3×5×7×4 = 420 / 15 105		— 2 = / — 2 =		2518. Greater. / 418. Least.
1.2.3.4 / 4.5.6.7	20 120 / 4×5×6×7 = 840 / 5×7×12 = 420 / 35		— 3 = / — 3 =		837. Greater. / 417. Least.
1.2.3 / 5.6.7	30 / 5×6×7 = 210		— 4 =		206. Least.
1.2 / 6.7	6×7 = 42		— 5 =		37. Least.

Examp. to 11.

Examples to 11.

1.2.3.4.5.6.7.8.9 / 3.4.5.6.7.8.9.10.11 / 2×2=4	Greater Number 19958398 / 15 105 1155 4620 / 3×5×7×11×4×6=27720—2=27718	Least Number.
1.2.3.4.5.6.7.8 / 4.5.6.7.8.9.10.11 / 2×2×3=12	6652797 / 35 385 4620 / 5×7×11×12×6=27720—3=27717	
1.2.3.4.5.6.7 / 5.6.7.8.9.10.11 / 2×2×3=12	1663196 / 35 385 4620 / 5×7×11×12×6=27720—4=27716	
1.2.3.4.5.6 / 6.7.8.9.10.11 / 2×2×3×5=60	332635 / 77 4620 / 7×11×60×6=27720—5=27715	
1.2.3.4.5 / 7.8.9.10.11 / 2×2×3×5=60	55434 / 77 4620 / 7×11×60×6=27720—6=27714	
1.2.3.4 / 8.9.10.11 / 2×2×3×5=60	7913 / 660 / 11×60×6= 3960—7= 3953	
1.2.3 / 9.10.11	90 / 9×10×11 = 990—8= 982	
1.2 / 10.11	10×11 = 110—9= 101	

Progressional till the last. For the Remains that are Progressional till the last Division, and then o, proceed as in the 4*th Case* before, to find the least Number proper to the given Divisors, with 1 the Difference between the Divisor and Remainer; and this least Number multiply by the Difference of the Divisors given, till the half of the greatest Divisor, and the other having but few Divisors, are soon had: or multiply the Prime Numbers, except the two last Divisors, and their Product by the Prime Numbers omitted from the Unit, multiplied with the Difference, and 2 and 6, according to the Compounds as aforesaid, taking in 4 among the Prime Divisors for

for 7; but halfing the Difference, and next omitting it in higher Divifors in this laft Multiplication, when in 7 the firft Divifor is next above the greater Half of the greater or laft Divifor, and in higher Divifors than 7 a Place further, and from the Product take the Difference. What elfe needful may be obferved in the Inftances following.

Examples to 7. *Examples to 7.*

Remains.		12 60 Difference 2 × 2 omitted.
1.2.3.4.0		3 × 4 × 5 × 4 = 240
3.4.5.6.7	2	2
Divifors.	omitted.	238 Number defired.

Remains.		20 Difference 3 × 2 × 3 omitted.
1.2.3.0		4 × 5 × 18 = 360
4.5.6.7	2 + 3	3
Divifors.	omitted.	357 Number defired.

Remains.		Difference 4 ÷ 2 × 2 × 3 omitted.
1.2.0		5 × 12 = 60
5.6.7	2 + 3	4
Divifors.	omitted.	56 Number defired.

1.0
6.7 is 7 the Number given.

Alfo 119 being found by the 4*th Cafe* before to be the Number fought, where 1 is the Difference between the Divifor and Remainer: If therefore 119 be multiplied by the Differences 2 and 3, there will be produced 238 and 357, as before.

Examples to 11. *Examp. to* 11.

1.2.3.4.5.6.7.8.0 — 15 105 420 840
3.4.5.6.7.8.9.10.11 — 3 × 5 × 7 × 4 × 2 × 6 = 5040
2 Difference.
2 omitted × 2 Difference = 4 — 5038 Number.

1.2.3.4.5.6.7.0 — 35 630 1260
4.5.6.7.8.9.10.11 — 5 × 7 × 18 × 2 × 6 = 7560
3 Difference.
2 + 3 omitted × 3 Difference = 18 — 7557 Number.

1.2.3.4.5.6.0 — 35 840 1680
5.6.7.8.9.10.11 — 5 × 7 × 24 × 2 × 6 = 10080
4 Difference.
2 + 3 omitted × 4 Difference = 24 — 10076 Number.

1.2.3.4.5.0 — 1050 2100
6.7.8.9.10.11 — 7 × 150 × 2 × 6 = 12600
5 Difference.
2 + 3 + 5 omitted × 5 Difference = 150 — 12595 Number.

1.2.3.4.0 — 1260 2520
7.8.9.10.11 — 7 × 180 × 2 × 6 = 15120
6 Difference.
2 + 3 + 5 omitted × 6 Difference = 180 — 15114 Number.

1.2.3.0 — 300
8.9.10.11 — 150 × 2 × 6 = 1800
2 + 3 + 5 + 7 omitted. — 7 Difference.
So 2 × 3 × 5 × 5 (omitting 7) = 150 — 1793 Number.

1.2.0 — 60
9.10.11 — 30 × 2 × 6 = 360
2 + 3 + 5 + 7 omitted. — 8 Difference.
So 2 × 3 × 5 (omitting 7) = 30 — 352 Number.

1.0
10.11 is 11 the Number given.

Also 2519 by the 4*th Case* before, being found to be the Number sought, where 1 is the Difference; if then 2519 be multiplied by the Differences 2, 3,4,5 and 6, the Numbers will be found as above 5038. 7557. 10076. 12595. and 15114.

Other Diversities may be observed in higher Divisors, but for that seldom more than 11 are propounded in any Question, this may suffice.

6. To find Remains disorderly.

6*ly*. To find the Numbers which divided by the given Divisors leave the Remains disorderly, let it be observed, that the greatest Divisor is the Excess of an *Arithmetical Progression*, and the Remain of that Divisor, if any, the first Term thereof if 0 remain; the greatest Divisor is both the first Term and the Excess, and the Number sought is the last Term of the *Progression*: add therefore the Excess to the first Term continually, till proving every Term by all the Divisors, a Number be found that will leave the Remains proposed.

Q. Of a Sum divided to 4 Children, &c.

Example 1. A Man distributeth a Sum of Money among his 4 Children equally, and hath 3 *l.* left for himself: but if that Sum had been equally parted among 7, he would have had 5 *l.* left: what was that Sum?

Answer.

Answ. The least Number of that Property is 19, which is found by adding 7 the greatest Divisor to 5 the first Number, and again, to 12 the second Term, because 12 divided by 4 leaves 0.

Q. Of a Number thought on.

Example 2. One thinking on a Number, desires me to tell him what it is, whereupon I bid him divide it by 3, 5, 7, and tell me the Remains, which done, he declares them to be 2, 3, 6: what then is the least Number of that Property?

Answer.

Answ. 83: For by adding 7 the Excess to 6, the first Term successively, I find no Numbers till 83 of that Property.

```
 6     2(2              5     (3
13     8 3 (27         12     19 (4
20     3               19     4
27
34     3                      (5
41     8(3 (16                19 (2
48     5                      7
55
62
69     1(6
76     8 3 (11
83     7
```

Other Examples.

To get other Numbers of like Property, multiply all the Divisors given one into another, and add the Product to the least Number found as above. So in the first Example 28, that is 4×7, added to 19; and in the second Example 105, that is 3 × 5 × 7; added to 83, shall produce other Numbers of like sort.

The Reason of leaving the Way used by others.

The *Appendix* added by *Alsted* out of *Geysius*, directeth to get the least Numbers by multiplying the Remains into certain Multiplicands, and dividing the Total of the Products by the Divisors multiplied one into another, the least Number shall be left remaining: but because sometime the least Number will be more than the Product of all the Divisors, that way is not so generally approved. For 32 divided by 4 and 7, leaves the Remains 0, 4; but 28, that is 4 × 7, dividing any Number, can never have 32 left a Remain: wherefore though the Way here above mentioned be tedious in many Divisors, yet being general and holding true in all Cases, it is to be chosen before the other, which by reason of the wonderful Variety without multiplicity of Rules and Exceptions, cannot be made good but in some particular Cases.

In division of Progressional Dividends, four things.

§. 2. In Division of *Progressional Dividends*, the work is to divide so that the Quotients may be *Arithmetically Progressional* in their Natural Order, but the Remains Nothing or Equal, Ordinate or Perturbate; in which latter only the Quotients are often interrupted in their *Progression*.

1. To find Nothing remain.

In every *Arithmetical Progression*, the Remains will be 0 when the first Term is equal to the Difference: For then the Difference will exactly divide all the Terms, as 3. 6, 9, 12, 15, &c. this is a Natural *Progression*; and 3 the first Term being equal to the Difference, dividing all the Terms, the Quotients will be in their Natural Order, 1, 2, 3, 4, 5, and 0 remain.

(0

Examples

(0 (0 (0 (0 (0

$\frac{3}{3}$(1 $\frac{6}{3}$(2 $\frac{9}{3}$(3 $\frac{12}{3}$(4 $\frac{15}{3}$(5

2. To find equal Remains.

In an *Arithmetical Progression*, when the Difference will not exactly divide the Terms, this is called sometimes an *Artificial Progression*, and shall have the Quotients *Progressional*, but all the Remains equal; as 3, 5, 7, 9, 11, &c. divided by 2, the Excess shall have the common Remain 1, but the Quotients 1, 2, 3, 4, 5, in their Natural Order.

Examples

(1 (1 (1 (1 (1

$\frac{3}{2}$(1 $\frac{5}{2}$(2 $\frac{7}{2}$(3 $\frac{9}{2}$(4 $\frac{11}{2}$(5

3. To find ordinate Remains.

In Division of an *Arithmetical Progression*, when the Divisor equal to the Number of Terms shall be but an Unit greater than the first Term in a Natural *Progression*, the Remains shall be Ordinate or Retrograde, *Arithmetically Progressional*; but the Quotients orderly, as 5, 10, 15, 20, 25, 30, divided by 6, makes the Quotients 0, 1, 2, 3, 4, 5, but the Remains 5, 4, 3, 2, 1, 0.

Examples

(4 (3 (2 (1 (0

$\frac{5}{6}$(0 $\frac{10}{6}$(1 $\frac{15}{6}$(2 $\frac{20}{6}$(3 $\frac{25}{6}$(4 $\frac{30}{6}$(5

4. To find perturbate Remains.

In Division of an *Arithmetical Progression*, when the Divisor equal to the Number of Terms shall be more than an Unit greater than the first Term in a Natural *Progression*, the Remains shall be Pertubate, yet have all the Natural Numbers to the Divisor, as 5, 10, 15, 20, 25, 30, 35, divided by 7; the Remains shall be 5, 3, 1, 6, 4, 2, 0, which are all the Numbers to 7.

Examples

(3 (1 (6 (4 (2 (0

$\frac{5}{7}$(0 $\frac{10}{7}$(1 $\frac{15}{7}$(2 $\frac{20}{7}$(2 $\frac{25}{7}$(3 $\frac{30}{7}$(4 $\frac{35}{7}$(5

In division into Progressional Quotients, two things.

§. 3. The Division into *Progressional Quotients Arithmetically*, is not properly where the Dividends are *Progressional*, as in the last Section, though *Geysius* placeth the 2 last sorts above here: but the Work is to find a Number which being divided, and the Remains continually by certain Divisors given, the Integers in the Quotients shall be in their Natural *Progression* descending to 1. As to divide 110 by 24, and the Remain 14 by 4, and the Remain 2 by 1; the Quotients will be 4, 3, 2.

In this Division is considerable the Invention of { Divisors. / Dividends.

1. To find the Divisors.

For the finding of Divisors, let be noted.

1*st*, The last Divisor will always be 1.

2*ly*, Therefore to 1 add the first Quotient, and the Total is the *Penult Divisor*.

3*ly*, To the Product of the first Quotient by the *Penult Divisor*, add the second Quotient and 1 for the *Antepenult Divisor*.

4*ly*, To the Product of the first Quotient by the *Antepenult Divisor*, add the Product of the second Quotient into the *Penult Divisor*; and the third Quotient and 1 for the *Proantepenult Divisor*.

Examples

As if the Quotients were desired to be 5, 4, 3, 2; then shall

1 be the *Ultimate Divisor*.

$5 + 1 = 6$ *Penult Divisor*.

$5 \times 6 + 4 + 1 = 35$ *Antepenult Divisor*.

$5 \times 35 + 4 \times 6 + 3 + 1 = 203$ *Proantepenult Divisor*.

2. To find the Dividends.

For the finding of the Dividends.

Add together the several Products of the Divisors, multiplied by their respective Quotients.

Examples

As if a Number be desired, which divided by 203, the Quotient shall be 5; and the Remain of that Division divided by 35, the Quotient shall be 4; and the Remain of that Division divided by 6, the Quotient shall be 3; and the Remain of that Division divided by 1, the Quotient shall be 2: Then shall 1175 be obtained thus.

$203 \times 5 = 1015$
$35 \times 4 = 140$
$6 \times 3 = 18$
$1 \times 2 = 2$
1175

(160 1175 203 (5 (2 36(0 35 (4 (2 20 6 (3 (0 2 1 (2

O. Of discovering three Things hidden. Suppose *A*, *B*, *C* have three Things, as a Ring, a Thimble, and a Bodkin; which they hiding, exchange one with another: how may the Order of their hiding be discovered?

Rule, how called. This Rule is called *Bis cæca*, or *twice Blind*, because it is grounded on blind or unknown Proportions, and therefore requires the more careful Observation of *What to be done.* these six Particulars.

1. Progressionals taken. 1. For the Number of Things so hidden or concealed, take Numbers Progressional, as 1, 2, 3, *&c.* So let the Ring be 1, the Thimble 2, and the Bodkin 3.

2. Divisors found. 2. According to the third Section above, find the peculiar Divisors for those Progressional Quotients, always 1 less than the Quotients; which in the Example above shall be 4, 1; that is, 1 the Last, and 3 + 1 the *Penult Divisor.*

3. Multiplicands found. 3. Multiplicands are to be found in a certain Order, thus; The last Multiplicand is taken at pleasure; from which Number substract the Divisors in their order, and the Remains shall be the other Multiplicands in their order: As in the Example above, taking 6 for the last Multiplicand for *C*, then shall the other Multiplicands be 2 and 5, that is, 6 — 4, and 6 — 1: So shall the three Multiplicands be, for *A* 2, for *B* 5, for *C* 6.

4. Progressionals multiplied. 4. Cause every one of the Persons hiding the Things, to multiply the Progressional Number of the Thing they have so hidden by their proper Multiplicands, and to tell you the Sum of the Products. So if *A* have the Thimble, or 2; and *B* the Ring, or 1; then shall *C* have the Bodkin, or 3: And then multiplying 2 by 2, and 5 by 1, and 3 by 6, the Product of *A* is 4, *B* 5, *C* 18, and the Total 27.

5. Total of the Products substracted. 5. Having the Total, substract the same from the Product of the last Multiplicand by the Sum of the Progressionals: So 6 being the last Multiplicand in the above-mentioned Example, it shall be multiplied in 6 the Progressionals; that is, 1 + 2 + 3 from this Product 36; the Total 27 substracted, leaves 9 remaining.

6. Remain divided, gives discovery. 6. This Remain divided by the first Divisor, the Quotient sheweth the Thing hid by *A*; the Remain of this Division divided by the next Divisor, sheweth the Thing hid by *B*: And so successively for all the rest till the last, where there are many Divisors, which last Thing shall be the Complement. Wherefore in the former Example, 9 the Remain divided by 4 for *A*, the Quotient shall be 2, and shew *A* to have the Thimble; the 1 remaining divided by 1, shall shew *B* to have the Ring, or first Thing; and by Consequence the other Thing remaining, which is the Bodkin, must be in the Custody of *C*.

Example in four Progressionals.

Example in four Things hidden. Persons *A*, *B*, *C*, *D*. Things hid 1, 2, 3, 4.
Divisors 24 . 5 . 1. that is, 1 the last Divisor; 5, which is 4+1, the *Penult Divisor*; and 24 made of 4×5+3+1, the *Antepenult Divisor.*
The last Multiplicand suppose 12; then the other Multiplicands shall be

	12		12		12		12
	24		5		1		0
for *A* —	12	*B*	7	*C*	11	*D*	12

Supposing *A* hath hid	2 .	*B*	3 .	*C*	1 .	*D*	4.
Then the Products are —	12		7		11		12
For *A* —	24 .	*B*	21 .	*C*	11 .	*D*	48

The Total 21 + 11 + 48 — 24 = 56.
The last Multiplicand 12. Progressionals 1, 2, 3, 4 = 10
Product of 12 × 10 = 120 — 56 = 64.

Then

Then dividing 64 by 24, the Quotient 2 shews *A* to have the second Thing hid; the Remain 16 divided by 5, gives 3 in the Quotient for *B*, and denotes he the third Thing hid; the Remain 1 divided by 1, shews *C* hath the first Thing; and by Consequence *D* must have the fourth Thing.

$$\overset{(16}{\frac{\cancel{64}}{\cancel{24}}}\Big(2\ A. \qquad \overset{(1}{\frac{\cancel{16}}{\cancel{5}}}\Big(3\ B. \qquad \frac{\cancel{1}}{\cancel{1}}\Big(1\ C. \qquad D\ 4.$$

Hence the Ground of other Methods, to find Things hid, and Numbers thought on.

The Things premised in the foregoing Sections of this Chapter being the Ground, as well of discovering the Dividends by the Divisors and Remains, concealing the Quotients, as by the natural Progression of Numbers from an Unit, and the Products of them by certain Multiplicands in one Sum, (concealing the Order of Multiplication) to find the Order of Multiplication, gave being principally to the several Inventions of other Methods for finding Numbers thought upon, and Things hidden. Some Examples whereof follow.

To find a Number thought on.

1. *To find a Number thought upon, or to tell a Man how many single Pence, or Pieces of Money he hath in his Purse.*

Bid the Person that thinketh, that he quadruple the Number thought on, and to the Product add 6, 8, 10, or any other Number at pleasure, and tell you the Half of the Total: For then if you take away half the Number which you willed him to add, there will remain double the Number at first thought upon.

Example.

The Number thought, suppose ——— 6
The Quadruple thereof ——— 24
To which if 10 be added, it makes ——— 34
The Half of it is ——— 17
From which 5 half the Number added being substracted, there remaineth ——— } 12
One half of which returneth the first Number 6

Otherwise.

Or bid him that thinketh, to double his Number; to which let him add 5, and then multiply the Total by 5; and having the Product, cut off the Right-hand Figure, or Cipher, and from the Residue substract 2.

Example.

The Number thought upon ——— 8
The double of it ——— 16
Addition of 5 makes it ——— 21
Multiplied by 5, produceth ——— 105
Right-hand Figure cut off, leaves ——— 10
From which 2 substracted, rests ——— 8

Otherwise.

Otherwise, bid him that thinketh to double the Number, and add 4 thereto, and multiply the Total by 5, and add to the Product 12, and multiply that Total by 10, and declare to you this last Product: Then from the same withdraw 320, and the Remainder in the hundred Place shall be the Number desired.

Example.

The Number thought ——— 6
Doubled is 12, to which 4 added is ——— 16
Multiplied by 5 produceth 80, and 12 added is - 92
That multiplied by 10, produceth ——— 920
From which 320 substracted, leaves ——— 600
The Remainder in the hundred Place is ——— 6

Otherwise.

Otherwise, bid him that thinketh, to triple his Number thought: then ask him if it be Even or Odd; if Odd, give him 1 to make it Even: and if 1 be given, reserve 1 in your Mind; and after the Number tripled is made Even, let him cast away half, and then triple that half: Then ask him again, if it be Even or Odd: if Odd, give him 1 again to make it Even; and for this 1 given upon the last tripling, reserve 2 in your Mind, (so that if you give 1 at both triplings, then have you 3 reserved in Mind); afterwards let him cast away half, and tell you how many Nines he can give you out of the other half; and for every 9 accompt 4, to which add the reserved Numbers if the Triples fell odd.

The

Example.

The Number thought	————	15	
The Triple	————	45	
Because it is Odd, 1 is added, and it is	————	46	1 Reserved.
The Half	————	23	
The Triple	————	69	
Because it is Odd, 1 is added, and it is	————	70	2 Reserved.
The Half	————	35	
Out of 35 can be given but 3 Nines, which at 4 for 9, returneth	————		12
			15 Total.

Otherwise. Otherwise, bid him that thinketh that he break the Number in two Parts, and square each part, and add the Products together; and then let him multiply the Parts one by another, and add the Product doubled to the former, and tell you the whole Sum; then take the Square Root thereof for the desired Number.

Example.

The Number thought ———— 7
The Parts suppose 3 and 4

The Square of $\left\{\begin{matrix} 3 \text{ is } 9 \\ 4 \text{ is } 16 \end{matrix}\right\}$ together — 25

The Parts 3 and 4 multiplied make — 12
Doubled is 24, added to 25 is ——— 49
The Square Root thereof is ——— 7

Otherwise. Otherwise, bid the Party that thinketh, that multiply the Number thought by what Number you please; then bid him divide the Product by another Number, and multiply that Quotient by some other Number; and that Product again divide by some other as often as you will: and in like manner take a Number at Pleasure, and secretly multiply and divide by the same Multipliers and Divisors as oft and in the Order he did, then bid him divide the last Quotient by the Number first thought, and in like manner do you; so will the Quotients be both alike; a Thing which seems admirable to those ignorant of the Cause: then bid him to his Quotient add his Number thought on, and demand the Sum (as if you knew nothing of his Quotient), from which substract your Quotient, and you have your desire.

Example.

The Number thought ———— 8
Divided by 4, the Quotient is ——— 2
Which multiplied by 8, is ——— 16
This divided by 4, is ——— 4
Which multiplied by 5, is ——— 20
This divided by the first 8, is ——— $2\frac{1}{2}$

$\frac{8}{4}(2$; 8 ; $\frac{16}{4}(4$; 5 ; $(4$; $\frac{20}{8}(2\frac{1}{2}$

In like manner, if 12 be taken and divided and multiplied by the same Numbers, and the last Product divided by 12, the Quotient will be $2\frac{1}{2}$ as the other.

$$\frac{12}{4}\Big(3 \times 8 = \frac{24}{4}\Big(6 \times 5 = \frac{30}{12}\Big(2\frac{1}{2}.$$

To find a Number thought on, without asking any Questions. 2. *To find a Number thought upon without asking any Questions, certain Operations being done.*

Bid him that thinketh add half of his Thought-Number thereto; and if it be an odd Number and cannot be halfed evenly, let him take the bigger half to add to it, and if so, keep 3 in Mind; then bid him take half the whole Sum and add thereto, and if it be an odd Number again as before, let him take the bigger Half, and for this odd one reserve 2 in Mind, so as if both be odd, 5 must be reserved: when this is done, let him substract from the last Total double the Number thought upon, and from the Remainer will him to cast away half if he can; if it be odd, then reject 1, which I reserve, and so perpetually halfing it till he come to 1: for then mark how many halfs there were after the double Number was substracted, and for the first half accompt 2, for the second 4, for the third 8, *&c.* and add unto these Numbers the Units rejected upon the last halfings (noting the first

1 rejected is but 1; the second is to be reckoned 2, the third 4, *&c.*) multiply this Sum by 4, and from the Product deduct the Numbers reserved upon the first halfings, if any.

Examples in even Numbers.

Example in even Numbers.

The Number thought	8	The Number thought	18		
The half added 4, is	12	The half added 9, is	27		
The half of 12 is 6, added is	18	The bigger half added makes	41	2 Reserved.	
Double of 8, is	16	Double of 18, is	36		
Substracted, leaves	2	Substracted, leaves	5		
The half	1	The first half	2	1 Rejected.	
		The second half	1		
For this one halfing is accompted 2, which multiplied by 4, returneth the first Number	8	For this second halfing is accompted 4, which with 1 rejected is 5; this multiplied by 4 is 20, and the 2 reserved deducted, there rests	18		

Examples in Odd Numbers.

Example in odd Numbers.

The Number thought	7		The Number thought	17	
Greater half 4 added, is	11	3 Reserv'd.	Greater half 9 added, is	26	3 Reserv'd.
Greater half 6 added, is	17	2 Reserv'd.	Half 13 added, is	39	
Double of 7, is	14		Double of 17, is	34	
Substracted, leaves	3		Substracted, leaves	5	
Lesser half	1	1 Rejected.	Lesser half	2	1 Rejected.
For this half 2 being taken, and the Rejected 1 is 3, multiplied by 4 is 12, from whence the Reserved 5 deducted, leaves	7		Second half	1	
			For this half 4 being taken, and the Rejected 1 is 5, multiplied by 4 is 20; from whence the Reserved 3 deducted, leaves	17	

3. *Two Numbers being proposed unto two Persons, to tell which of those Number is taken by each of the Parties.*

To find which of two Numbers taken.

Admit the two Numbers proposed be one Even and the other Odd, as 11 and 12; and when they have privately accepted which they please thereof, triple the one and double the other, and add their Products together: Contrariwise, double that Number you tripled, and triple that Number you doubled, and add their Products together, the one will be Even and the other Odd; then bid A to triple the Number he took, and bid B to double the Number he took, and add both their Products together; and if A can half the Sum evenly, he took the Even Number, otherwise B accepted thereof.

Numbers proposed, 11, 12.

Example.

A taketh	11	A taketh	12
B	12	B	11
A tripleth	33	A tripleth	36
B doubleth	24	B doubleth	22
Total Odd	57	Total Even	58

4. *To discover whether Even or Odd Numbers be taken when they are not proposed.*

To find if the Number taken be Even or Odd.

Bid A to triple his Number and B to double his, and add both the Products together, and if they can give you the Even half, A took an Even Number, and B an Odd; if not, understand the contrary.

Example.

A	B		A	B	
4	3		3	4	
3	2	Even.	3	2	Odd.
12 +	6	18	9 +	8 =	17.

5. *To know several Numbers thought upon by one or sundry Persons.*

To find several Numbers thought on.

Bid them add the first and second together, and tell you the Total; likewise the Total of the Second and Third, and so further if there be more Numbers, and then tell you the Total of the first and last Numbers added together: For then by

by the Rule of *Falshood* or *double Position*, as before in the second Part of this 4*th* Book, *Chap.* 14. Resolution may be had.

For Odd.

Or if the many Numbers thought on be Odd, as 3 Numbers, 5, 7, *&c.* having the Totals given as before, place them in order, and add together all those that stand in the odd Places, *viz.* the First, Third, Fifth, *&c.* keep this Number apart from which you must make Substraction. In like manner add all those Numbers together which are in the even Places, *viz.* the Second, Fourth, *&c.* and substract this Total from the former, the Remain shall be double the first Number; which found, the rest are easily known, because the Sum of the First and Second is given.

For Even.

But if the many Numbers thought on be Even, as 4 Numbers, 6, 8, *&c.* then having the Totals of each two Numbers as before, with the Total of the second and last Numbers, add the Numbers in the odd Places, except the First, and take the Sum from the Sum of the Numbers in the even Places, the Remain shall be double the second Number thought upon: And this being known, the rest are easily obtained.

Example in Odd.

Example in five Numbers.

Suppose the Numbers thought ——— 3 . 6 . 4 . 8 . 10
First and Second, are ——————— 9
Second and Third ——————— 10
Third and Fourth ——————— 12
Fourth and Fifth ——————— 18
First and Last ——————— 13
Numbers in the odd Places, are 9 . 12 . 13. together ——— 34
Numbers in the even Places, are 10 . 18. together ——— 28
Difference ——————— 6
The Half is the first Number 3.
This taken from 9, the First and Second leaves 6, *&c.*

Example in Even.

Example in six Numbers.

Suppose the Numbers thought ——— 4 . 6 . 7 . 9 . 10 . 11
First and Second, are ——————— 10
Second and Third ——————— 13
Third and Fourth ——————— 16
Fourth and Fifth ——————— 19
Fifth and Sixth ——————— 21
Second and last ——————— 17
Numbers in the even Places are 13 . 19 . 17 . together ——— 49
Numbers in the odd Places are, 16 . 21 . together ——— 37
Difference ——————— 12
The Half is the second Number 6.
This taken from 10, the First and Second, leaves 4, *&c.*

To find Digits thought on, or Points of Dice.

6. *To declare one or more of the Digits thought upon, or the Points cast by two or more Dice.*

Let the first Number be doubled, and thereto add 5; multiply the Sum by 5, then add 10, and the next Number thought upon; multiply this Sum by 10, and thereunto add the next Number, and so proceed. Now if he give you the last Sum, then if he thought but upon one Number, substract only 35 from it, and the Remain in the Place of Tens is the Number thought upon: If he thought on 2, then take 350, and the Remainers in the 100 and 10 Places are the desired Numbers, *&c.* And because the Right-hand Figures, where above 3 are thought upon, will be the same Numbers thought upon, the more to conceal the Secret, you may ask for half the last Sum, or bid him put 14, 15, or some other Number thereto, which afterward may be easily deducted.

Example.

Numbers thought, suppose — 4 . 6 . 8 . 7 . 9.
Double of 4 is — 8
Addition of 5 makes it — 13
Multiplication of 5 produceth — 65
Addition of 10 makes it — 75 — 35 = 40
Next Number thought added — 81
Sum multiplied by 10, is — 810 — 350 = 460
Next Number 8, added, is — 818
Sum multiplied by 10, is — 8180 — 3500 = 4680
Next Number 7 added, is — 8187
Sum multiplied by 10, and 9 added, makes it — 81879 — 35000 = 46879

7. *To tell what Numbers remain, after certain Operations done, without asking any Questions.* *To find what Remains, &c.*

Let him that thinketh on a Number, multiply it by what Number you please: to the Product bid him add another Number, (which you must be sure may be equally divided by that Number before multiplied by) then let him divide the Sum by the Number he first multiplied by, and from the Quotient substract the Number thought; and this Remainer shall be equal to your Quotient, if you divide that Number which was added by that which multiplied.

Example.

Number thought, suppose — 8
Multiplied by 5, is — 40
Adding 20, it is — 60 So 20, divided by 5, yieldeth 4
Which divided by 5, gives — 12
Number thought taken away, leaves remaining — 4

8. *A Ring hidden among 9 or 10 Persons; how to discover the Person that hath the Ring, and upon which Hand, Finger, and Joint.* *To find what Hand, Finger and Joint a Ring is on.*

When the Ring is disposed among the Company, you being absent, cause the Persons to sit down in a Row; and let one of them who is privy to the Ring's Disposal, double the Number of the Person, and thereto add 5; then let him multiply the Addition by 5, to the Product bid him add the Number of the Finger: And lastly, to the right Hand adjoin the Number of the Joint, and add to the whole Sum, for Secrecy-sake, 7, 8, 9, *&c.* which done, require the Total; whence take the Number last added, and from the Residue substract 250, and you shall have 3 Figures left; the First whereof to the left Hand shall signify the Person, the middle Number the Finger, and the Third the Joint. And if after Substraction of 250 there rest 0 in the Place of Tens, then is the Ring on the Tenth or little Finger of the left Hand; and so must 1 be abated from the Place of Hundreds.

Example.

Suppose *A*, *B*, *C*, *D*, *E*, *F*, or 6 Persons.
And *E* the 5*th* Person had the Ring on his left Hand, on the middle Joint of the 9*th* Finger, accompting from the Thumb of the Right.
Then 5 doubled is 10, and 5 added makes — 15
This multiplied by 5, produceth — 75
The Finger added, it is — 84
The Joint 2 adjoined, makes it — 842
And if for Secrecy 7 be added, the Total is — 849—257=592
From this given Total 7, and 250 taken, there remaineth 592, representing — *Person.* the 5, *Finger.* the 9, *Joint.* the 2.

Otherwise, observing an Order of the Persons, and likewise of their Hands as before, calling the Right the First, and the left the Second; after the Number of the Person is doubled, 5 added, and that Sum multiplied by 5 as before, bid the Party add 10, and the Number of the Hand; which Sum bid him also to multiply by 10, and then add the Number of the Finger; and then again multiply by 10, and add the Number of the Joint, and what Number you please afterward for secrecy-sake: Then demanding the Total, deduct from thence the Number last added, and 3500, and the remaining Figures represent the Person, Hand, Finger and Joint desired. *Otherwise.*

Suppose

Example.

Suppose the *6th* Person hath the Ring on the left Hand, third Finger, and second Joint.

Then 6 doubled is 12, and 5 added makes	17
This multiplied by 5 is 85, and 10 added is	95
The Hand added is 97, and multiplied by 10 is	970
The Finger added is 973, and multiplied by 10 is	9730
The Joint added, makes the Total	9732
From this 3500 taken, leaves	6232

Representing the *6th* Person, second Hand, third Finger, and second Joint.

To find the Points cast on Dice.

9. *Two or more Dice being cast, how by Art to discover the Number of Points that may arise.*

Suppose one had cast three Dice, bid him add the Points that were upmost together, and put one of the Dice apart with the same Side upmost; let him add to the Sum the Points under the other two, then bid him cast those two Dice, and add the Points cast to the former Sum; and put one of the two Dice away, not changing the Side; and marking the Points under the other Dye, add it to the former Sum: Lastly, throwing that one Dye, whatever appears upwards, add it to the former Sum, and let the Dice remain without Alteration. This done, coming to the Table, note what Points appear upward on the three Dice; to which add 3 times 7, (for every Dye cast, always 7 is to be added) and this addition of 7 for every Dye, and the Points lying almost on the Dice, shall be equal to all the Operations made by the other Party privately.

Example.

As if three Dice being cast, there should appear 4, 6, 3, which added together make 13; then laying by one of the Dice, as suppose 4, and adding to 13 the Points under 6 and 3, which will be 1 and 4, (for always the Point, and his Opposite, or that above and underneath, makes 7) the Sum will be 18: Then throwing the 2 Dice, suppose there appear 5 and 2, which makes (being added) 25; and laying by 5, and adding the Point under 2, which will be 5, it makes the former Sum 30: Then throwing the one Dye, suppose there appear 4, then is the Sum 34. So the Points lying upward, are 4, 5, 4; which when I find, I add for each Dye 7, and that 21 with 13, the Points lying upward, make up 34 as before.

To find the Number of Pieces of Money &c. in the Hand.

10. *If one hold in each Hand as many pieces of Money, Stones, &c. as in the other, how to find the Number.*

Bid him that holds the same, that he put out of one Hand into the other what Number you think convenient (provided it may be done): then bid him take out of the Hand that he put the Number into as many as remain in the other Hand, and put into that Hand; for then be assured that in the Hand which was put the first taking away, there will be found just double the Number taken away at first.

Example.

As admit in each Hand were 10 pence, then suppose 4 were taken out of the right Hand and put into the Left; then was there 14 in the Left, and but 6 remaining in the Right; then if 6 be taken from 14 and put to the other Hand, there will be left but 8 the Double of 4, the Number first substracted.

To find how many Counters, &c. three Persons have taken.

11. *Three Persons having taken Counters, Cards or other Things; to find how many each hath taken.*

Cause the third Party to take a Number which may be divided by 4; and as often as he takes 4, let the Second take 7, and the First 13; then cause them to put all together and declare to you the Total, which divide by 3, and the Quotient is the Double of the third Number, or Sum which the third Person did take.

Otherwise.

Or cause the First to give to the Second and Third as many as each of them hath; then let the Second give to the First and Third as many as each of them hath; Lastly, let the Third give to the First and Second as many as each of them hath, and ask how much one of them hath (for then they will have all alike); so half that Number is the Number the third Person had at first; which once known, all is soon known.

Example the first Way.

Example by the first way: Suppose the Third took 8 Counters, which is twice 4 then must the Second take 14, which is so many times 7, and so consequently the First 26, which is twice 13; all added together is 48, which divided by 3 yieldeth 16, the half of which 8 is the third Person's Number.

Example

Example the latter Way.

	A	B	C
Example by the latter way: Numbers supposed to be taken	26	14	8.
A giving to *B* and *C* like Sums, that is, *B* 14 and *C* 8, they have	4	28	16
B then giving to *A* and *C* like Sums, that is, *A* 4 and *C* 16, they have	8	8	32
C then giving to *A* and *B* like Sums, that is, *A* 8 and *B* 8, they have	16	16	16

To find the Points in three Cards.

12. *Three Cards chosen out of a Pack, to find how many Points they contain if the Pack be full.*

Let him that hath chosen the 3 Cards, accompt the Points in each Card, and bid him take as many Cards as will make up 15 to each Number of the Points on the Cards first chosen, then will him to give you the remaining Cards; for 4 of them being rejected, the rest shew the Number of Points that were on the 3 Cards so chosen.

Example.

As if the Cards were 7, 6, 5: now 7 wanting 8 of 15, therefore he took 8 Cards to make up the Points on the first Card 15; so is there 9 Cards taken from 52 (the whole Pack) for the first Card, then 6 wants 9 of 15; therefore taking 9 Cards with the Card of 6 Points, that is, 10 more from the whole Pack, so will but 33 Cards remain of the 52; then 5 doth want 10 of 15, therefore 11 Cards more taken from 33 there will be left but 22; which Number of Cards given, substract 4 from, and the Remain 18 is the Number of the Points 7, 6, 5, on the Cards first chosen.

If more than 3 Cards be taken.

But when 4, 5, 6, or more Cards be chosen, and the Number of Cards out of which they are taken be more than 52, and the Term be 15, 14, 12, or such like; then multiply the Term by the Number of Cards taken at first, and to the Product add the Number of Cards so taken; then this Total substract from the whole Number of Cards, the Remain shall be the Number which must be taken from the Cards which remain to make up the Game: if there remain 0 after Substraction, then the Number of Cards which remain, do declare the Number of Points in the Cards first chosen: if the Substraction cannot be made, then substract the Number of Cards from that Number, and the Remainer added to the Cards that did remain, will be the Number of Points in the Cards first taken.

Examples.

As if the Cards were 8, 6, 3, 2, and the Term given 14. I see the first or 8 wants 6 of 14, the second or 6 wants 8 of 14, and 3 wants 11 of 14, and 2 wants 12, which taken, the Party delivers you the rest of the Cards; and if the Pack were whole, they are but 11; for 7, 9, 12 and 13 are 41: then do you multiply 14 by 4 (the Number of the Cards chosen at first) which makes 56, to which 4 (the said Number of Cards) added is 60, from which 52 substracted, leaves 8, which with 11 is 19, and so many were the Points of the Cards, *viz.* 8, 6, 3, 2.

Admit the Cards were 7, 10, 5, 8, and the Term given 12, then doth 7 want 5 of 12, ten 2, five 7 and eight 4, which with the 4 Cards first taken make 22, this taken from 52 leaves 30: Now multiplying 12 by 4, is produced 48, to which 4 added, the Total is 52, which substracted from 52 leaves 0: therefore doth 30 (the Cards left) represent the Numbers first taken, *viz.* 7, 10, 5, 8, which together make up 30.

Again, suppose 3 Cards be taken, as 7, 9, 5, their differences to 15 are 8, 6, 10, which Numbers of Cards taken from 52, the Remain is but 25; then multiply 15 by 3, to the Product 45 add 3 (for the first Cards taken) it is 48, which taken from 52, there rests 4; this taken from 25 (the remaining Cards) leaves 21 the Sum of 7, 9, 5.

To find which of 3 Persons hath things hidden.

13. *Three Things and three Persons proposed; to find which of them hath either of the three Things.*

Suppose the Persons be *A. B. C.* and the 3 Things be a Ring, a Thimble, and a Bodkin, and the Persons mutually consent to change the Things among themselves; then take 24 Counters (if more be taken the Matter may seem the more secret, but no more are needful) and lay them before the Parties, and causing them to sit or stand in a Row, give *A* with the Ring 1 Counter, *B* with the Thimble 2 Counters, and *C* with the Bodkin 3 Counters, leaving the rest of the Counters with them: retire apart, and let them change the Things, but not the Counters given them, and bid them, that he that hath the Ring after the Change, take up so many Counters as you gave him at first, and him that then shall have the Thimble to take up 2 for every Counter given him at first, and he that shall have the Bodkin to take up for every one given him at first 4: Then returning, consi-

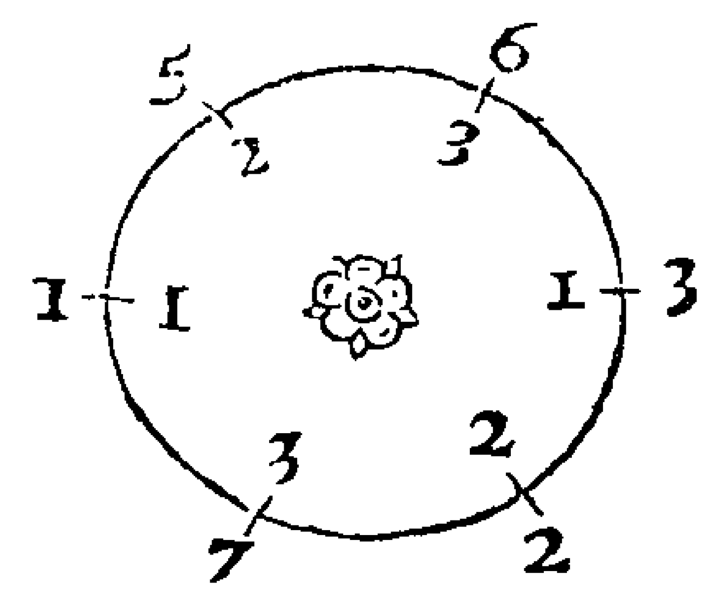
der the remaining Counters left out of the 24; for if the Parties have followed your Directions, there will be left either 1, 2, 3, 5, 6 or 7, and no other Number. Now if 1 remain, then hath there been no Exchange, but as you delivered them so they remain; if 2 remain, then hath *A* the Thimble, *B* the Ring, and *C* the Bodkin: If 3 remain, then hath *A* the Ring, *B* the Bodkin, and *C* the Thimble, and so further as is expressed in the following Table.

By the Table.

Remaining Counters.	*Persons.*	*Things hid.*	*Remaining Counters.*	*Persons.*	*Things hid.*
	A	Ring.		*A*	Thimble.
1	*B*	Thimble.	5	*B*	Bodkin.
	C	Bodkin.		*C*	Ring.
	A	Thimble.		*A*	Bodkin.
2	*B*	Ring.	6	*B*	Ring.
	C	Bodkin.		*C*	Thimble.
	A	Ring.		*A*	Bodkin.
3	*B*	Bodkin.	7	*B*	Thimble.
	C	Thimble.		*C*	Ring.

Otherwise by the Circle.

Otherwise, the same may be wrought without the Table by the help of a Circle, so divided into 6 Parts, wrote with 1 within, and 1 without; 2 within, and 5 without, *&c.* as followeth.



So if the Number of Counters remaining of the 24, be found in the upper Semicircle without; then that which is opposite within shews the First, and the next the Second, *&c.* as counting the Ring, Thimble and Bodkin, 1, 2, and 3: And suppose 6 Counters remain, 6 being found without at top, the opposite Figure within is 3; which imports that the first Man hath the Bodkin, or third Thing; and so going to the left Hand, the second Man hath the Ring, or first Thing, *&c.* But if the Remainer be found at the Bottom, as 2, then the Opposite within is 2, declaring the first Man to have the second Thing, or Thimble; and so going backward, contrary to the Former, the second Man hath the first Thing, or the Ring, *&c.* As if the former Table were thus figured as at *A*, or in Transmutations of Names as at *B*.

Table how otherwise set.

A

	A	1		*A*	2
1	*B*	2	5	*B*	3
	C	3		*C*	1
	A	2		*A*	3
2	*B*	1	6	*B*	1
	C	3		*C*	2
	A	1		*A*	3
3	*B*	3	7	*B*	2
	C	2		*C*	1

B

	1	*A*		1	*B*
1	2	*B*	5	2	*C*
	3	*C*		3	*A*
	1	*B*		1	*C*
2	2	*A*	6	2	*A*
	3	*C*		3	*B*
	1	*A*		1	*C*
3	2	*C*	7	2	*B*
	3	*B*		3	*A*

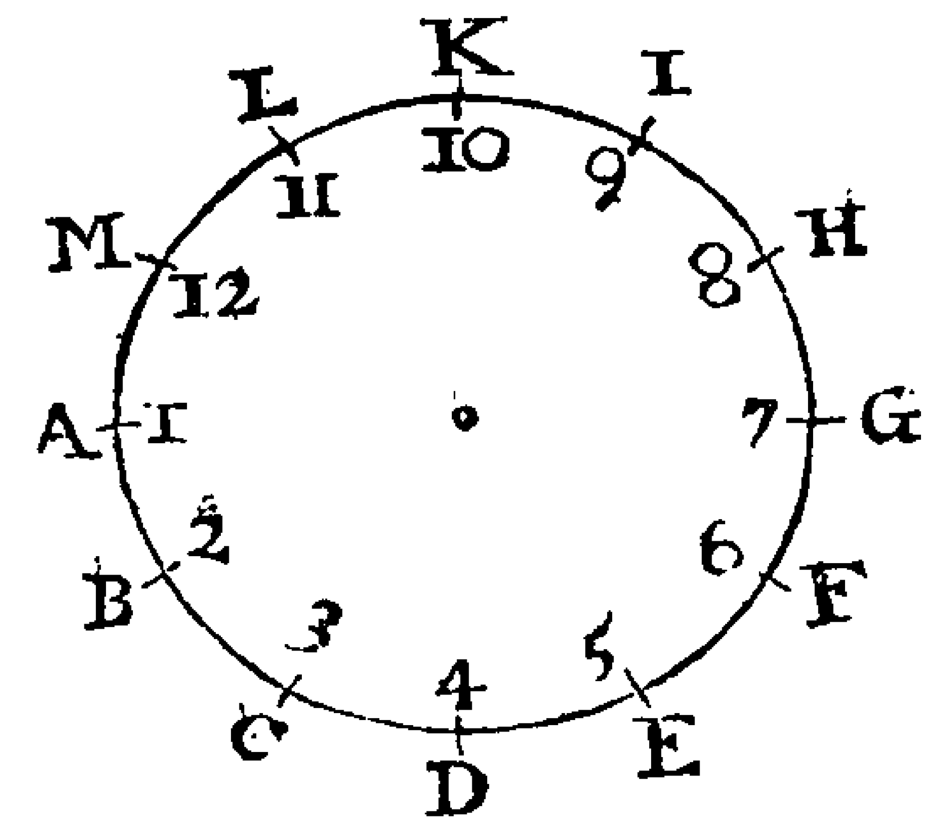
To find which Number on a Circle was thought on.

14. *Many Numbers disposed circular (or otherwise) to find which of them any one thinks upon.*

Suppose that having ranked 10 or 12 Things in a Row, as A, B, C, D, E, F, G, H, I, K, L, M, or Circular, (as the Figure sheweth); and one hath thought upon G, which is the 7*th*, then ask the Party at what Letter he will begin to accompt, (for accompt he must, otherwise it cannot be done); which suppose at K, that is the 10*th* Place; to which 10 add the Number of the Circle 12, which together is 22; so bid him to accompt 22 from K backwards, beginning his Accompt

compt with that Number he thought, so shall he accompt I to be 8, H 9, &c. And so will the Accompt of 22 end on the Letter G exactly, the Thing or Number sought.

Example.



To divide Liquors equally without Fusion.

To close up this Chapter, it is worthy note; 1*st.* That when a Question is propounded, of dividing equally among a Number of Persons, certain Liquors in Cask without Fusion; if the Number of Vessels in which the Liquors are, will not be equally divided by the Number of Persons, without a Remain, the Resolution is impossible: but if 0 remain upon the Division, break the Quotient into as many several Parts as Partners; so as being added together, they will make the Whole, and according to these Parts shall the Cask be divided.

Q. Of 21 Cask with Wine divided.

Example. *A*, *B*, and *C*, bought a Quantity of Wines, each paid alike, and was to have alike: It happened (some being sold) there was to be parted among them at last 21 Hogsheads, of which 7 were full, 7 half full, and 7 empty: how must they share the Vessels with the Wine in them, that each of them may have as many Vessels, and as much Wine one as another?

Answer.

Answ. Because 21 will be equally divided by 3, the Quotient 7 is to be broken into Parts, as 3, 3, 1, or 2, 2, 3; either of which Partitions make 7, and each of the Parts less than the Half: So may *A* have 3 Vessels full, 3 empty, and 1 half full; or 2 full, 2 empty, and 3 half full: then will he have 7 Vessels and $3\frac{1}{2}$ Hogsheads of Wine in Quantity, *B* the like; and then will be left for *C*, after the first Division, 1 full Vessel, 1 empty, and five half full; or after the second Division, 3 full, 3 empty, and 1 half full. And because this, as other-like Questions, will be truly resolved by more Divisions than one, the Proposition, however placed by some, is most proper for *Technology*.

	Full.	*Empty.*	*Half.*		*Full.*	*Empty.*	*Half.*
A	3 .	3 .	1	*A*	2 .	2 .	3
B	3 .	3 .	1	*B*	2 .	2 .	3
C	1 .	1 .	5	*C*	3 .	3 .	1

To equalize the Sums for which divers Things sold at different Rates.

2*ly.* That in Questions about equalizing the Sums for which different Numbers of Commodities may be sold at different Rates, besides a Progression given, an occult Progression in the Proposition is necessary to be discovered for the Resolution; which is often done by adding the different Rates, and thereby dividing the least Term of the given Progression, and by the Integers in the Quotient multiplying the highest Rate for the Increase of the Progression sought from an Unit. Yet it is questionable whether it will hold generally; but the Artist when it fails, may supply the defect by his Ingenuity.

Q. Of Apples sold differently, yet the Sums alike.

Example. Suppose 3 Women carry Apples or Oranges to Market; as *A* 20, *B* 30, and *C* 40: how can it be that they can sell as many for a Penny the one as the other, and when they had sold all, have one as much Money as another and no more, the Rates they sell at increasing as 3 to 1?

Answer.

Answ. 3 and 1 added make 4, which dividing 20 give 5; this multiplied into 3 the highest Rate, produceth 15 for the Increase of the Progression sought: So if *A* sold 1 Apple for 1 Penny, then adding 15 to 1, *B* shall sell 16 for Pence apiece; and adding 15 to 16, *C* shall sell 31 for Pence apiece; and the rest of their Apples being all sold at 3 Pence apiece, shall make up each of their Monies 4 *s.* 10 *d.* or 58 *d.*

If the Sum be not propounded, the Resolution divers.

But it is to be further noted, that if the whole Sum of the Money made be not propounded in the Question, these as other Propositions in *Technology*, are capable of being truly resolved by several Numbers, even as many as the last Parcel at the highest Price is more than an Unit, if the first Parcel at the lowest Price

begin

begin at the Unit. And therefore in the former Example, because 31 wants 9 of 40, the Question may be resolved 9 ways: Nevertheless the Sum of Money will be different in every of them, as here followeth.

Examples.

Price	A . B . C		A . B C		A . B . C	
1 d	1 . 16 . 31	58 d.	2 . 17 . 32	56 d.	3 . 18 . 33	54 d.
3	19 . 14 . 9		18 . 13 . 8		17 . 12 . 7	
d.	A . B . C		A . B . C		A . B . C	
1	4 . 19 . 34	52 d.	5 . 20 . 35	50 d.	6 . 21 . 36	48 d.
3	16 . 11 . 6		15 . 10 . 5		14 . 9 . 4	
d.	A . B . C		A . B . C		A . B . C	
1	7 . 22 . 37	46 d.	8 . 23 . 38	44 d.	9 . 24 . 39	42 d.
3	13 . 8 . 3		12 . 7 . 2		11 . 6 . 1	

Proof of Technology. The Practice of most of the Works in this Chapter carry the Evidence of their Proof in their Operation, the Conclusions proving themselves true, so as other Demonstrations would be vain.

CHAP. V. *Progression Geometrical.*

Geometrical Progression. BEsides what hath been said of this sort of *Progression*, *Chap.* 1. *Part* 1. and *Chap.* 1. *Part* 3. of this 4*th* Book, it is necessary to add, that in every *Geometrical Progression*, the Consequent divided by the Antecedent, shews the *Ratio* in the Quotient; so as to beget new Proportionals of this sort, is to multiply the Antecedent by the *Ratio*: but this is to be understood of Integers in Fractions, and Decimals the contrary.

New Proportionals, how gotten.

Ratio, *improperly called the* Excess. The *Ratio* (called in some Authors the *Excess*, but improperly, the Excess being the Difference of 2 Numbers in *Arithmetical Progression*) is alike between all the Terms; so as if the second Number contain the first 2, 3, or 4 times or more, then shall the Third contain the Second so many times also, and the Fourth the Third, &c.

To double it,&c. *what.* To double, triple or multiply how often soever any *Ratio* is, so often to put together the Space or Distance between the Terms.

Common way of Proceeding. The common Proceedings by most in this *Progression* is by the first Term, the *Ratio*, and the Number of Terms, to place all the Terms orderly as they increase, and collect their Total by common Addition.

Q. *Of Wheat sold, increasing the Price of every Bushel.* *Example.* A Farmer selleth a Quarter of Wheat (containing 8 Bushels) to be paid for the first Bushel 2 Farthings, for the second 8 Farthings, and so increasing by a fourfold Proportion or *Ratio*: the Question is, what was to be paid for the Wheat by that Agreement?

Answer. *Answ.* 43690 Farthings or 45 *l.* 10 *s.* 2 *d.* $\frac{1}{2}$. For by setting down orderly, as at *A. B.* or *C.* all the Terms, and adding them together, the Sum is so found.

	d			B q
A. Least Term or Extream	$0\frac{1}{2}$	1	Bushel of Wheat.	2
	2	2		8
	8	3		32
	2:8	4		128
Ratio 4.	10:8	5		512
	2:2:8	6		2048
	8:10:8	7		8192
Greatest Term or Extream	34:2:8	8	Terms or Places.	32768
Total or Sum	$45:10:2\frac{1}{2}$			43690

q 4) 43690 (2 q; 1092 (2 d; 12; 91,0; $45:10:2\frac{1}{2}$

C

Terms or Places.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 .

Least Term 2.8.32.128.512.2048.8192.32768. Greatest Extream.

Ratio 4. Sum 43690 q.

And

And in Species thus, $\alpha \;.\; \beta \;.\; \frac{\beta q}{\alpha} \;.\; \frac{\beta c}{\alpha q} \;.\; \frac{\beta qq}{\alpha c} \;.\; \frac{\beta qc}{\alpha qq} \;.\; \frac{\beta cc}{\alpha qc} \;.\; \frac{\beta qqc}{\alpha cc} \;.\; \therefore$

In the Computation of this second sort of *Progression* is to be observed as in the Sections following. *What to be noted.*

§ 1. That as in *Arithmetical Progression*, so in *Geometrical*, there are 5 principal Things, *viz.* *1st. The five Principals.*

1. The first Term or least Extream, which in the Example above is 2, and noted in Species with α as the least Term of an *Arithmetical Progression.* *1. The first Term, &c.*

2. The last Term or greatest Extream, being the last of the *Progression*, noted also sometime as in *Arithmetical Progression* with ω: but if all the Terms be set down, then it is according to the Power of the Multiplication of the second Term, to be divided by the next inferior Power of the First. So as in the former Example 32768 the last Term is expressed by $\frac{\beta qqc}{\alpha cc}$. *2. The last Term, &c.*

3. The Number of Terms or Places in the whole *Progression*, which in the Instance above is 8, and in Species noted commonly as in *Arithmetical Progression* by *T*, and sometimes by *N*. *3. The Number of Terms, and Species thereof.*

4. The *Ratio* marked sometime with *R*, sometime with *r*, sometime with *Rat.* and sometime with $\frac{R}{S}$ or R to S; and when α is 1, is always equal to β, and so sometime marked therewith. This in the foregoing Example is *Quadruple*, or as 4 to 1; the Number of *Ratio*'s in every *Progression*, or the Number of Spaces or Distances between the first and last Terms is always less by 1 than the Number of Terms, and so marked with T—1, or N—1. *4. The Ratio, and several Species thereof.*

5. The Sum or Total of all the Terms in the *Progression*, which above in the Example is 43690. and for this the Note in Species is Z, in common with *Arithmetical Progression.* *5. The Sum or Total, &c.*

§. 2. By any 3 of the 5 Principals, the other two may be found. As, *2ly. To find the two unknown.*

1. *To find the first Term* (or α) *of a Geometrical Progression.* *To find the first Term.*

1. If the last Term, the Number of Terms, and the *Ratio* be given; that is, ω . T . R . and in the former Instance 32768 . 8 . and 4. to find 2. *1. Data. ω.T.R.*

Then either 1*st*, divide the second Principal by the Fourth so many times lacking one, as there be Units in the Third: *Rules.*

Or, 2*ly*, divide the second Principal by the Fourth, exalted to the next inferior Power of the Third, taking the Third for an Index.

Examples.

1 . 2 . 3 . 4 . 5 . 6 . 7 = 8 — 1

$\frac{\omega}{R}$ to T—1 = α 4) 32768 (8192 (2048 (512 (128 (32 (8 (2

$\frac{\omega}{\text{R figurat to T} - 1} = \alpha$ $\frac{32768}{4.16.64.256.1024.4096.16384}$ (2

1 . 2 . 3 . 4 . 5 . 6 . 7 = 8 — 1

2. If the last Term, the Number of Terms, and the Sum be given; that is, ω . T . Z . and in the former Instance 32768 . 8 . & 43690 . to find 2. *2. Data. ω.T.Z.*

Then without respect to the third Principal, take the greatest Common Divisor between the Second and Fifth, marked sometimes in Species with M: So 2 shall be the Number sought, because it is the greatest Common Divisor between 32768 and 43690. *Rule.*

Example.

$M)\frac{\omega}{Z} = \alpha$ 43690 (1 / 32768 / 10922 (3 / 2 $2)\frac{32768}{43690}\left(\frac{16384}{21845}\right.$

3. If the last Term, the *Ratio*, and the Sum be given; that is, ω . R . Z. and in the former Instance 32768 . 4 . & 43690 . to find 2. *3. Data. ω.R.Z.*

Then because the second and fifth Principals are given, take the greatest Common Divisor between, as before in the next precedent Proposition. *Rules.*

Or, (2.) multiply the second Principal by the Fourth, divide the Product by the Fourth, lacking an Unit: From the Quotient take the Fifth, and multiply the Remain by the Divisor: So shall 32768 be multiplied by 4; and the

Product 131072 be divided by 3 from the Quotient $43690\frac{2}{3}$; if 43690 be taken, and the Remain $\frac{2}{3}$ multiplied by 3, produceth 2.

Example. $\frac{\omega R}{R-1} - Z \times R - 1 = \alpha$

$$\begin{array}{r} 32768 \\ 4 \\ \hline 3)131072(43690\frac{2}{3} - 43690 = \frac{2}{3} \times 3 = 2 \end{array}$$

4. Data. T.R.Z. 4. If the Number of Terms, the *Ratio*, and the Sum be given; that is, T. R. Z. and in the former Instance 8 . 4 . & 43690 . to find 2.

Rule. Then from the fourth Principal multiplied into it self, according to the Units in the Third, substract 1; multiply the Remain by 1 less than the Fourth, sometime called the *Antecedent of the Ratio*, and multiply 1 less than the Fourth by himself, marked in Species sometime by *D*, and called the *Difference of the Terms of the Ratio:* Then as the Product of the Remain, by the Antecedent of the *Ratio*, is to the Product of that Antecedent into himself; or as the fourth Principal figurate to T—1, is to the Fourth lacking 1; so is the fifth Principal given to the First required.

Example. R figurate to T.—1 in R — 1. D :: Z . α

Or,

R figurate to T. — 1 . R — 1 :: Z . α

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
4.16.64.256.1024.4096.16384.65536
(65535×3) (3×3)
As 196605 . 9 :: 43690 . 2
Or, 65535 . 3 :: 43690 . 2

$$\begin{array}{r} 65536 \\ 1 \\ \hline 65535 \end{array}$$

2. *To find the last Term (or ω) of a Geometrical Progression.*

To find the last Term.

1. Data. α.T.R. 1. If the first Term, the Number of Terms, and the *Ratio* be given; that is, α . T . R . and in the former Instance, 2 . 8 . and 4 . to find 32768.

Rules. Then either, (1.) multiply the first Principal figurately by the Fourth, according to the Units in the Third lacking 1; and so shall 2 be multiplied into 4, till the Index be 7, that is T—1.

Or, (2.) figurate the fourth Principal to the Third lacking 1, and multiply by the First.

Examples. α in R to T — 1 = ω

1 . 2 . 3 . 4 . 5 . 6 . 7
2.8.32.128.512.2048.8192.32768.

R figurate to T — 1×α = ω

1 . 2 . 3 . 4 . 5 . 6 . 7
4.16.64.256.1024.4096.16384

$$\begin{array}{r} 2 \\ \hline 32768 \end{array}$$

2. Data. α. T. Z. 2. If the first Term, the Number of Terms, and the Sum be given; that is, α . T . Z . and in the former Instance. 2 . 8 . and 43690, to find 32768.

Rule. Then divide the fifth Principal by the First, and from the Quotient take the greatest Figural Number, whose Index is 1 less than the Third, and multiply this Figural Number by the First: So shall 43690, divided by 2, give in the Quotient 21845; out of which the greatest second Sursolid, whose Index is 7, (that is, 1 less than 8) that can be taken, is 16384; which multiplied by 2, produceth 32768 the second Principal desired.

Example. $\frac{Z}{\alpha}$ — $\square$ Power, Index $\}$ T—1

Ergo, Power × α = ω

$\frac{43690}{2}$(21845
16384 × 2 = 32768
qqc
7 = 8 — 1

3. Data. α R.Z. 3. If the first Term, the *Ratio*, and the Sum be given; that is, α . R . Z . and in the former Instance, 2 . 4 . and 43690, to find 32768.

Rule. Then multiply the fifth Principal by the Fourth lacking an Unit, to the Product add the First, and divide the Total by the Fourth. So shall 43690, multiplied by 3, and 2 added, make the Total 131072, which divided by 4, gives in the Quotient 32768.

Example. $\frac{ZR - 1 + \alpha}{R} = \omega$

$$\begin{array}{r} 43690 \\ 3 \\ \hline 131070 \end{array} + 2 = \frac{131072}{4}(32768$$

4. Data. T.R.Z. 4. If the Number of Terms, the *Ratio*, and the Sum be given; that is, T.R.Z. and in the former Instance 8 . 4 . and 43690, to find 32768. Then

Then by the *Data* find the first Principal, according to the 4*th* Proposition of the 1*st* above-mentioned; and then take the Figurate Number of the Fourth Principal, according to the Units in the Third, lacking 1, and the *Analogy* will be; As 1 to this Figurate *Ratio*; so shall the first Principal found, to the Second required. Rule.

R figurate to T.—1 in R—1. D :: Z. α
Or R figurate to T.— 1. R — 1 :: Z. α

Example.

1.2.3.4.5.6.7.8
4.16.64.256.1024.4096.16384.65536
1
65535

As 196605.9 :: 43690.2.
Or, 65535.3 :: 43690.2.

1. R figurate to T — 1 :: α. ω — As 1.16384 :: 2.32768.

3. *To find the Number of Terms* (or T) *of a Geometrical Progression.* *To find the Number of Terms.*

1. If the first Term, the last Term, and the *Ratio* be given; that is, α. ω. R. and in the former Instance, 2.32768, & 4. to find 8. 1. Data. α. ω. R.

Then multiply the first Principal by the Fourth successively, till the Second be produced, and to the Number of the several Multiplications add 1: So 2 multiplied by 4, shall in 7 Multiplications produce 32768; therefore to 7 add 1, and 8 is obtained. Rule.

N of Mult. of α R to ω + 1 } = T

Example.

2.8.32.128.512.2048.8192.32768
1.2.3.4.5.6.7+1=8

Or if the second Principal be divided by the First, and the Fourth be figurate to that Quotient, the Index of this figurate *Ratio* shall be the Number of Terms lacking 1. Rule.

α. 1 :: ω. R figurate to T—1

Example.

2) 32768 (16384
4.16.64.256.1024.4096.16384
1.2.3.4.5.6.7+1=8

2. If the first Term, the last Term, and the Sum be given; that is, α. ω. Z. and in the former Instance, 2.32768. & 43690, to find 8. 2. Data. α. ω. Z.

Then without respect to the fifth Principal; as the first Principal to 1, so shall the second Principal be to the *Ratio* multiplied into it self, according to the Distance of the Term given from the first Term. From hence therefore, if a Root be extracted of the highest Power therein, the Index of that Power and 1 more, shall be the Number sought: So shall 2 to 1, be as 32768 to 16384; therefore 16384 being the second Sursolid of 4, hath the Index 7, to which 1 added is 8. Rule.

α. 1 :: ω. R figurate to T—1. — As 2.1 :: 32768.16384. Example.

Ergo, √ qqc. 16384 (4
7 + 1 = 8

Or divide the fifth Principal by the second, and by the Remain of that Division, the Remain of the first taken from the Second; then by the Sum of both the Quotients multiply the First till the Second be produced, and to the Number of the several Multiplications add an Unit. So shall 43690 divided by 32768 give 1 in the Quotient, and 10922 Remain, which shall be Divisor to 32766, that is the Remain after 2 is taken from 32768: this last Division giveth 3 in the Quotient, to which 1 added is 4, this multiplied into 2 the first 7 times, produceth 32768; therefore 7 and 1 make 8 the Number desired. Rule.

Example.

$$\frac{Z}{\omega} + \frac{\omega - \alpha}{\text{Remain}} \times \alpha \text{ to } \omega + 1 \left.\begin{matrix}\text{N. of Mult. of}\\ \end{matrix}\right\} = T \quad \frac{10922}{\frac{43690}{32768}}(1 \quad \frac{32768}{\frac{2}{32766}} \quad \frac{32766}{10922}(3$$

1+3=4×2.8.32.128.512.2048.8192.32768
1.2.3.4.5.6.7+1=8

3. If the first Term, the *Ratio*, and the Sum be given; that is, α. R. Z. and in the former Instance 2, 4, and 43690, to find 8. 3. Data. α. R. Z.

Then multiply the first Principal by the Fourth, till a Number be produced next greater than the Fifth, and the Number of Multiplications shall be the Third. So 2 by 4 multiplied till 131072 be produced, the Number of the several Multiplications will be 8, the Number of Terms sought. Rule.

N

Example. N of Mult. of α R to $\square^{-}$ Z } = T 2×4=8.32.128.512.2048.8192.32768.131072
1 . 2 . 3 . 4 . 5 . 6 . 7 . 8

Rule. Or by Analogy, the first Principal to the Fifth, shall be as the Difference of the Terms of the *Ratio* to the *Ratio* figurate according to the Units in the Third lacking 1, and the Remain multiplied by the Antecedent of the *Ratio*; or as the *Ratio* lacking 1, to the *Ratio* figurate to T lacking 1; therefore if from the Quotient of this Number and 1 added, divided by the same Antecedent, if the upper Way be taken, or the Quotient and 1 of the lower way a Root be extracted of the highest Power therein, the Index of this Root shall be the Number of Terms.

Example. α . Z :: D . R figurate to T. — 1 in R — 1. 4—1=3×3=9.
Or, α . Z :: R—1 . R figurate to T.—1.
As 2 . 43690 :: 9 . 196605.
Or 2 . 43690 :: 3 . 65535.
Ergo 3) 196605 (65535 + 1 = 65536.
And √qcc 65536 (4 . Index 8.

4.Data. ω.R.Z. 4. If the last Term, the *Ratio*, and the Sum be given; that is, ω . R . Z. and in the former Instance 32768 . 4 . and 43690 . to find 8.

Rule. Then multiply the greatest common Divisor between the second and fifth Principals by the Fourth till the Second be produced; and to the Number of the several Multiplications add an Unit. So 2 the common Divisor between 32768 and 43690, shall be multiplied by 4 the *Ratio* 7 times, which will produce 32768: therefore 1 added to 7 gives 8, the Number of Terms in this Example.

Example. M) $\frac{\omega}{Z} = \alpha$ 2) $\frac{32768}{43690}$ ($\frac{16384}{21845}$

Ergo, N of Mult. of α R to ω + 1 } = T 2×4=8.32.128.512.2048.8192.32768
1 . 2 . 3 . 4 . 5 . 6 . 7+1=8

Otherwise. Or, because this Common Divisor is the first Term, any of the Ways in the second and third Propositions precedent by α . ω . Z . or α . R . Z . may be used.

To find the Ratio. 4. *To find the* Ratio (or R) *of a Geometrical Progression.*

1.Data. α.ω.T. 1. If the first Term, the last Term, and the Number of Terms be given; that is, α . ω . T . and in the former Instance, 2 . 32768 . and 8. to find 4.

Rule. Then divide the second Principal by the First, and from the Quotient extract a Root, whose Index shall be less than the Third by an Unit: For the *Analogy* is, As the First to 1; so is the Second to the *Ratio* figurate to the Distance of the Term given from the First. Wherefore 16384, the Quotient of 32768 divided by 2, shall be a second Sursolid, which hath 7 for the Index, that is 1 less than 8, and 4 for the Root the *Ratio* desired.

Example. $\sqrt{\frac{\omega}{\alpha}}$ Index T—1=R $\frac{32768}{2}$ (16384 (√qqc . 4 Index . 7=8—1

α.1 :: ω . R figurate to T—1 4.16.64.256.1024.4096.16384.
2.1 :: 32768 . 16384 1 . 2 . 3 . 4 . 5 . 6 . 7

2. Data.α.ω.Z. 2. If the first Term, the last Term, and the Sum be given; that is, α . ω . Z. and in the former Instance, 2.32768. and 43690, to find 4.

Rule. Then divide the fifth Principal by the Second, and by the Remain of that Division, divide the Remain of the First taken from the Second, and add both the Quotients together: So shall 1, the Quotient of 43690, divided by 32768, and 3 the Quotient of 32766, (that is, ω—α) divided by 10922, (the Remain of the first Division) make together 4 the *Ratio* sought.

Example. $\frac{Z}{\omega} + \frac{\omega - \alpha}{\text{Remain.}} = R$ $\frac{43690}{32768}$ (1, Remain 10922; 32768 − 2 = 32766; $\frac{32766}{10922}$ (3 1+3=4

3.Data.α.T.Z. 3. If the first Term, the Number of Terms, and the Sum be given; that is, α . T . Z . and in the former Instance, 2.8. and 43690, to find 4.

Rule. Then divide the fifth Principal by the First, and from the Quotient take the greatest figural Number, whose Index is 1 less than the Third, the Root of this figural Number shall be the *Ratio*. So 43690 divided by 2, giveth 21845; out of which the greatest second Sursolid, whose Index is 7, (that is 1 less than 8) that can be taken is 16384, the Root whereof is 4.

Z

Example.

$\frac{Z}{\alpha} - \square -$ Power $\}$ T$-$1 Index $\}$

Ergo, $\sqrt{}$ Power $=$ R

$\frac{43690}{2}\Big($ 21845 ; 16384 $\left\{\begin{matrix}\sqrt{qqc} & . & 4 \\ \text{Index} & . & 7 = 8 - 1\end{matrix}\right.$

4.16.64.256.1024.4096.16384.
1. 2 . 3 . 4 . 5 . 6 . 7 .

4. If the last Term, the Number of Terms, and the Sum be given; that is, ω. T. Z. and in the former Instance, 32768 . 8 . and 43690, to find 4. 4. Data. ω. T. Z.

Then by the greatest common Divisor, between the second and fifth Principals, divide the Second: And from the Quotient extract a Root, whose Index shall be less than the Third by an Unit. So 2 the common Divisor between 32768 and 43690, shall divide 32768, and the Quotient 16384 shall be a second Sursolid, whose Index is 7, that is 1 less than 8, and 4 the Root is the desired *Ratio*. Rule.

Example.

$\text{M}\Big)\frac{\omega}{Z} = \alpha$ $\quad$ $2\Big)\frac{32768}{43690}\Big(\frac{16384}{21845}$

Ergo $\sqrt{\frac{\omega}{\alpha}}$ Index T $-$ 1 $=$ R $\quad$ 16384 $\left\{\begin{matrix}\sqrt{qqc} & . & 4 \\ \text{Index} & . & 7 = 8 - 1\end{matrix}\right.$

5. *To find the Sum* (or Z) *of a Geometrical Progression.* *To find the Sum.*

1. If the first Term, the last Term, and the Number of Terms be given; that is, α. ω. T. and in the former Instance 2.32768, and 8, to find 43690. 1. Data. α. ω. T.

Then divide the second Principal by the First; from the Quotient extract a Root, whose Index is less than the Third by an Unit; and by an Unit less than this Root divide the Remain of the First taken from the Second, and to this Quotient add the Second; so shall 32768, divided by 2, give 16384: the Index less than 8 by 1 is 7; the second sursolid Root therefore of 16384 is 4, from which 1 taken, leaves 3 for Divisor to 32766, the Remain of 32768; when 2 the First is substracted, the Quotient of this Division 10922 and 32768, make up 43690 the desired Sum. Rule.

Example.

$$\frac{\omega - \alpha}{\sqrt{\frac{\omega}{\alpha}}\ \text{Index} - 1\ \text{T} - 1} + \omega = Z \qquad \frac{32768}{2}\Big(16384\ \sqrt{qqc}\ 4 - 1 = 3\Big)32766\Big(\begin{matrix}10922 \\ 32768 \\ \hline 43690\end{matrix}$$
$$32768 - 2 = 32766$$

2. If the first Term, the last Term, and the *Ratio* be given; that is, α. ω. R. and in the former Instance, 2.32768 and 4, to find 43690. 2. Data. α. ω. R.

Then multiply the second Principal by the Fourth; from the Product take the First, and divide the Remain by 1 less than the Fourth. Rules.

Or, substract the First from the Second, and divide the Remain by 1 less than the Fourth, and to the Quotient add the Second.

So by the first of these shall 32768 be multiplied by 4; and 2 taken from the Product, leaves 131070 to be divided by 3: And by the latter way 32766 shall be divided by 3; and to the Quotient 10922 shall be added 32768, and both ways bring forth 43690.

Examples.

$$\frac{\omega R - \alpha}{R - 1} = Z \qquad 32768 \times 4 = 131072 - 2 = \frac{131070}{4 - 1 = 3}\Big(43690$$

$$\frac{\omega - \alpha}{R - 1} + \omega = Z \qquad 32768 - 2 = \frac{32766}{4 - 1 = 3}\Big(10922 + 32768 = 43690$$

And by way of *Analogy*: As the Difference of Terms of the *Ratio*, to the *Ratio* multiplied into it self according to the Units in the Third lacking 1, and the Remain multiplied by the Antecedent of the *Ratio*; or the *Ratio* lacking 1 to the *Ratio* figurate to T$-$1: So shall the First be to the Fifth: Or the same *Ratio* so figurate and multiplied as aforesaid, multiplied into the First, and divided by the said Difference, shall be equal to the Fifth. Otherwise.

Example

α . 1 :: ω . R figurate to T$-$1.
For seeing, As 2 . 1 :: 32768 . 16384 $\quad$ 4$-$1$=$3$\times$3$=$9

Ergo, D. R figurate to T.$-$1 in R$-$1 :: α . Z .
9 . 196605 :: 2 . 43690 .

Or, R—1 . R figurate to T.—1 :: α . Z .
3 . 65535 :: 2 . 43690 .

3. Data. α.T.R. 3. If the first Term, the Number of Terms, and the *Ratio* be given; that is, α . T . R . and in the former Instance, 2 . 8 . and 4, to find 43690.

Rule. Then multiply the first Principal by the Fourth continually, so many Times as there be Units in the Third; and from the last Product take the First, and divide the Remain by 1 less than the Fourth. So 2 by 4 shall be multiplied 8 times; and taking 2 from 131072 the last Product, and the Remain 131070 divided by 3, giveth 43690 as desired.

Example. $\frac{\alpha \text{ in R to T} - \alpha}{R - 1} = Z$

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
2×4=8.32.128.512.2048.8192.32768.131072
2
4—1=3)131070(43690

Rule. Or if the Fourth be multiplied into himself, according to the Units in the Third lacking 1, these Products multiplied by the First, shall be equal to the Fifth lacking the First.

Example. $\left.\begin{matrix}\text{Sum of}\\ \text{R figurate to T}-1\times\alpha+\alpha\end{matrix}\right\} = Z$ 4+16+64+256+1024+4096+16384=21844
2
Ergo, 43688+2=43690 43688

4. Data. ω.T.R. 4. If the last Term, the Number of Terms, and the *Ratio* be given; that is, ω . T . R . and in the former Instance 32768, 8 and 4, to find 43690.

Rule. Then continue the Division of the second Principal by the Fourth, so many times as there be Units in Third lacking 1: and take this last Quotient from the Product of the Fourth multiplied into the Second; this Remain divide by 1 less than the Fourth, so 32768 divided 7 times by 4 bringeth 2 in the Quotient, which taken from 131072, and the Remain divided by 3, giveth the Sum desired 43690.

Example. $\frac{\omega R - \frac{\omega}{R} \text{ to } T - 1}{R - 1} = Z$

1 . 2 . 3 . 4 . 5 . 6 . 7 .
4) 32768 (8192 (2048 (512 (128 (32 (8 (2.
32768×4=131072—2=131070(43690
4—1=3

Data and Quesita in every Question. Here also, as before in *Arithmetical Progression*, is to be observed, that by every 3 of these Principals both the other 2 may be found; and also that in all Questions duly propounded, 3 of the 5 are given to find sometime the one, sometime both the other: For though the common Sort of Questions, as the selling of an Horse by the Nails in his Shoes, a Coat by the Buttons, or an House by the Doors, &c. be all alike to the former Example first mentioned; yet may one Question be varied in the propounding 20 several Ways, according to these precedent Propositions, and by them accordingly may Resolution be had. But all these Calculations are for whole Numbers; if Fractions or Decimals be used, Alterations must be made according to their Nature and Use.

Common Sort of Questions.

Every Question may be varied 20 ways.

Alterations of Fractions and Decimals.

3ly. To find any middle Term, 3 Cases. §. 3. Next after the finding of the 5 Principals, it is necessary to know how to find any of the middle Terms or Means that lie between the two Extreams, in effecting of which there are 3 Cases considerable.

1. If the least Extream be an Unit. *Case* 1. If the least Extream of the *Progression* be an Unit, then square the Term given, and the Product shall be the Double of the given Term lacking one: as if I square the third Term, I get thereby the Fifth; So if the fourth Term be squared, the seventh is produced.

If two of the middle Terms be given to find another, then add the Number of the Terms given, and the Sum lacking 1 shall be the Number of the Term produced by multiplying the Terms one into another: As to find the 8th Term, the fourth and fifth Terms shall be multiplied one into another, for 4 and 5 makes 9, that is one more than 8.

Examples.

Example.

	1	2	3	4	5	6	7	8
Example.	1	4	16	64	256	1024	4096	16384.

$$\begin{array}{r} 16\times16 \\ \hline 96 \\ 16 \\ \hline 256 \end{array} \quad 3+3=6 \quad 5=3+3-1 \qquad \begin{array}{r} 64\times256 \\ \hline 1024 \\ 1536 \\ \hline 16384 \end{array} \quad 4+5=9 \quad 8=4+5-1$$

Caſe 2. If the leaſt Term be not an Unit, and yet by the *Ratio* of the *Progreſſion* may be divided to the Unit; then any one Term ſquared produceth juſt the Double of the given Term; and two of the Terms multiplied one into another, ſhall produce the Term the Number of their Terms added point at. *2. If the leaſt Term be no Unit, yet may be divided to 1.*

As to find the 8*th* Term, multiply the 4*th* Term into it ſelf: and to get the 7*th* Term, let the Third and Fourth be multiplied, for 3 and 4 make 7. Examples.

	1	2	3	4	5	6	7	8
Example.	3	9	27	81	243	729	2187	6561.

$$\begin{array}{r} 81\times81 \\ \hline 81 \\ 648 \\ \hline 6561 \end{array} \quad 4+4=8 \quad 8=4+4 \qquad \begin{array}{r} 27\times81 \\ \hline 567 \\ 162 \\ \hline 2187 \end{array} \quad 3+4=7 \quad 7=3+4$$

Caſe 3. If the leaſt Term be not an Unit, nor by the *Ratio* of the *Progreſſion* can be brought to an Unit; then the beſt way to avoid multiplicity of Rules, is to proceed by the following Analogy, which indeed will ſerve in all Caſes, that is, As an Unit to the *Ratio* multiplied into it ſelf, according to the Diſtance of the Term ſought from the firſt Term; ſo is the firſt Term, to the Term ſought. *3. If the leaſt Term be no Unit, nor can be divided to 1.*

As to know the 8*th* Term of a *Geometrical Progreſſion*, whoſe leaſt Extream is 2, and *Ratio* 3, the Diſtance between the firſt Term and the 8*th* Term is 7, the *Ratio* figurate to 7 is 2187, which multiplied by 2 is 4374 the 8*th* Term deſired. Example.

1 : R figurate to T—1 :: α . ω
1 . 2187 :: 2 . 4374.

1	2	3	4	5	6	7	8
2	6	18	54	162	486	1458	4374.
3	9	27	81	243	729	2187.	

Here among theſe *Geometrical Proportions* may have Place the *Lemma*'s ſet down by Mr. *Briggs* in his Latin Piece, that demonſtrate the Original of *Logarithms*, making the Index of Unity 0, though they might have been placed before among Figural Numbers or *Logarithms*. *Lemma's of Mr.* Briggs.

Lemma 1. If in a Rank or Series of Numbers continually Proportional from an Unit, (as thoſe following in *A*) any two of them be taken, as *C* 8 and *D* 32, with their Indices 3 and 5; two other Series of Numbers continually Proportional from an Unit, being made from the 2 Numbers taken out of the firſt Series, *viz.* from 8 and 32, as at *E* and *F*, the firſt Numbers in each Series next to the Unit being the former Numbers marked with *C* and *D*; and both Series being continued to 3 (the Index of the Number *C* in the Column *A*) ſhall be the Index of a Proportional Number in the Column *F*; and 5 in the like manner (the Index of the Number *D*) and the Index of a Number in the Column *E*; the laſt Proportional Number in the Columns *E* and *F* ſhall be equal to the Number found in the firſt Series marked with *A*, whoſe Index is 15, the Product of the 2 Indices to the laſt Numbers in the Columns *E* and *F*. 1. Lemma.

Q		*A*	*B*	*P*	*E*	*M*
0		1	0	0	1	0
		2	1		8	1
		4	2		64	2
	C	8	3	1	512	3
		16	4		4096	4
1	*D*	32	5		32768	5
		64	6	2		
		128	7		*F*	*N*
		256	8		1	0
		512	9	3	32	1
2		1024	10		1024	2
		2048	11		32768	3
		4096	12	4		
		8192	13			
		16384	14			
3		32768	15	5		

A The first Series of Proportional Numbers.
B The Indices of that Series of Numbers.
E The second Series of Numbers given.
M The Indices of that Series, to which those in *P* are equal.
F The third Series, whose Indices are equal to those in *Q*.

2. Lemma. *Lemma* 2. If in a Series of Numbers continually proportional from an Unit, any one of them be divided continually by his Side or Root as often as it can, the Number of Divisions shall be the Index of the Number divided, shewing the Distance of that Number from the Unit, or the Number of Intervals between Unity and the Dividend.

As of 729 by the Root 3, the Index is 6.

Indices 0 . 1 . 2 . 3 . 4 . 5 . 6 .
Proportionals 1 . 3 . 9 . 27 . 81 . 243 . 729. 3) 729 (243 (81 (27 (9 (3 (1
1 . 2 . 3 . 4 . 5 . 6

3. Lemma. *Lemma* 3. In a Series of Numbers continually proportional, any two of them being given, with the Index of one of them, to find the Index of the other, or its distance from Unity; as the given Numbers 8 and 32 in the Series above marked with *A* in the first *Lemma*, and the Index of the greater 5; Let then another Series of Numbers continually proportional be made by the Multiplication of 8, whose Index is sought by it self, and of the Product by it self continually, till you come to a Number whose Index is equal to 5, the Index given at *E*; The last Product by the first *Lemma*, shall be the same with the Product made by the continual Multiplication of 32, till the Index of the Product in this last Series be the Index sought: And therefore if the Number 32768 be divided by 32, the Side in the third Series, according to the Directions of the second *Lemma*, the Quotient will be 3, *viz.* 1024 . 32 . 1 . And so 3 shall be the Index, as well of this Product 32768 in this third Series, as of 8 in the first Series, from whence the 2 given Numbers were taken.

4. Lemma. *Lemma* 4. In a Series of Numbers continually proportional from Unity; If one Number multiply another, the Product will be one of the Numbers in that Series continued, and the Index thereof will be the Sum of the Indices given: As in the Series at *A*, if 4 multiply 256, the Product will be 1024, and the Index thereof 10, *viz.* the Sum of 2 & 8, the Indices annexed to 4 and 256.

5. Lemma. *Lemma* 5. If one Number be multiplied by another, the Number of Places in the Product shall be equal to the Number of Figures in both the Factors, unless the Product made of the first Figures toward the left Hand, in both the Numbers given, may be expressed by one Digit; as often as this shall happen, the Number of Places in the Product will be less by 1 than the Number of Figures in both the Factors: As if 68 be multiplied by 26, the Product 1768 is expressed with 4 Figures; but if 68 be multiplied by 14, the Product 952 is expressed by 3 Figures.

Thus

Logarithms diſcovered.

Thus came to be diſcovered the Number of Places, or Decimal Indices in every Figural Number which are the Logarithms, as was ſaid before in the firſt Chapter of *Logarithms.*

4ly. To find out Means. Two Caſes.

§. 4. After finding out the middle Terms in any *Geometrical Progreſſion*, is convenient to know how to find one or more mean Proportionals between two Numbers given: In finding which are conſtituted 2 *Caſes*.

1. If one Mean be ſought.

Caſe 1. If one Mean Proportional be deſired between two given Numbers. Then multiply them together, and take the Square Root of the Product.

Common Way.

As to find a Mean Proportional between 4 and 9, they produce, being multiplied, 36; therefore 6 the Square Root thereof ſhall be the Proportional Mean between 4 and 9.

So between 6 and 24, is 12 found to be the Mean Proportional.

Example.

$4 \times 9 = 36 (6^{\surd}$ $\qquad$ $6 \times 24 = 144 (12^{\surd}$

$4 . 6 . 9$ $\qquad$ $6 . 12 . 24$

How by Logarithms.

Becauſe Extraction of Roots is very eaſy by Logarithms, as for the Square Root to divide by 2, for the Cube by 3, &c. and oftentimes Fractions are given, or happen in the Work of Mean Proportionals: It is beſt for the Artiſt to work by Logarithms. Wherefore,

If the Data are alike.

1. When the Logarithms of the Numbers propounded are of the ſame Kind, add them together, and half the Sum ſhall be the Logarithm of the Mean Proportional required, and of the ſame Kind with the Logarithms of the Numbers given.

Examples.

Examples.

Integers, The Mean Proportional between 4 and 25, is 10: becauſe 10 is the Square Root of 100, the Product of 4 by 25.

Fractions, The Mean Proportional between $\frac{1}{4}$ and $\frac{4}{25}$ is $\frac{1}{5}$: for the Fractions multiplied are $\frac{4}{100}$, the √ʒ whereof is $\frac{2}{10}$ or $\frac{1}{5}$,

Decimals, The Mean Proportional between 0,25 (that is $\frac{1}{4}$) and 0,16 (that is $\frac{4}{25}$) is 0,20 (that is $\frac{2}{10}$ or $\frac{1}{5}$) as before. Their Logarithms follow.

Integers.

0,60205,99913	Log. of 4
1,39794,00087	Log. of 25
2,00000,00000	Log. of 100
2) 1,00000,00000	Log. of 10

Fractions.

—0,60205,99913	Log. of $\frac{1}{4}$
—0,79588,00174	Log. of $\frac{4}{25}$
—1,39794,00087	Log. of $\frac{4}{100}$
2) —0,69897,00043	Log. of $\frac{1}{5}$

Decimals.

—1,39794,00087	Log. of 0,25″
—1,20411,99827	Log. of 0,16″
—2,60205,99914	Log. of 0,04
2) —1,30102,99957	Log. of 0,2

If the Data are unlike.

2. When the Logarithms of the Numbers propounded are of divers Kinds, and the defective Log. the Log. of a common Fraction; then take the leſſer Log. out of the Greater, and one half of the Remain ſhall be the Logarithm of the Mean Proportional, and of the ſame kind with the greater Logarithm.

Examples.

As the Mean Proportional between $\frac{3}{4}$ and 12 ſhall be 3, becauſe the Product $\frac{36}{4}$ hath the Square Root $\frac{6}{2}$ or 3.

And the Mean Proportional between $\frac{1}{25}$ and 4, ſhall be $\frac{2}{5}$; for ſo is the Square Root of their Product $\frac{4}{25}$.

1,07918,12460	Log. of 12
—0,12493,87366	Log. of $\frac{3}{4}$
0,95424,25094	Log. of $\frac{36}{4}$
2) 0,47712,12547	Log. of 3

—1,39794,00084	Log. of $\frac{1}{25}$
0,60205,99913	Log. of 4
—0,79588,00171	Log. of $\frac{4}{25}$
2) —0,39794,00085	Log. of $\frac{2}{5}$

But if the Defective Log. be the Log. of a Decimal Fraction, then add the Logarithms together, and divide the Sum by 2, with the Addition and Diviſion proper thereto, ſet forth at large in the Part treating of Logarithms.

As the Mean Proportional between 4 and $\frac{1}{25}$ turned into a Decimal, (that is, 0,04″) is 4′, equal to the former $\frac{2}{5}$.

0,60205,99913 . Log. of 4.
—2,60205,99913 . Log. of 0,04
2) —1,20411,99826 . Log. of 0,16
—1,60205,99913 . Log. of 0,4

Observations. Mean Proportionals under this first Case thus found, make evident,

1. What the third Proportional is equal to. How noted in Species. 1. That among Numbers geometrically continually Proportional, every third Proportional shall be equal to the Square of the Second divided by the First; which is the reason of their being marked in Species as before, by $\alpha . \beta . \frac{\beta q}{\alpha}$. As 4 . 12 . 36 . for the Square of 12 is 144; which divided by 4, giveth 36.

2. If the First be a Square, so the Third. 2. That if the First be a square Number, so shall the Third, because the Product of the Extreams shall be equal to the Product of the Mean multiplied into himself; as 4 and 36 be both Squares, and 4 multiplied into 36, shall be equal to 12×12, that is 144.

3. Complement between single Squares is the Mean. 3. That the Complement between single Squares, is the Mean Proportional: As if the Segments of the Side 12 be 10 and 2, then 20 the Complement is the Mean Proportional between 100 and 4: For $100 + {}^{20}_{20} + 4$, the Squares of the Segments and the two Complements are equal to the Square of the whole Line 144.

2. If more Means than 1 be sought. To get 2. *Case* 2. If two or more mean Proportionals be desired between 2 Numbers given.

Then for 2 Means, multiply the greater Extream by the Square of the Lesser, and the Cube Root of the Product is the lesser of the two Means sought: contrariwise, multiply the Square of the greater Extream by the Lesser, and extract the Cube Root of the Product for the greater Proportional Mean. Or if the greater Extream be divided by the Lesser, the Cube Root of the Quotient shall be the *Ratio* to multiply the least Extream, &c.

Example. As if between 2 and 54, two proportional Means be sought, the Lesser will be 6 and the Greater 18: For 54 multiplied by 4, the Square of 2, produceth 216, whose Cube Root is 6 the lesser Mean: and 2916 the Square of 54 multiplied by 2 produceth 5832, whose Cube Root is 18 the greater Mean; so if 2 divide 54 the Cube Root of 27, the Quotient is 3, which multiplying 2 is 6, and 6 is 18.

Least Extream . 2 . Square . 4.	$54 \times 4 = 216$ (6 . Least Mean.
Greatest Extream. 54 . Square . 2916.	$2916 \times 2 = 5832$ (18 . Greatest Mean.
And 2) 54 (27 $\sqrt{\phi}$ 3 $\times 2 = 6$	$3 \times 6 = 18$

Proportionals 2 . 6 . 18 : 54 . Triple Ratio.

To get 3. To get 3 Means between 2 Numbers given, proceed by the first Case to get the middle Proportional between the 2 given Extreams; then between that middle Proportional and the least Extream, a mean Proportional is to be found in like manner, and also between the greatest Extream and that middle Proportional.

Example. As between 2 and 512 to get 3 Means Pproportional, the middle Proportional is found to be 32, by multiplying 2 in 512 and taking the square Root of the Product: Likewise between 2 and 32, the Mean is found to be 8, and between 32 and 512 the Mean is 128.

Extreams { 2	$2 \times 512 = 1024$	(32 $\sqrt{\mathfrak{z}}$ middle Proportional.
Extreams { 512	$2 \times 32 = 64$	(8 $\sqrt{\mathfrak{z}}$ Proportional between 2 and 32.
	$32 \times 512 = 16384$	(128 $\sqrt{\mathfrak{z}}$ Proportional between 32 & 512.

Proportionals 2 . 8 . 32 . 128 . 512 . Quadruple Ratio.

To get many. When many Means are desired to avoid multiplicity of Rules, the best way is to get the *Ratio*: for seeing the two Extreams and Number of Terms are given, the *Ratio* is obtained by the first Proposition for finding the fourth Principal before spoken to; and when the *Ratio* is gotten, the least Extream multiplied continually thereby, produceth the Means desired.

Example in 4, the common Way. As to get 4 proportional Means between 2 and 2048, the Extreams given are the first and last Terms of a *Geometrical Progression*, between which if 4 Means be gotten, the whole Number of Terms will be 6; therefore the Quotient of 2048 divided by 2, which is 1024, shall be a Sursolid Number whose Index is 5, that is 1 less than 6: the Number of Terms and the Sursolid Root of 1024 which is 4, shall

shall be the *Ratio* sought, by which 2 successively multiplied shall produce the Means desired.

$\sqrt{\frac{\omega}{\alpha}}$ Index T—1=R 2)2048(1024 $\sqrt{qc}$. 4. Index 5=6—1

Proportionals . 2 . 8 . 32 . 128 . 512 . 2048 . Ratio 4.

To avoid the Fractions that may arise in this Case as in the First, the Practice may be by *Logarithms*, which those expert therein will find easy and commodious. Wherefore, *How by Logarithms.*

1. When the Logarithms of the Numbers given are both of the same kind, and the Numbers Integers, take the Log. of the lesser Number out of the Log. of the Greater, and the third Part of the Remainder added to the Log. of the Lesser, shall give the Log. of the lesser mean Proportional required; and if to this Log. the third Part be added, the Sum shall be the Log. of the other Mean Proportional required. *If the Data are alike, both Integers.*

Example. 2 Mean Proportionals between 2 and 54, are 6 and 18, as before was found. Example.

	1,73239,37598	. Log. of 54 (27 $\sqrt{\phi}$. 3.
	0,30102,99957	. Log. of 2
3)	1,43136,37641	. Log. of 27
	0,47712,12547	. Third part of the Difference, or $\sqrt{\phi}$.
	0,77815,12504	. Log. of 6, The lesser Mean.
	1,25527,25051	. Log. of 18, The greater Mean.

But when the Logarithms are both of common Fractions, take the Log. of the greater Fraction from the Log. of the Lesser: and the third Part of the Remainer added to the Log. of the greater Fraction, shall be the Log. of the greater Mean; and being added to this, shall make the Log. of the lesser Mean. *Both Common Fractions.*

As to find 2 Means between $\frac{4}{45}$ and $\frac{3}{10}$, the Log. of $\frac{3}{10}$ shall be taken from the Log. of $\frac{4}{45}$, and the third Part of the Difference added to the Log. of $\frac{3}{10}$, shall be the Log. of $\frac{1}{5}$ the greater Mean; and the same third Part added to the Log. of $\frac{1}{5}$, shall be the Log. of $\frac{2}{15}$ the lesser Mean. Example.

Fractions	—1,05115,25225	. Log. of $\frac{4}{45}$ the lesser Fraction.
	—0,52287,87453	. Log. of $\frac{3}{10}$ the greater Fraction.
	—0,52827,37772	. Log. of $\frac{8}{27}$
3)	—0,17609,12591	. Third part of the Difference.
	—0,69897,00044	. Log. of $\frac{1}{5}$ the greater Mean.
	—0,87506,12635	. Log. of $\frac{2}{15}$ the lesser Mean.

Proportionals $\frac{3}{10}$. $\frac{1}{5}$. $\frac{2}{15}$. $\frac{4}{45}$. *Ratio* $\frac{2}{3}$.

Thus also working with the *Logarithms* of Decimals will the Effects be answerable, always using the Substraction and Division proper to Decimal Logarithms. *Both Decimals.*

As to find 2 Means between 0,048 and 0,75, the Log. of 0,75 taken from the other, leaves the Log. of 0,064, whose third Part or Cube Root is 4′, the Log. of which added to the Log. of 0,75, gives the Log. of 0,3 for the greater Mean, and again, added to this Log. gives the Log. of 0,12 for the lesser Mean. Example.

Decimals	—2,68124,12374	. Log. of 0,048‴ . The lesser Decimal.
	—1,87506,12634	. Log. of 0,75″ . The greater Decimal.
	—2,80617,99740	. Log. of 0,064‴ .
3)	—1,60205,99913	. Third part of the Difference.
	—1,47712,12547	. Log. of 0,3′ . The greater Mean.
	—1,07918,12460	. Log. of 0,12″ . The lesser Mean.

Proportionals 0,75″ . 0,3′ . 0,12″ . 0,048‴ . *Ratio* 0, 4′.

2. When the Logarithms of the Numbers given are of divers Kinds, and the defective Log. is the Log. of a common Fraction, then add them together; and if out of the third Part of the Sum you may substract the Log. of the Fraction given, the Remain of this Substraction shall be the abundant Log. of the lesser Mean sought, to which add the third Part aforesaid, and the Sum shall be the abundant Log. of the other Mean required. *If the Data are unlike, and one a common Fraction.*

But

But if when the given Logarithms are added, you may ſubſtract the third Part of the Sum out of the Log. of the given Fraction; then the Remainder of this Subſtraction ſhall be the Defective Log. of the leſſer Mean ſought: Out of which Log. if again you may ſubſtract the ſaid third Part, the Remain thereof ſhall be the Defective Log. of the other Mean required.

And if you may ſubſtract the Log. of the firſt Proportional found as above, out of the ſaid third Part, the Remainder thereof ſhall be the abundant Log. of the latter Proportional required.

Examples. As between $\frac{3}{4}$ and 48 will be found 3 the leſſer Proportional, and 12 the Greater. And betwen $\frac{1}{4}$ and 2 are found 2 Means, $\frac{1}{2}$ the Leſſer, and 1 the Greater.

—0,12493,87366	Log. of $\frac{3}{4}$	—0,60205,99913	Log. of $\frac{1}{4}$
1,68124,12374	Log. of 48	0,30102,99957	Log. of 2
1,80617,99740	Log. of 64	0,90308,99870	Log. of 8
3) 0,60205,99913	3*d* Part of the Sum.	3) 0,30102,99956	3*d* Part of the Sum.
0,47712,12547	Log. of 3.	—0,30102,99956	Log. of $\frac{1}{2}$.
1,07918,12460	Log. of 12.	0,00000,00000	Log. of 1.
Prop. $\frac{3}{4}$. 3 . 12 . 48 .	Ratio 4.	Prop. $\frac{1}{4}$. $\frac{1}{2}$. 1 . 2 .	*Ratio* 2.

When the Defective Log. given is the Log. of a Decimal, then ſubſtract the Log. of the Decimal from the Log. of the Integer, and take the third Part of the Difference from the Log. of the Integer, the Remain ſhall be the greater Mean ſought. Take again the third Part out of the Log. of the Mean thus found, and this Remain ſhall be the Log. of the leſſer Mean required, always uſing the Subſtraction proper to Decimal Logarithms.

One a Decimal.

As in the laſt Examples, and two others that follow.

Examples.

1,68124,12374	Log. of 48	0,30102,99957	Log. of 2.
—1,87506,12634	Log. of 0,75″	—1,39794,00087	Log. of 0,25″
1,80617,99740	Log. of 64	0,90308,99870	Log. of 8.
3) 0,60205,99913	Third Part	3) 0,30102,99957	Third Part.
1,07918,12461	Log. of 12	0,00000,00000	Log. of 1.
0,47712,12548	Log. of 3	—1,69897,00043	Log. of 0,5′
Prop. 0,75.3.12.48.	*Ratio* 4.	Prop. 0,25. 0,5. 1.2.	*Ratio* 2.

0,82930,37728	Log. of 6,75″	0,00000,00000	Log. of 1.
—1,39794,00087	Log. of 0,25″	—1,09691,00130	Log. of 0,125‴
1,43136,37641	Log. of 27.	0,90308,99870	Log. of 8.
3) 0,47712,12547	Third Part.	3) 0,30102,99956	Third Part.
0,35218,25181	Log. of 2,25″	—1,69897,00044	Log. of 0,5′
—1,87506,12634	Log. of 0,75″	—1,39794,00088	Log. of 0,25″
Prop. 0,25. 0,75. 2,25. 6,75.	*Ratio* 3.	Prop. 0,125. 0,25. 0,5.1.	*Ratio* 2.

More Means gotten in like manner. In like manner obſerving the Nature of the given Logarithms, whether of *Integers*, *Fractions*, or *Decimals*, and accordingly taking their Sum or Difference, may as many Means be gotten as ſhall be deſired: Thus,

Divide this Sum or Difference by 1 more than the intermediate Numbers ſought; this Quotient, which will always be the Log. of the *Ratio*, multiply ſeverally by 2, 3, 4, *&c.* till you come to the Diviſor, and theſe Products ſeverally muſt be added to, or ſubſtracted from one of the Logarithms of the given Numbers, or one of them therefrom as the Caſe requires, and the Remains ſhall be the Logarithms of the Means required.

Example in 5 Means. As to get 5 intermediate Numbers between $\frac{3}{4}$ and 48 the Sum; but if $\frac{3}{4}$ be turned into a Decimal, then the Difference of their Logarithms is to be divided by 6, that is 1 more than the Means ſought: the Quotient of this Diviſion taken from the Log. of 48, ſhall leave the Log. of 24; double the Quotient taken away, ſhall leave the Log. of 12; and ſo for the reſt as below appeareth: And the like will be effected, if the Log. of $\frac{3}{4}$ be taken ſeverally from theſe Products.

And other Proportionals. And if A, the ſixth Part of the Logarithms of the Extreams, be multiplied by 4, and the Product 4 A, or P, be added to the Log. of 48, the Sum ſhall be the 4*th* Propor-

Proportionl above 48. But if the Log. of $\frac{3}{4}$ be added to P, the Sum shall be the 4*th* Proportional below $\frac{3}{4}$, and so Numbers above and under each Extream are found.

Extreams	B	—0,12493,87366	. Log. of $\frac{3}{4}$.	I
	H	1,68124,12374	. Log. of 48.	K
	6 A 6)	1,80617,99740	. Log. of 64 the Sum	L
	1 A	0,30102,99957	. Log. of 2.	M
	2 A	0,60205,99914	. Log. of 4.	N
	3 A	0,90308,99871	. Log. of 8.	O
	4 A	1,20411,99828	. Log. of 16.	P
	5 A	1,50514,99785	. Log. of 32.	Q
Means	C	0,17609,12589	. Log. of $1\frac{1}{2}$.	R
	D	0,47712,12546	. Log. of 3.	S
	E	0,77815,12503	. Log. of 6.	T
	F	1,07918,12460	. Log. of 12.	V
	G	1,38021,12417	. Log. of 24.	W

α I . R . S . T . V . W . K ω

$\frac{3}{51}$. $\frac{3}{32}$. $\frac{3}{16}$. $\frac{3}{8}$. $\frac{3}{4}$. $1\frac{1}{2}$. 3 . 6 . 12 . 24 . 48 . 96 . 192 . 384 . 768 .

For $4A + B = \frac{I}{P} = \alpha$ Log.—1,32905,87194 $\frac{1}{16}$) $\frac{3}{4}$ ($\frac{3}{64}$

And $4A + H = KP = \omega$ Log. 2,88536,12202 $48 \times 16 = 768$

Observations. The finding of Mean Proportionals under this second Case in the manner aforesaid, makes evident,

1. Complements between Cubes, the 2 Means. 1. That the greater Complement between the Cubes of the Segments of any Line is the greater Mean, and the lesser Complement the lesser Mean. As if the Segments of 12 be 10 and 2, their Cubes are 1000 and 8, the greater Base 100, the Lesser 4, the Altitude of the Greater 10, of the Lesser 2, the greater Complement is made of 100 by 2, the Lesser of 4 by 10: So are the Cubes of the Segments with the 3 greater and 3 lesser Complements, equal to the Cube of the whole Line 1728.

Greater Cube.		*Greater Compl.*		*Lesser Compl.*		*Lesser Cube.*	
1000	.	200	.	40	.	8	
		200		40			
		200		40			
1000	+	600	+	120	+	8	=1728

2. Product of 3 Proportionals, the Cube of the Mean. 2. That the Product of 3 *Geometrical Proportionals* continued, is the Cube of the Mean: So 8 multiplied into 40, and the Product 320 into 200, shall be 64000 the Cube of 40.

3. Product of the Extreams by the Squares alternate, are the Cubes of the Means. 3. That if 4 Numbers be Geometrically continued in Proportion the Product of the Extreams by their Alternate Squares, are the Cubes of the Means, the Greater of the Greater, and the Lesser of the Lesser: As 1000 multiplied into 64 (the Square of 8) is 64000 the Cube of 40, the lesser Mean: And 8 multiplied into 1000000 (the Square of 1000) is 8000000, the Cube of 200 the greater Mean.

4. Analogy in 4 Proportionals of this sort. 4. That in 4 such Proportionals, as the First to the Fourth; so is the Cube of the First to the Cube of the Second.

As 8 . 1000 :: 512 . 64000.

5. Such Proportionals like Solids, their least Terms are Cubes. 5. That if the Extreams of such 4 Proportionals are like Solids, being reduced to their least Terms they will be Cubes. As 16 . 24 . 36 . 54 . here 16 and 54 are like Solids, because made of 3 sides Proportional, (as $2 \times 4 \times 2 = 16$, $3 \times 6 \times 3 = 54$) reduced to their least Terms, are 8, 12, 18, 27, where 8 and 27 are Cubes.

5ly. The State of the Question duly considered. 6. 5. In *Geometrical Progression*, as before in other Places hath been warned, the Artist must carefully consider the State of the Question, and upon what point the Resolution depends, and when Decimals or Fractions are given or happen in the Work, to work accordingly; and if one Question be included in another, to resolve them orderly: *Operation as the Data requires.* Also when some one of the 3 Principals are not very obvious in the Question propounded, but given under some Occult Phrases; *Principal occult to be discovered.* the skill of the Artist is concerned to discover it, and sometime to dispose the first Person in

Persons to be orderly placed. in the Question to the last Place in the Progression, or such other Term as the State of the Question requireth.

Q. Of a Sum divided to 6 Men. *Example* 1. A certain Sum of Money was to be divided among 6 Men, so as *A* was to have $\frac{1}{3}$ of the whole, *B* $\frac{1}{3}$ of the remainder, and so to every one of the rest, and at last there was left 4 *l.* The Question is, what each Man received, and what was the Sum divided amongst them?

Answer. To which for Answer, upon perusal of the Question there appeareth to be clearly given the third Principal 6, and occultly the least Extream and *Ratio*, (*viz.* the first and fourth Principals) to find the fifth and each Particular Term: Wherefore to make the least Extream plain, and accordingly the *Ratio*, I consider that to the 4 *l.* remaining must be put 2, that it being divided into 3 Parts, and one third Part taken away, no more than 4 may be left, and this third Part of 6 being 2, must be the sixth Man's Portion, and accordingly the least Extream of the *Progression*; which 6 is also soon perceived to be the Remainder before, and consequently but $\frac{2}{3}$ Parts of a former Remain, that of necessity must be 9, and so the fifth Man's Portion found to be 3 the $\frac{1}{3}$ of 9; and 3 in comparison to 2 the sixth Man's Portion, is in the *Ratio Sesquialter*: Wherefore multiplying by the *Ratio* $1\frac{1}{2}$ or otherwise proceeding by the former Propositions, the several Terms and Sum with the Remain added, may be obtained for a full Resolution.

Remains.			*Terms.*	*Persons.*	*Portions.*		
l.	*s.*	*d.*			*l.*	*s.*	*d.*
4	00	00	1	6 . *F*	2	00	00
6	00	00	2	5 . *E*	3	00	00
9	00	00	3	4 . *D*	4	10	00
13	10	00	4	3 . *C*	6	15	00
20	05	00	5	2 . *B*	10	02	06
30	07	06	6	1 . *A*	15	03	09

Sum of the Progression 41 : 11 : 03 Money Received.
Remain added 4 : 00 : 00
Sum to be divided 45 : 11 : 03

Q. Of Alms to 4 Beggars. *Example* 2. A Beggar meeting a Gentleman on the Road, requesteth an Alms: to whom the Gentleman replies, he hath but little Money about him, but if the Beggar will give him as much as he hath, he will give him 6 Pence: whereunto the Beggar at first unwilling, being afterward assured he should thereby be a Gainer did consent, and received his Alms. The Gentleman afterward meeting with the second, third and fourth Beggars, did likewise, and then had left himself no Money at all, having given the last Beggar but 4 Pence: The Question is, what Money he had at first, and what were the Alms he gave?

Answer. *Answ.* He had at first $5\frac{1}{2}$ *d.* so the first Beggar received one half Penny for an Alms, the second a Penny, the third two Pence, and the last as much.

In the stating this Question, as in the former, the third Principal 4 is given plain, the least Extream and *Ratio* more occult; yet the last Beggar's Alms being ascertained to be 4 *d*, it is sufficient to declare them, seeing by the Form of the Question it appeareth to have been doubled, therefore the half of Necessity must be the Remain after the third Beggar had his 6 *d* deducted, which 6 *d.* and 2 *d.* make 8 *d.* that had been doubled of the former Remain. The like is to be understood of the Rest, whereby the *Ratio* of the Alms is found to be double except the last, which is distracted on purpose to make the Question intricate and uncouth: and sometimes such Questions as these receive an easier Resolution by the Rule of false *Position* than by these Rules of *Progression*, though they seem hence to take their Foundation.

Terms of the *Progression*	1 . 2 . 3 . 4
Beggars in their Order	4 . 3 . 2 . 1
Money received by the Beggars	4 . 6 . 6 . 6
Money in the Almoner's Purse, and doubled by the Beggars	2 . 4 . 5 . $5\frac{1}{2}$
Proper Alms received by the Beggars	2 . 2 . 1 . $0\frac{1}{2}$

For $5\frac{1}{2} \times 2 = 11 - 6 = 5 \times 2 = 10 - 6 = 4 \times 2 = 8 - 6 = 2 \times 2 = 4$

6ly. Mean Proportionals helpful to discover the distinct Quantities in mixt Bodies. §. 6. The finding of Mean Proportionals besides many other Uses, is necessary to the discovery of the true and distinct Quantities of each sort in mixed Bodies, and

and reſolveth Queſtions of that Nature, placed by ſome under *Alligation*, but cannot thereby be well reſolved without borrowing help from hence.

Q. Of the Weight of Silver mixt.

Example 1. Let an Ingot of the fineſt Silver, from which a Mark being cut off, be melted with a Mark of Copper inſtead thereof; and afterward a Mark of that mixt Metal cut off, and the Remainder melted with another Mark of Copper: Again, the third time a Mark of this Mixture being cut off, and the Remainder melted with another Mark of Copper; and finding by the Saie, that this laſt Mixture is $8\frac{5}{128}$ Ounces fine: The Queſtion is, how many Marks the Ingot weighed?

Anſwer.

Anſw. 8 Marks; for the fineſt Silver (as the Ingot was before Allay) being 12 Ounces fine, two Mean Proportionals are to be ſought between the ſame and $8\frac{5}{128}$ (the fineneſs of the Ingot after Allay), which to do, firſt $8\frac{5}{128}$ and 12 muſt be reduced to one Denominator, and then leaving the Denominators, the Proportions will reſpect their Numerators only. And as the Differences between the Proportionals reſpect the ſeveral Allays, ſo ſhall the Difference between the 2 greateſt Proportionals, reſpect the greateſt Fineneſs and leaſt Allay; wherefore the Analogy is, as that Difference to 1, ſo is the greateſt Proportional to the Weight deſired.

$8\frac{5}{128}$ and $\frac{12}{1}$ reduced, are $\frac{1029}{128}$ and $\frac{1536}{128}$.

Proportionals . 1029 . 1176 . 1344 . 1536.

$1536 - 1344 = 192$. Then as 192 . 1 :: 1536 . 8 Marks.

Q. Of Wine mixt with Water, how much drawn out at a time.

Example 2. A Merchant hath a Piece of Wine of 128 Gallons, out of which he draweth certain Gallons, and filleth up the Veſſel again with Water; the ſecond Time he draweth out as much as he did at the Firſt, and filleth it up again with Water; and the like he doth the third and fourth Time, and in the end findeth that there was left in the Veſſel $75\frac{1}{32}$ Gallons of Wine, beſides the Water that was put in: how many Gallons was drawn out at a Time?

Anſwer.

Anſw. 16 Gallons: Here the Draught being repeated 4 times, 3 Mean Proportionals are to be gotten, (for they are always to be 1 leſs than the Mixtures) 128 and $75\frac{1}{32}$ then reduced as before, ſhall have their Numerators 2401 and 4096, between which the 3 Proportionals are 2744 . 3136 . 3584. Then becauſe the Inquiry here, is not as in the laſt Queſtion for the Quantity at firſt, but for the Draught out of that Quantity, the Analogy is, As the Denominator to 1, ſo is the Difference between the higheſt Proportionals to the Number ſought.

$75\frac{1}{32}$ and $\frac{128}{1}$ reduced, are $\frac{2401}{32}$ and $\frac{4096}{32}$

Proportionals 2401 . 2744 . 3136 . 3584 . 4096.

$4096 - 3584 = 512$. Then as 32 . 1 :: 512 . 16 Gallons.

Of the Wine drawn out every time.

And in like manner, if the Quantity of Wine drawn out every Time in the 16 Gallons were deſired: by an orderly deduction of the leſſer Proportional out of the next Greater, and dividing the Difference by the common Diviſor 32, the Quotients will determine the Deſire.

			Gall.	
So	$3584 - 3136 = 448$	Therefore 32) 448 (	14 .	2 Draught.
And	$3136 - 2744 = 392$	Therefore 32) 392 (	$12\frac{1}{4}$.	3 Draught.
And	$2744 - 2401 = 343$	Therefore 32) 343 (	$10\frac{23}{32}$.	4 Draught.

If the Price of the Wine be given, to find the Quantity.

And if the Prices of the Wine before and after Mixture had been given, to find the Quantity: (as ſuppoſe a Gallon before mixture were worth 5 *s.* and after Mixture 2 *s.* $\frac{3813}{4096}$ to know how many Gallons in the whole Piece): then after the Mean Proportionals are gotten between the 2 Prices, the Analogy is, As the Difference between the 2 higheſt Prices to the firſt Draught, ſo is the higheſt Price to the whole Quantity.

$2\frac{3813}{4096}$ and $\frac{5}{1}$ reduced, are $\frac{12005}{4096}$ and $\frac{20480}{4096}$

Proportionals . 12005 . 13720 . 15680 . 17920 . 20480.

Prices . $2\frac{3813}{4096}$. $3\frac{179}{512}$. $3\frac{53}{64}$. $4\frac{3}{8}$. 5. and $5 - 4\frac{3}{8} = \frac{5}{8}$

Then as $\frac{5}{8}$. 16 :: 5 . 128 . Gallons in the whole.

Proof of Geometrical Progreſſion.

The ordinary Proof of *Geometrical Progreſſion*, is according to the firſt Example of this Chapter, to place every Term in order, and collect the Sum by common *Addition*; what other Operations the Sections include, carry Evidence of their truth in their Operation, or may be examined one by another, where there are varieties of Reſolution.

CHAP.

CHAP. VI. *Transmutation.*

Transmutation whence, and what it serves for.

GEometrical Progression begat Transmutation, which serveth to shew what Number of Changes may be made by any Number of Persons or other things in their Places or Positions: As to know what Number of Changes in the Sound may be made by 5, 6, 7, 8, or more Bells, or different Notes upon 4, 6, 8, or 10 stringed Instruments, and how often the Gammaut may be varied, &c.

What it is like.

Transmutation, sometime like *Multiplication*, regardeth the Place of the Numbers, or Numbers themselves, to increase the one by the other; and sometime the Place in which the Person or Numbers stands, is not so much respected as the Variety and Multiplicity of Changes.

Changes, the Sorts.

Changes are Simple or Intermixt.

Simple Changes, what.

Simple Changes are (without relation to any Consort) the Number of the several Positions or Places that a Company of Persons, or other things, may be set or placed in order one beside another; and every time some one or other of the Persons or Things changing their Station, is removed into the Place of another, and all of them never found standing alike.

To find such.

To get such a Number, multiply every Number from the Unit successively into each other's Product unto the Number assigned.

Q. Of the Sittings of 7 Scholars.

Example. Seven Scholars taken out of a Free-School to be sent to the University, were to be entertained at some College at Commons for a certain Sum of Money, with two Meals a Day, so long and no longer than that sitting altogether on a Form at every Meal, they might sit diversly, and never the whole 7 to be alike in Situation: The Question is, how long they were to stay there, or how many several Positions or Sittings there might be made by them in an unlike Posture?

Answer.

Answ. 5040 Sittings, which at 2 Meals a Day, amounts to almost 7 Years, fully to 6 Years 330 Days; for 1 multiplied by 2 is 2, and 2 by 3 is 6, and 6 by 4 is 24, and 24 by 5 is 120, and 120 by 6 is 720, and 720 by 7 is 5040.

Sittings. (330
And 5040 / 2 *Meals.* (2520 / 365 (6 Years.

A marvellous Number of Changes by the 24 Letters.

Hence it is no marvel that by 24 Letters there ariseth and is made such variety of Languages in the World, and such a prodigious Number of Words in each Language; seeing the Diversity of Syllables produceth that Effect, by the interchangable placing of Consonants single, double, treble, &c. with and among the Vowels. The Number of simple Changes in 24 by multiplying every Product successively into the next Number, is found to be 620448401733239439360000.

Intermixt Changes, what.

Changes intermixt, or Counterchanges, are the Number of Varieties which a Company may make amongst themselves, part with part, or some of them to match the rest in Consort, and regard their Companions, and not the Places they stand in as the simple Changes did, which changed none of their Company, but the Place they stood in.

To find such.

To get the Number of such a Variety, let the whole Number given be broken into as many Parts as there be Units therein and under; or beside those Parts, set the Complement of the Units that make up the whole Number; then get the Varieties that any of those Parts make to match with one of the Complements, and multiply this Number by the whole Number given.

Varieties like the Numbers in the Table for Extraction of Roots.

The Varieties that the Parts make with 1 of the Complements, are like the Numbers in the Table for Extraction of Roots; for 1 makes no Changes, 2 makes but 2 simple Changes and none intermixt; 3 therefore begins the Dance: And of 3, if any 2 be given to match with the other 1, the Changes will be but 3, because 3 times 1 is 3: But if 1 of 3 be to match with any of the other 2, there will be 6 Changes; for 1 may be joined with the Second, or with the Third, and so the Second with the First or Third, and the Third with the First or Second.

Thus

Thus any Number lacking 1, in opposition to any 1 thereof, can make but so many intermixt Changes as there be Units in the whole Number given: but in opposition to 2, 3, or more, order their Changes accordingly, by multiplying the whole Number into those Tabellary Numbers, only as the given Number lacks 1, take the Numbers in the Table to 1 less than the Number given: As for 4, they must be 3, 3; for 5 the next Numbers, 4, 6, 4; and for 6 the next, 5, 10, 10, 5, &c. that is so many of the Tabellary Numbers as will serve for the Number of Parts.

Example. Suppose in 5 Numbers any 4 of them will find out the other 1, or any 3 of them the other 2, or any 2 of them the other 3, or any 1 of them the other 4: how many intermixt Changes there must be in any or all these Cases is the Question? Example.

For Resolution, 5 disposed with his Parts and Complements, stand thus:

Parts	4 . 3 . 2 . 1
Complements	1 . 2 . 3 . 4

Then 4 to 1 cannot change; therefore 1 multiplied by 5, the Number makes 5, the Variety of Rules or Operations that must be in that Case.

But if any 3 of 5 be given, to find any of the other 2; then any 4 Numbers consorting but 3 together at a time, making 4 Changes, this Tabellary Number 4 is to be multiplied by 5 the whole Number: So shall 20 be the Number of Changes or Rules necessary for the finding any of the other 2 desired; as was before evident both in the Chapters of *Arithmetical* and *Geometrical Progression*.

If 2 of 5 be given, to find any of the other 3: Then because any 2 of 4 associating together will make 6 Changes, this Tabellary Number 6 shall be multiplied by 5 the whole Number, and the Product 30 shall in this Case be the Number of Changes or Rules requisite.

Lastly, If any 1 of 5 will find out the rest, then doth 1 match himself 4 times: This Tabellary Number 4 therefore is to be multiplied by 5, and the Product 20 is the Number of Changes desired; and the whole Process appears thus:

	5
Parts of 5, the whole Number given	4 . 3 . 2 . 1
Complements of the Parts to 5	1 . 2 . 3 . 4
Varieties of the Parts to 1, or Tabellary Numbers	1 . 4 . 6 . 4
Products of the Varieties by the whole Number	5 . 20 . 30 . 20

Hence notice came to be taken, how many Weights were necessary to weigh any aliquot Number of Pounds by: As to weigh any even Number of Pounds between 1 ℔ and 40 ℔; take 4 Weights in triple Proportion continued, as 1, 3, 9, 27, which make up the Total 40; and to weigh thereby 21 ℔, put 9 to the thing weighed, and 3 and 27 to the Counter-poise: For $3+27=30-9=21$, &c. *What Weights will weigh certain even Number of Pounds.*

So to weigh any aliquot Number of Pounds, between 1 ℔ and 121 ℔, take 5 Weights in like Proportion, as 1, 3, 9, 27, 81, which make together 121.

The best Demonstration to prove the Truth of both Sorts of Changes, is to take Letters or Species, and place them orderly in their Places or Counter-places as the Case requires. So the simple Changes in 4 is seen to be 24, and the Intermixt 4, 12, 12, as hereafter followeth. *Proof of* Transmutation.

Simple Changes in 4.

	a . b . c . d	b . a . c . d	c . a . b . d	d . a . b . c
	a . b . d . c	b . a . d . c	c . a . d . b	d . a . c . b
1 . 2 . 3 . 4	a . c . b . d	b . c . a . d	c . b . a . d	d . b . a . c
1 . 2 . 6 . 24	a . c . d . b	b . c . d . a	c . b . d . a	d . b . c . a
	a . d . b . c	b . d . a . c	c . d . a . b	d . c . a . b
	a . d . c . b	b . d . c . a	c . d . b . a	d . c . b . a

Intermixt Changes in 4.

	a.b.c.d	a.b.			b
	a.b.d.c	a.c.	d	a.	c
	a.c.d.b	b.c.			d
	b.c.d.a	a.b.			a
4		a.d.	c	b.	c
Parts —— 3 . 2 . 1		b.d.			d
Complem's. 1 . 2 . 3		a.c.			a
Varieties – 1 . 3 . 3		a.d.	b	c.	b
Products – 4 . 12 . 12		c.d.			d
		b.c.			a
		b.d.	a	d.	b
		c.d.			c

CHAP. VII. *Anatocifm.*

Anatocifm. TO clofe up this Part of continued Proportions, comes in the latter Brood of *Geometrical Progreffion*, a Child more like the Father than the former of *Tranfmutation.*

Whence derived. How taken. Includes both Intereft and Annuities. *Anatocifm*, derived from ἀνὰ and τίκτω, importeth a bringing forth, renewing or increafing, is here taken for the annual Increafe of Intereft or Ufury; and under the Title of *Intereft* and *Annuities* commonly paffes in moft Authors, becaufe Propofitions of both have their Refolutions near of kin to each other, and are proper enough to be joined together.

Intereft what. Principal what. *Intereft* is the Sum reckoned for the Loan or Forbearance of fome Principal Sum lent for a certain Time, according to fome certain Rate, and therefore called *Principal*, becaufe it is the Sum that procreates the Intereft, or from which the Intereft is reckoned.

Intereft the forts. Intereft is twofold, *Simple* and *Compound.*

Simple Intereft what, and how found. *Simple Intereft* is counted from the Principal only; and fo the *Rule of Three*, or *Rule of five Numbers* before feen, being fufficient to find out at any Rate, and for any Time whatfoever, any Portion of *Simple Intereft* required, no further inquiry is to be made thereabouts here.

Compound Intereft, what. *Compound Intereft* is that which is counted from the Principal and Simple Intereft forborn, called alfo *Intereft upon Intereft.*

Annuities, what. Why the Cafes thereof differ from Intereft. *Annuities*, are yearly Paiments, or Annual Rents, paiable Half-yearly or Quarterly: And their Cafes differ fomewhat from thofe of Intereft, becaufe if an Annuity be forborn, the Paiments increafe as well as the Intereft; but if a Sum of Money be lent, that Principal only is to be reftored with the Intereft.

What to be noted in both. In the Queftions and Cafes concerning *Intereft* and *Annuities*, is to be noted;

1. The 5 Principals of a Geom. Prog. to be well known. 1*ft.* That the Learner be well acquainted with the five Principals of *Geometrical Progreffion*, and the way to find any 2 of them by the other 3, as before to be feen in the 5*th* Chapter of this Part.

2. How to conftitute a Progreffion by any Ratio. 2*ly.* How by any fort of *Ratio*, as well as Multiplex exampled in there, a Progreffion may be conftituted, which is to be done by this *Analogy*: As the Antecedent of the *Ratio*, is to the Confequent thereof; fo is the firft Term of a *Progreffion Geometrical*, to the Second; and by confequence fo the Second to the Third, the Third to the Fourth, *&c.*

Examples. As if the *Ratio* were 2 to 5, beginning at 8: Then as 2 . 5 :: 8 . 20 the fecond Term. And as 2 . 5 :: 20 . 50, the third Term, *&c.*

So if the *Ratio* be at 4, 5, 6, 7, or 8 *per Cent.* As fuppofe 8, and the firft Term of the Progreffion be 5; Then as 100 . 108 :: 5 . 5,4. And if of both Examples Progreffions be inftituted, feven Terms thereof will be, as hereunder follow.

1	2 . 5 :: 8 .	20	1	100 . 108 ::	5 .	5,4.
2		20	2			5,4
3		50	3			5,832
4		125	4			6,29856
5		312,5	5			6,8024448
6		781,25	6			7,346640384
7		1953,125	7			7,93437161472

3*ly*. The *Ratio* of any former Term in a Series of such continued Proportionals, unto any of the Terms following, is equal to the *Ratio* of the first Term to the Second, multiplied into it self, according to the Distance of that latter Term from the former. *3. What the Ratio of one Term to another is.*

As the *Ratio* of 5,4 to 6,8024448, which is the Third from it, is as the *Ratio* of 100 to 108 triplicated, or as the Cube of 100 to the Cube of 108. Example.

(C.100) (C.108)
1000000 . 1259712 :: 5,4 . 6,8024448.

4*ly*, It is most convenient, first to reduce the *Ratio* of Interest, so that the Antecedent may be 100 or 1; and to use Decimals for common Fractions and Logarithms for the rest of the Work, because otherwise many laborious Extractions of Roots are required. 4. *Ratio of the Interest, how best to be reduced.*

As if the *Ratio* be of Pence $\overset{d.}{14{,}4}$. or of Shillings $\overset{s.}{1{,}2}$. then for $\overset{l.}{1}$ the *Ratio* will be as 100 to 106, commonly called 6 *per Cent.* or 6 on the 100: For $\overset{d.}{240}$, (the Pence in a Pound Sterling) to $\overset{d.}{254{,}4}$. (the Pence a Pound, and the Interest of that Pound in a Year at the same Rate): Or $\overset{s.}{20}$, to $\overset{s.}{21{,}2}$. is as 100, to 106; and 100 . 106 :: 1 . 1,06 . So is 1 . 1,06 α . β . and β is procreated of the Pension or Paiment α whatsoever it be at that Rate in one whole Year. Example.

5*ly*, If the Paiment be half Yearly or Quarterly, that is, in Days 182,5 . or 91,25 . For β, otherwise *R*, commonly called the Rate when the first Term is an Unit: let the Log. of the Yearly Procreat be multiplied accordingly by $\frac{1}{2}$ or $\frac{1}{4}$ or $\frac{182{,}5}{365}$ or $\frac{91{,}25}{365}$: So the Quotient dividing the Numerator by the Denominator, shall be the Log. of the *Ratio* for half Years or Quarters: or in Decimals take $\sqrt{q}\,1{,}06$. or $\sqrt{qq}\,1{,}06$ instead of β. For it is vulgarly taken amiss, when half the yearly Interest is taken for the half Year, and the Quarter thereof for a Quarter of a Year: And though *Simple Interest* be commonly so taken, yet in *Compound Interest* it is not so, because the Increase is continual from the first beginning to the end of 3 Months, and so to 6 Months, and 12 Months. Then if the Increase in 3 or 6 Months should come to the Quarter or Half of the Whole, it would come to more than the whole at the Years end. *5. To get the Log. of the Rate for the Paiment Yearly, Half-yearly, &c.* *Vulgar Error.*

As let the *Ratio* be as 100 to 106, the Log. of 100 is 2,00000,00000; the Log. of 106 is 2,02530,58653, the Difference between them is 0,02530,58653, and this is the Log. of the Rate for 1 Year: And in like manner the Difference between the Log. of any Principal Money, and the Log. of that Principal, and the Increase or Decrease added together, shall be the Log. of the Rate for 1 Year. And accordingly this Difference multiplied or divided as the Case may require by the Time or Term, be it Years, Months, Weeks or Days, may not unfitly be termed the Log. for the Time, seeing it performs the Office thereof: so for 2 Years it must be multiplied by 2, for 3 Years by 3; if for $\frac{1}{2}$ a Year by $\frac{1}{2}$, for $\frac{1}{4}$ by $\frac{1}{4}$, for a Month by $\frac{1}{12}$, for a Week by $\frac{7}{365}$, for a Day by $\frac{1}{365}$, according to the Time required. Examples.

Rate at 6 l. per Centum.

2,00000,00000 .	Log. of 100 *l.* Principal.
2,02530,58653 .	Log. of 106 . Principal and Interest.
0,02530,58653 .	Difference or Log. of the Rate for 1 Year.
0,01265,29326 .	Log. of the Rate for $\frac{1}{2}$ Year.
0,00632,64663 .	Log. of the Rate for $\frac{1}{4}$ Year.
0,00210,88221 .	Log. of the Rate for a Month.
0,00048,53179 .	Log. of the Rate for a Week.
0,00006,93311 .	Log. of the Rate for a Day.

6*ly*,

6. The Number of Ratio's, and Number of Terms to be well observed.

6*ly*, Becauſe in theſe *Progreſſions*, the Number of *Ratio's* is leſs by 1 than T, the Number of Terms or Paiments, the Number of *Ratio's* ſhall be T—1. And if α be the firſt Paiment and 1, then the Log. of β multiplied by T—1 ſhall be the Log. of ω the laſt Term; and the Log. of β multiplied by T, ſhall be the Log. of $\beta\omega$, that is, of β multiplied into himſelf according to the Number of Paiments, and β—1 ſhall be the Intereſt of the firſt Paiment: But if α be no Paiment but the Principal Stock put out to Intereſt, then ſhall $\beta\omega$ be the laſt Paiment for the Increaſe of α forborn ſo long time.

Example.

As ſuppoſe a *Progreſſion*, whoſe *Ratio* as 1, to 1,06, were inſtituted thus:

α	. 1.
β	. 1,06
βq	. 1,1236
βc	. 1,191016
βqq	. 1,26247696
Z	. 5,63709296

Then if β be the firſt Paiment, the ſame Number ſhall be both of the Rates and Paiments, of which the Fourth ſhall be 1,26247696 for ω, and accordingly T ſhall be 4: but if α be the firſt Paiment, the ſame Number ſhall be both of the Terms and Paiments, of which the Fourth ſhall be 1,191016, for ω, leſs than the other by one Degree, which being multiplied by β, ſhall give the Sum 1,26247696 for $\beta\omega$. and ſo ſhall T. be 5.

And by Logarithms, if α be the firſt Payment.

0,02530,58653	. Log. of β = 1,06
4	. T—1
0,10122,34612	. Log. of ω = 1,26247696
0,02530,58653	. Log. of β = 1,06
5	. T
0,12652,93265	. Log. of $\beta\omega$ = 1,3382255776.
	The 5*th* Term if β be the Firſt.

7. Whence ariſe the 2 firſt Theorems.

7*ly*, Wherefore $\beta\omega$ being procreated of α the Penſion, or 1 *l.* let out at Intereſt according to T, there ariſe hence theſe two Theorems, and by marking *Quantamlibet Summam librarum*, or any Sum of Money imployed with Q℔. may briefly be expreſſed in Species thus:

Species Q℔. for what uſe.

l. *Let out.*

Theorem 1. *Theor.* 1. 1. $\beta\omega$ :: Q℔ . Q℔ with the Gain according to T.

Theorem 2. *Theor.* 2. $\beta\omega$. 1 *l.* :: Q℔ and Gain according to T. To the preſent Value.

Example.

For in the foregoing Inſtance, becauſe 1 to 1,26247696 is as 1 to the 1 *l.* let out, and 0,26247696 the Gain let run for 4 Years: It ſhall be on the contrary, that 1,26247696 to 1. ſhall be as 1 *l.* let out, and the Intereſt let run for 4 Years, to 1 *l.* the preſent Value.

Otherwiſe.

But if β be the firſt Paiment, then ſhall $\beta\omega$ accordingly be ω the laſt Paiment, α being none, or β multiplied into the laſt Paiment where α is the Firſt; and then the Theorems ſhall be thus:

Let out.

Theorem 1. *Theor.* 1. 1 *l.* ω :: Q℔ . Q℔ with the Gain according to T.

Theorem 2. *Theor.* 2. ω. 1 *l.* :: Q℔ and Gain according to T. To the preſent Value.

8. Whence two other Theorems ariſe.

8*ly*, Foraſmuch as $\frac{\beta\omega-\alpha q}{\beta-\alpha}$ that is, $\frac{\beta\omega-1}{\beta-1}$ = Z the Sum of all the Terms of the *Progreſſion* (whoſe laſt is ω), and is therefore the Procreat of 1 *l.* Paiment let run according to the Number of Terms T. Hence ariſe 2 other Theorems.

Theorem 3. *Theor.* 3. $\beta-1$. $\beta\omega-1$ :: Q℔. Penſion let run according to T.
To the Penſions with the Intereſt to be paid in the End.

Theorem 4. *Theor.* 4. $\beta\omega-1$. $\beta-1$ :: Q℔ to come according to T.
To the equivalent Penſion to be paid according to T.

Example.

For in the Inſtance above, ſeeing (β—1 that is) 0,06 dividing ($\beta\omega$—1 that is) 0,3382255776, the Quotient ſhall be (Z that is) 5,63709296, the Sum of the Progreſſion. And becauſe 0,06 to 0,3382255776, is as 1 *l.* Penſion let run 5 Years to 5,63709296, the Penſions and Intereſt for forbearance then due: It ſhall

shall be on the contrary, that 0,3382255776, to 0,06, shall be as 5,63709296, (that is 1 *l.* future according to T) to the equivalent Pension, *viz.* 1 *l.*

And according to the first Paiment so is ω, and $\beta\omega$ to be accompted, and these Theorems to be understood and altered, as was above observed in the two first Theorems.

9thly, Seeing $\beta\omega$ is procreate of 1 *l.* let out according to T: And $\frac{\beta\omega-1}{\beta-1}$ is the Procreate of the Pension of 1 *l.* let run according to T; which in ready Money equalleth the Price of the Pension: *9. Whence arise two other Theorems.*

Say $\beta\omega$. 1 *l.* :: $\frac{\beta\omega-1}{\beta-1}$. $\frac{\beta\omega-1}{\beta-1 \text{ in } \beta\omega}$. Whence therefore according to T, shall $\frac{\beta\omega-1}{\beta-1}$ the Price of the Pension be procreated. Hence also arise 2 Theorems.

Theor. 5. $\beta-1$ in $\beta\omega$. $\beta\omega-1$ :: Qt Pension according to T. To the Price of the same in ready Money. Theorem 5.

Theor. 6. $\beta\omega-1$. $\beta-1$ in $\beta\omega$ :: Qt Present. To the Pension to be bought according to T. Theorem 6.

For in the former Instance, $\beta-1$ (being 0,06) multiplied in $\beta\omega$, (which is 1,3382255776) makes 0,080293534656. And because 0,080293534656 to 0,3382255776 (that is $\beta\omega-1$) is as 1 *l.* Pension in 5 Years to 4,212364— (the Price thereof in ready Money); it shall be on the contrary, that 0,3382255776, to 0,080293534656, shall be as 4,212364— paid present for the Annuity of 1 *l.* bought for 5 Years. Example.

And in these, as in the 2 Theorems last before, let ω and $\beta\omega$ be understood according to the first Paiment: And seeing 1 *l.* is taken for Qt in every of the six Theorems, and Explanation thereof, for Annuities of greater Value, multiply the 4*th* Number, or Number found, by the *Analogy*, by the Number of Pounds employed: Or let the Third in the *Analogy* be the same Number. And for Example-sake, further followeth the Work of a Pension for 10 Years, payable Half-yearly, at the Rate of 6 *per Cent.* that is 1 to 1,06; and T. the Number of Paiments 20. *Farther Example.*

Log. of 1,06, is .	0,02530,58653 .	
	in ½	
	0,01265,29326½ .	Log. of β = 1,0295, &c.
	20 .	T
	0,25305,86530 .	Log. of $\beta\omega$ = 1,7908, &c.
	—2,47129,17111 .	Log. of $\beta-1$ = 0,0295, &c.
	—2,72435,03641 .	Log. of $\beta-1$ in $\beta\omega$.
	—1,89812,15755 .	Log. of $\beta\omega-1$ = 0,7908, &c.
It is therefore	—1,89812,15755 .	
	—2,72435,17111 .	*l.*
	1,17376,98644 .	Log. of the Price 14,922 + for 1 *l.* Pension.
And	—2,72435,17111	
	—1,89812,15755 .	*l.*
	—2,82623,01356 .	Log. of the Pension 0,067023, for 1 *l.* Price.

To these found Logarithms, add the Logarithm of Qt.
Or multiply those Values found by Qt.

10thly, Questions both of Interest and Annuities, as others in *Geometrical Progression*, (upon whom their Resolution depends) may be varied many ways: So that the finding of the five Principals of a *Geometrical Progression*, before seen in the Chapter thereof, and the true knowledg of the six Theorems last above-mentioned, are sufficient to resolve any Propositions concerning them; the Principal Varieties whereof follow, first distinctly, and afterward intermixt; and in both are chiefly wrought by Logarithms, all other Ways for any considerable Time being tedious and troublesome. *10. Questions may be varied many ways. On what the Work depends. Best done by Logarithms.*

Touching Compound-Interest distinctly.

Of Compound Interest, what to be noted.

Compound-Interest respecteth four Things: By any three of which given, the other may be found out.

1. The Sum to be received. What it agreeth and answereth to, and how called.

1. The Sum or Amountment of any principal Sum lent or forborn, for a certain Time or Term, to be received with Interest upon Interest, after a certain Rate: This answereth to ω, or the Second of the five Principals in a *Geometrical Progression*, and agreeth with the first Theorem, and is sometime called the *Profit* or *Loss*.

2. The Sum lent. What it agreeth and answereth to, and how called.

2. The principal Sum lent, or ready Money to be paid for a Sum of Money to be forborn a certain Time or Term, with Interest upon Interest, after a certain Rate: This answereth to α, or the first of the five Principals in a *Geometrical Progression*, and agreeth with the second Theorem, and is sometime called *Abatement* or *Rebatement*, and sometime *Discount*.

3. The Rate. What it answereth to.

3. The Rate, according to which the principal Sum lent or forborn increaseth or decreaseth, as the Question concerneth Profit or Loss. This answereth to R, or the Fourth of the five Principals in a *Geometrical Progression*.

4. The Time or Term. What it answereth to.

4. The Time or Term in which the Increase or Decrease is to be, whether Years, Quarters, Months, Days, or what else. This marked by T, or N, answereth to the Third of the five Principals in a *Geometrical Progression*.

Prop.1. To find the Sum to be paid.

Proposition 1. To know what Sum ought to be paid for a Sum forborn a certain Time, or Term, at any Rate, with Compound Interest.

Q. Of 200 l. forborn 7 Years, what it comes to.

As to know what Sum shall be paid for 200 *l.* forborn 7 Years, with Interest upon Interest, after the Rate of 6 *l.* *per Cent. per Annum:* Or what comes 200 *l.* to in 7 Years, by Interest upon Interest, at the Rate of 6 on the 100?

Answer.

Answ. *l.* 300,726, and somewhat more: For seeing here is given α . T . R . to find ω . that is α . 200 . T . 7 . and R . as 1 to 1,06, the 300,726, *&c.* may be found by the former Rules in *Progression*, only there the *Ratio* was figurate to T—1, but here to T, as by the Decimals appeareth.

By Decimal.

Thus	1,06	R
	1,1236	
	1,191016	
	1,26247696	
	1,3382255776	
	1,418519112256	
	1,50363025899136	
	200	α
	300,72605179827200	ω

By the first Theorem.

And by the first Theorem.

1 . 1,50363025899136 :: 200 . 300,72605179827200.

Or

Or thus the common Way, *By the Common Way.*

2 00	α
1,06	R
1200	
2000	
212,00	1
1,06	
127200	
212000	
224,7200	2
1,06	
13483200	
22472000	
238,203200	3
1,06	
1429219200	
2382032000	T
252,49539200	4
1,06	
151497235200	
252495392000	
267,64511155200	5
1,06	
16058706931200	
26764511552000	
283,703822451200	6
1,06	
1702222934707200	
2837038224512000	
ω 300,72605179827200	7

The Work by Logarithms is both eafy and fpeedy thus :

To the Product of the Log. of the Rate (which as aforefaid is the Difference between the Log. of the Principal Money, and the Log. of the Principal and Simple Intereft in 1 Year) multiplied by the Term or Time (called the *Log. for the Time*) add the Log. of the Sum fo lent or forborn. *By Logarithms.*

	2,00000,00000	. Log. of 100
	2,02530,58653	. Log. of 106
Difference	0,02530,58653	. Log. of 1,06 . R. or the Rate for 1 *l.* for 1 Year.
	7	. T.
Product	0,17714,10571	. Log. of 1,5036, *&c.* or Log. for the Time.
	2,30102,99957	. Log. of 200 . α
Sum	2,47817,10528	. Log. of 300,726, *&c.* ω.

If the fame Sum of 200 *l.* were forborn only for half a Year, then half the Difference or Log. of the Rate fhall be the Log. for the Time, which added to the Log. of 200, fhall be the Log. of 205,91, *&c.* and not full 206 *l.* as is commonly reckoned at *Simple Intereft.* *Example for half a Year.*

0,01265,29326	. Log. for $\frac{1}{2}$ Year.
2,30102,99957	. Log. of 200
2,31368,29283	. Log. of 205,91, *&c.*

If the Sum of the former Example be forborn 3 Years 5 Months and 4 Days: then firft the Log. of the Rate is to be multiplied for the 3 Years by 3, and then by $\frac{5}{12}$, or the Log. of the Month by 5, and then by $\frac{4}{365}$, or the Log. of the Day by 4; and to thefe is to be added the Log. of 200, as before. *Example for 3 Years, 5 Months, and 4 Days.*

0,02530,58653	.	Log. of the Rate in 1 Year.
3	.	Years.
0,07591,75959	.	Log. for 3 Years.
0,01054,41105	.	Log. for 5 Months.
0,00027,73244	.	Log. for 4 Days.
0,08673,90308	.	Log. for the Time.
2,30102,99957	.	Log. of 200.
2,38776,90265	.	Log. of 244,213, &c.

If the Rate be unusual, or the Sum uneven.

If occasion be to accompt the Interest at an unusual Rate, or the Sum for which the Interest be reckoned be an uneven Sum; there is no Difference in the Method of proceeding from the former, but alike easy by Logarithms.

Example.

As if 56 *l.* 13 *s.* 6 *d.* be forborn 4 Years and 8 Months, and Compound Interest be reckoned for the forbearance after the Rate of 5 *l.* 10 *s.* *per Cent.* the Sum to be paid will be *l.* 72,7617, &c.

2,02325,24596	.	Log. of 105,5. or 105 *l.* 10 *s.*
2,00000,00000	.	Log. of 100.
0,02325,24596	.	Log. of 1,055. the Rate.
0,09300,98384	.	Log. for 4 Years.
0,01550,16397	.	Log. for 8 Months, or $\frac{2}{3}$ of a Year.
0,10851,14781	.	Log. for the Time.
1,75339,15288	.	Log. of 56,675 . or 56 *l.* 13 *s.* 6 *d.*
1,86190,30069	.	Log. of 72,7617, &c.

Example in a more unusual Rate.

If the Rate be more unusual, as at 5 *l.* 10 *s.* for 56 *l.* 13 *s.* 6 *d.* *per Annum*, and it be desired to know what 98 *l.* 10 *s.* 6 *d.* would amount to after that Rate in 3 Years an half and 10 Days; the Operation will be in like manner, but otherwise than by the use of Logarithms extraordinary Difficult.

56 : 13 : 6	1,79361,57939	.	Log. of *l.* 62,175, or 62 *l.* 03 *s.* 6 *d.*
5 : 10 : 0	1,75339,15288	.	Log. of 56,675, or 56. 13. 6.
62 : 03 : 6	0,04022,42651	.	Log. of the Rate for 1 Year.
	0,12067,27953	.	Log. for 3 Years.
	0,02011,21325	.	Log. for $\frac{1}{2}$ Year.
	0,00110,20346	.	Log. for 10 Days.
	0,14188,69624	.	Log. for the Time.
	1,99354,64435	.	Log. of 98,525 . or 98 *l.* 10 *s.* 6 *d.*
	2,13543,34059	.	Log. of 136,5945, &c.

Prop. 2. *To find the ready Money.*

Prop. 2. To know what Sum in ready Money ought to be paid for a Sum of Money to be received after a certain Time or Term, by *Compound Interest.*

O.Of l. 300,726 &c. payable 7 Years hence, what now worth.

As to know what *l.* 300,726, &c. to be paid 7 Years hence is worth ready Money, at the Rate of 6 *l.* *per Cent. per Annum.* Or what Sum in 7 Years did amount to *l.* 300,726, &c. by Interest upon Interest, after the Rate of 6 on the 100.

Answer.

Answ. 200 *l.* For this being the Converse of the 1*st Prop.* here is ω . T . R . given to find α, that is, ω . 300,726, &c. T . 7 . and R . as 1 to 1,06, and so as before noted *Prop.* 1. only figurating the *Ratio* to T, may be found by the former Rules in *Progression*, or the second Theorem: For 1,50363025899136 . 1 :: 300,726051798272 . 200. But the work as aforesaid, being most easy by Logarithms, the Rule thereof is thus:

By Progression. By the second Theorem. By Logarithms.

Substract the Log. for the Time out of the Log. of the Sum to be paid.

2,02530,58653	.	Log. of 106.
2,00000,00000	.	Log. of 100.
0,02530,58653	.	Log. of 1,06, the Rate for 1 *l.* in 1 Year.
7	.	Years T.
0,17714,10571	.	Log. for the Time, or 1,50363, &c.
2,47817,10528	.	Log. of 300,726, &c. ω.
2,30102,99957	.	Log of 200 . α.

For other Rates or Times.

Thus whatever Rate or Time of forbearance it be, when the Log. for the Time is gotten, let it be taken from the Log. of the Sum to be paid, if Increase be accompted.

As *l.* 244,213, *&c.* will be payable at 3 Years 5 Months and 4 Days hence; yet it may be received presently, if allowance be made for the Time or Term, after the Rate of *6 per Cent. per Annum:* what will be payable in ready Money? *Example for 3 Years, 5 Months, 4 Days.*

Answ. 200 *l.* For the Log. for the Time 0,08673,90308, taken from the Log. of 244,213, *&c.* will leave the Log. of 200.

0,02530,58653	.	Log. of the Rate in 1 Year.
3	.	Years.
0,07591,75959	.	Log. for 3 Years.
0,01054,41105	.	Log. for 5 Months.
0,00027,73244	.	Log. for 4 Days.
0,08673,90308	.	Log. for the Time.
2,38776,90265	.	Log. of 244,213, &c.
2,30102,99957	.	Log. of 200.

But in all the Questions found in other Authors under this Proposition, if there have been a Loss or Decrease, the Log. for the Time shall be added to the Log. of the Sum of Money not lost. *Example in Loss.*

As 200 *l.* is remaining at the end of 7 Years, by Loss or Discount, after the Rate of *6 per Cent. per Annum:* what was the Sum at the beginning of the 7 Years?

Answ. l. 300,726, &c.

0,02530,58653	.	Log. of the Rate in 1 Year.
7	.	Years.
0,17714,10571	.	Log. for the Time.
2,30102,99957	.	Log. of 200.
2,47817,10528	.	Log. of 300,726, &c.

And because this Proposition is converse to the First, (as before noted) the Resolution of this last Question is done by the first Proposition. And if under the first Proposition such a Question as this had been set, *viz. l.* 300,726, &c. is to be received at the end of 7 Years; but losing the Compound Interest at 6 *per Cent.* it will be paid presently: what shall be paid for the same at present? The Work by this second Proposition, as above, will appear to be 200 *l.* So as Questions like this last are proper to this Proposition, and those like the other to the first Proposition, however transposed in other Books. *This second Proposition converse to the first. What Questions are proper to both.*

Prop. 3. To know after what Rate any Sum hath increased or decreased by Compound Interest in a certain Time or Term. *Prop. 3. To find the Rate.*

As to know after what Rate 200 *l.* in 7 Years will increase to *l.* 300,726, &c. Or suppose 200 *l.* by Interest upon Interest in 7 Years did amount to *l.* 300,726, &c. what was the *Ratio* compared to 100? *Q. Of the Rate by which 200 l. in 7 Years increased to 300,726 &c.* Answer.

Answ. 6 *l. per Cent. per Annum.* And so by the former Rules in *Progression* may be found, here being given α . ω . T . to find R . that is α . 200 . ω. 300,726, &c. and T . 7 . For 200 . 1 :: 300,726, &c. 1,503630, &c. differing from the way there only in this, that here the Number found by the *Analogy*, is the *Ratio* figurate to T, thereto T—1, as before noted in the two precedent Propositions. *By Progression.*

By Logarithms the Work is thus. Divide the Difference of the Logarithms of the two Sums propounded, by the Term or Time given; to the Quotient add the Log. of the intended or supposed Antecedent of the Rate, and the Total shall be the Log. of the Antecedent and Consequent: Then deducting the absolute Number of the Antecedent, from the absolute Number of the other, the Remain shall be the Rate desired. *By Logarithms.*

2,47817,10528	.	Log. of *l.* 300,726, &c. ω.
2,30102,99957	.	Log. of 200. α.
7) 0,17714,10571	.	Difference, or Log. for the Time.
0,02530,58653	.	Quotient, or Log. of the Rate.
2,00000,00000	.	Log. of 100. Antecedent of the Rate.
2,02530,58653	.	Log. of 106, Antecedent and Consequent.
		6. on 100 the Rate.

If the Time be an Half-year, Quarter, &c.

If the Term or Time given, befides the whole Number of Years, be fome odd Parts of a Year, as half a Year, a Quarter, Months, Days, or fuch-like, then reduce them all into one Denomination; and as a Fraction, with 365 (the Days in a Year) *&c.* abbreviate them to their leaft Terms; and thereby, or rather by the Decimal thereof, divide the Log. of the Difference as above.

Example.

As if it were defired to know after what Rate 200 *l.* in 3 Years, 5 Months, and 4 Days, will increafe to *l.*244,213, *&c.* Becaufe the Time reduced into Days will not be abbreviated, but both Terms thereof large, the Decimal is rather to be chofen, which is 3,42762556; by which the Difference divided maketh the Quotient, and fo the Rate as above.

	2,38776,90265	.	Log. of *l.*244,213, &c.
	2,30102,99957	.	Log. of 200.
3,42762556)	0,08673,90308	.	Difference.
	0,02530,58653	.	Quotient.
	2,00000,00000	.	Log. of 100, Antecedent.
	2,02530,58653	.	Log. of 106, Antecedent and Confequent.
			6. Rate.

Prop. 4. To find the Time or Term.

Prop. 4. To know in what Time or Term a Sum of Money, will increafe or decreafe by Compound Intereft, to a Sum propounded according to a certain Rate.

Q. Of the Time in which 200 l. increafed to 300,726, &c.

As to know in what Time 200 *l.* after the Rate of 6 *per Cent. per Annum*, by Intereft upon Intereft, will increafe to *l.* 300,726, *&c.* or when will *l.* 300,726, *&c.* be payable for 200 *l.* lent, after the Rate of 6 *l.* on the 100 for a Year?

Anfwer.

Anfw. At the end of 7 Years after the Loan, as may be found by the former Rules in *Progreffion*, with a little Difference, for here is given α. ω. R. to find T. that is, α. 200. ω. 300,726, *&c.* and R. 1,06. So 200 multiplied by 1,06 till 300,726, *&c.* be produced, the Number of Multiplications 7, is the Number defired; For 1 needs not be added as in *Progreffion.* The Work is fhort by Logarithms, thus.

By Progreffion.

By Logarithms.

Divide the Difference of the Logarithms of both Sums given by the Log. of the *Rate*; and the Quotient fhall be the Number defired.

	2,47817,10528	.	Log. of *l.* 300,726, *&c.* ω
	2,30102,99957	.	Log. of 200. α
	0,17714,10571	.	Difference, or Log. for the Time.
Log. of the Rate	0,02530,58653	(	7 Years T.

If any Remain be on Divifion.

If after the Divifion any Thing remain, divide that Remain by the Log. of the Rate for a Month: And if upon that Divifion any thing remain, divide it by the Log. of the Rate for a Day.

Example.

As 200 *l.* hath increafed to *l.*244,213, *&c.* and I defire to know in what Time it hath fo increafed by Compound Intereft, after the Rate of 6 *per Centum per Annum.*

Anfwer.

Anfw. In 3 Years, 5 Months, and 4 Days: For after the Firft Divifion, which bringeth 3 Years in the Quotient, there is left remaining 1082,14349; this divided by 210,88221, the Log. of the Rate for a Month, bringeth 5 in the Quotient, and leaveth remaining 27,73244; which divided by 6,93311, the Log. of the Rate for a Day, giveth 4 in the Quotient.

2,38776,90265	.	Log. of *l.* 244,213, &c.
2,30102,99957	.	Log. of 200.
0,08673,90308	.	Difference.

(1082 14349 (27 73244

8673,90308 (3 Years . 1082,14349 (5 Months . 27,73244 (4 Days.

2530,58653 210,88221 6,93311

Touching

Touching Annuities distinctly.

Annuities respect 5 Things, by any 3 whereof the other may be found as before in *Progression*. *Of Annuities, what to be noted.*

1. The Sum, or Arrerages of the Rent of a Lease, or Annuity, or Pension forborn a certain Term or Time, to be paid with Interest upon Interest after a certain Rate. This answereth to Z, or the fifth of the 5 Principals in a *Geometrical Progression*, when the Annuity is proposed as the first; otherwise it answereth to ω or the second Principal, when the ready Money or Price is given for the First, and agreeth with the third of the 6 Theorems in this Chapter. *1. The Arrerages. What it agreeth and answereth to.*
2. The Sum or Price, which in ready Money will buy or purchase an Annuity for a certain Time or Term, after a certain Rate by Interest upon Interest; this answereth to α, or the first of the 5 Principals in a *Geometrical Progression*, where the Arrerages are proposed as the Second, and agreeth with the 5*th* of the six Theorems abovementioned. *2. The Price. What it agreeth and answereth to.*
3. The Annuity or Pension that any Sum of ready Money will buy or purchase for a certain Time, after a certain Rate with Interest upon Interest. This answereth to α, or the first of the 5 Principals in a *Geometrical Progression*, where the Arrerages are proposed as the Fifth, and agreeth both with the 4*th* and last of the aforesaid 6 Theorems. *3. The Annuity or Pension. What it agreeth and answereth to.*
4. The Time or Term any Annuity or Pension, after a certain Rate by Compound Interest, may be detained to amount to a Sum propounded; This marked by T or N, answereth to the Third of the 5 Principals of a *Geometrical Progression*. *4. The Time or Term. What it answereth to.*
5. The Rate by which any Annuity or Pension by Compound Interest did increase to a Sum propounded; this answereth to R. or the fourth of the 5 Principals of a *Geometrical Progression*. *5. The Rate. What it answereth to.*

And according to the different State of the Question, under one or other of these Classes, is Resolution attainable by some or other of the Rules proper thereto: For as before in *Progression*, one of these 5 being sought, the Question may be varied 4 several Ways; and where the *Data* are all of those 5 Principals, the Rules there may be used with what alteration is necessary for Fractions and Decimals: otherwise because the Second of these is none of the former, some new Rules will be necessary, wherefore all the Varieties shall be exampled in.

Proposition 1. To know what Sum of Money ought to be paid for the Arrerages of Rent reserved upon a Lease, or for an Annuity or Pension forborn a certain Time or Term, after a certain Rate by Compound Interest. *Prop. 1. To find the Arrerages.*

The *Data* for the Resolution of Questions under this Classis, will be either, *Data under the first Prop.*

2 . 3 . 4 . The Price, Annuity, and Term.
2 . 3 . 5 . The Price, Annuity, and Rate.
2 . 4 . 5 . The Price, Term, and Rate.
or 3 . 4 . 5 . The Annuity, Term, and Rate.

The last being easier than the first, shall be first in Example, and the rest in their retrograde Order. *Of these 4, the last easiest.*

Variety 4. If an Annuity of 5 *l. per Annum* be detained 7 Years: what will the Amount thereof be by Interest upon Interest, after the Rate of 6 *l. per Cent. per Annum?* or what are the Arrerages of an Annuity of 5 *l.* a Year with the Compound Interest, after the Rate of 6 on the 100 for a Year forborn 7 Years? *Variety 4. Q. Of 5 l. per Annum, forborn 7 Years, what it comes to.*

Answ. l. 41,969188249280: for seeing here is given the Annuity, Term and Rate, or α . T . R . 3 of the 5 Principals of a *Geometrical Progression*, to find Z. that is, α . 5 . T . 7 . and R . 6 on the 100, which is as 1 to 1,06 by the Rules in the third Proposition for finding the 5*th* Principal there, 41,969, &c. is found thus: *Answer. By Progression.*

	5 α 1,06 R	1	
	30 50		
1	5,30 1,06	2	
	3180 5300		
2	5,6180 1,06	3	
	337080 561800		
3	5,955080 1,06	4	
	35730480 59550800		
4	6,31238480 1,06	5	
	3787430880 6312384800		
5	6,6911278880 1,06	6	
	401467673280 669112788800		
6	7,092595561280 1,06	7	T
	42555573367680 7092595561280 0		
7	7,51815129495680	8	

Collection of the Terms.

5,
5,30
5,6180
5,955080
6,31238480
6,6911278880
7,092595561280

Z 41,969188249280

$$,06 \Big) \frac{5\ \alpha}{2,51815129495680} (41,969188249280.$$

Otherwise. Otherwise by figurating the *Ratio* as in *Progression* also mentioned, but different herein, that here an Unit the Antecedent of the *Ratio* is to be added to the Sum, as,

1,
1,06
1,1236
1,191016
1,26247696
1,3382255776
1,418519112256
1,50363025899136

Sum of the first 7 Terms with 1, 8,3938376498 56
5 α Annuity.
41,969188249280 Z Arrerages.

And the third Theorem thus;

By the third Theorem. 0,06 . 0,50363025899136 :: 1 . 8,3938376498 56
5 α
41,969188249280 Z

By Logarithms. But as before in Interest, so in Annuities, the Work is easy and speedy in Logarithms, thus;

To

To the Log. for the Time, add the Log. of so much Principal Money as at Simple Interest in 1 Year will raise such an Annuity, and the Sum will be the Log. of that Principal Money, and the Arrerages; then substract the Principal out thereof, and the Residue is the Sum desired.

The Principal Money that at Simple Interest, after any Rate, will raise the Annuity propounded, is gotten either by the *Golden Rule direct* without, or rather with Logarithms, as in the former Instance. If 6 *l.* require 100 *l.* Principal to raise the same, then shall 5 *l.* require 83 *l.* 6 *s.* 8 *d.*

As 6 . 100 :: 5 . 83 : 6 : 8.
Or, 2,00000,00000 . Log. of 100
0,69897,00043 . Log. of 5
2,69897,00043 . Log. of 500
0,77815,12504 . Log. of 6
1,92081,87539 . Log. of 83,33333, &c.

Then the rest of the Resolution is thus:

2,02530,58653 . Log. of 106
2,00000,00000 . Log. of 100
0,02530,58653 . Log. of 1,06 . Rate . R or β
7 . Years . T
0,17714,10571 . Log. of 1,50363, &c. Time or $\beta\omega$
1,92081,87539 . Log. of 83,33333, &c. Principal in 1 Year, to raise 5 *l.* at Simple Interest.
2,09795,98110 . Log. of 125,30251, &c. Principal & Arrerages.
41,96918, &c. Z . Arrerages.

Or if the Log. of the Rate lacking 1, be taken from the Log. for the Time lacking 1, and to the Remain the Log. of the Annuity be added, the Sum will be the Log. of the Arrerages desired. *Otherwise.*

—1,70211,18155 . Log. of 0,50363, &c. Time—1. or $\beta\omega$—1
—2,77815,12504 . Log. of 0,06 Rate—1. or β—1
0,92396,05651 . Log. of 8,39383, &c.
0,69897,00043 . Log. of 5 α . Annuity.
1,62293,05694 . Log. of 41,96918, &c. Z . Arrerages.

If the Rents or Annuities be payable Half-yearly or Quarterly, as most commonly they are; then after the Principal Money that will raise such a Paiment is gotten, the rest of the Work differs nothing from the former. *If the Paiments by Half-yearly or Quarterly.*

To get this Principal Money, let the Log. of the Rate be halfed or quartered, according as the Rent or Annuity becomes payable; and for this Half or Quarter, get the *Geodetical* or Decimal as of another Log. Or commonly thus, enter the Table of Logarithms next after the Antecedent of the *Ratio* (neglecting the Index) with the halfed or quartered Log. till the next lesser Log. to that propounded be found; and there by the Difference of the next Lesser to the next Greater, and the next Lesser to the propounded Log. the *Geodeticals* or Decimals sought may be found, as in the Chapter of the *Reduction of Logarithms* was taught. And to the left Hand of this *Geodetical* or Decimal thus found out, is to be prefixed the absolute Number of the lesser Log. in the Table exceeding the Antecedent of the *Ratio* lacking the same Antecedent: And this shall be the first Number of the *Rule of Three*, to find the Principal Money required, to which the Antecedent shall be the Second, and the Paiment propounded the Third: Or if Operation be made by Logarithms, the Log. of that *Geodetical* or Decimal, with the absolute Number prefixed as aforesaid, shall be the Log. to be substracted from the Sum of the other two Logarithms. *How to get the Principal Mony.*

Example. The Rent of a Lease of 5 *l. per Annum*, payable Half-yearly, is in Arrear 7 Years: what do the Arrerages thereof amount to, after the Rate of 6 *l. per Cent. per Annum* by Compound Interest? *Q. Of 5 l. per Ann. payable Half-yearly, what the Arrears in 7 Years.*

Answ. *l.* 42,5873, &c. For here the Log. of the Rate as before being 0,02530,58653 (because the Paiments are Half-yearly) is to be halfed; and with this half, which is 0,01265,29326, if the Table of Logarithms be entred *Answer.*

next after 100 (the Antecedent of the *Ratio*) it will be found to fall between the Logarithms of 102 and 103, and to be more than the Log. of 102, the Index neglected 0,00405,27608: If therefore Ciphers be adjoined as many as ſhall be neceſſary, and Diviſion made by 423,70529 (the Difference between the Log. of 102 and 103) the Decimal 9565, &c. will be gotten to be placed on the right Hand of 2, the Abſolute Number exceeding the Antecedent of the *Ratio*; and having gotten the Decimal, the Shillings, Pence, &c. therein, are ſoon found as before in *Reduction of Decimals.* If otherwiſe, the Learner be more expert at Common Fractions, then the Difference between the leſſer Log. found as aforeſaid, and the Log. propounded, multiplied by 20, and divided by the Difference between the leſſer and greater Logarithms found in the Table, giveth the Quotient in Shillings; the Remain of this Diviſion, if any, multiplied by 12, and divided by the former Diviſor, giveth the Quotient in Pence, &c.

2,02530,58653	. Log. of 106
2,00000,00000	. Log. of 100
0,02530,58653	. Log. of 1,06. Difference.
2) 0,01265,29326	. Log. of 1,029565, &c. Half.
2,00860,01718	. Log. of 102
2,01283,72247	. Log. of 103
0,00405,27608	. Difference of the Half and Log. of 102.
0,00423,70529	. Difference between the Log. of 102 and 103.

(1970
213822(1
2756054 7(1
2394131 956(5
405,27608,0000,&c.(,9565,&c.
423,7052 9999
423,705 222
423,70 55
423,70

2 prefixed is *l.* 2,9565,&c.

		20
s.	19	1300
		12
		2600
	1	300
d.	1	5600
		4
q.	2	2400

Or for the *Geodetical* thus:

405,27608
20
8105,52160

(551210
3868 4687(9
8105,52160(19*s.*
423,705299
423,7052

5512109
12
66145308

(23774779
661 45308(1 *d.*
423,70529

This Principal Money found (omitting the Remain of the firſt Diviſion as inconſiderable) the reſt of the Reſolution to the laſt foregoing Queſtion, is as followeth.

As 2,9565 . 100 :: 2,5 . 84,5594, &c. by Decimals.

Thus	2,00000,00000	. Log. of 100. by Logarithms.
	0,39794,00087	. Log. of 2,5.
	2,39794,00087	. Log. of 250.
	0,47077,78833	. Log. of 2,9565 . or 2 *l.* 19 *s.* 1½ *d.*
	1,92716,21254	. Log. of 84,5594, &c.
	0,17714,10571	. Log. for the Time 14 half Years, or 7 Years.
	2,10430,31825	. Log. of 127,1461, &c.
		42,586 &c. Z.

Or thus	—1,70211,18155	. Log. of 0,50363, &c. Time—1 . or $\beta\omega$—1.
	—2,47077,78833	. Log. of 0,029565, &c. Rate—1 . or β—1.
	1,23133,39322	. Log. of 17,03467, &c.
	0,39794,00087	. Log. of 2,5 α .
	1,62927,39409	. Log. of 42,586, &c. Z .

As

Unusual Rates and uneven Times, as in Interest.

As well in Annuities as before in Interest, the Log. of the Rate if unusual, and the Log. for the Time if odd Months, Days, *&c.* once gotten, the rest of the Work is in like manner, so that Examples thereof need not be added here.

Variety 3. *Q. Of an Annuity sold, what the Arrears.*

Variety 3. If an Annuity to endure 7 Years be sold for *l.* 27,91190719813863, *&c.* ready Money: what will the Arrerages thereof being forborn all the Term amount to, at the Rate of 6 *l. per Cent. per Annum*, by compound Interest?

Answer. *By the first Theorem. By Progression. By Logarithms.*

Answ. l. 41,96918824928o. Here being given the Price, the Term and Rate, because the Price or ready Money propounded, is by the first Proposition of Interest as α and Z, the Arrerages sought as ω to α, Resolution may be had by the first Theorem, seeing the Amountment of a Sum of Money, purchasing an Annuity in any Number of Years by Interest upon Interest, shall equal the Arrerages of that Annuity so long detained: And because this variety gives α . T . R . to find ω . by the Rules in the first Proposition, for finding the second Principal of a *Geometrical Progression*, the said 41,969, *&c.* may thereby be obtained, only differing in figurating the Rate as before noted in Interest, but briefly by Logarithms thus.

Add the Log. for the Time, to the Log. of the Price or ready Money, and the Sum shall be the Log. of the Arrerages desired.

1,44578,95123	Log. of 27,9119, *&c.* Price.
0,17714,10571	Log. of 1,5036, *&c.* Time as before.
1,62293,05694	Log. of 41,9691, *&c.* Arrerages.

Variery 2. *Q. Of Arrerages of* 5 *l. per Ann. valued ready Money.* Answer. *By Decimals.*

Variety 2. The Arrerages of an Annuity of 5 *l. per Annum*, after the Rate of 6 *l. per Cent. per Annum* by Compound Interest, are valued to be worth *l.* 27,9119, *&c.* in ready Money: how much do those Arrerages amount to?

Answ. l. 41,9691, *&c.* as before: here are given the Price, the Annuity, and the Rate; so the Work without Logarithms is thus.

Get (as before under the fourth *Variety*) the Principal Money, that at Simple Interest after the Rate propounded, will raise such an Annuity; from this Number take the given Price, and by the Remainder divide the Number so gotten as aforesaid, then multiply this Quotient by the Price, and the Product is the desired Arrerages.

But to avoid those long and tedious Multiplications and Divisions that are in Decimals, Logarithms are used thus.

By Logarithms.

After the Principal Money, that at Simple Interest will raise the Yearly Annuity propounded, is gotten, taken there from the given Price, and the Log. of the Remainder out of the Log. of that Principal, then to the remaining Log. add the Log. of the Price; and the Sum is the Log. of the Arrerages.

1,92081,87539	Log. of	*l.* 83,33333, *&c.* Principal, as before.
		27,91190, *&c.* Price.
1,74367,76968	Log. of	55,4214, *&c.* Remainer.
0,17714,10571	Log. of	1,5036, *&c.* Time.
1,44578,95123	Log. of	27,9119, *&c.* Price.
1,62293,05694	Log. of	41,9691, *&c.* Arrerages.

Variety 1. *Q. Of* 5 *l. per Ann. for* 7 *Years bought, what the Arrears.* Answer. *A difficult Proposition. By Progression.*

Variety 1. An Annuity of 5 *l. per Annum* for 7 Years, was purchased for *l.* 27,9119, *&c.* ready Money: how much was the Arrerages reckoned to amount to by Compound Interest, if all had been let alone till the end of the Term?

Answ. l. 41,9691, *&c.* as before: The Rate being a principal Key in this Work, and none of the *Data* in this Variety, but the Price, Annuity, and Term; the Log. for the Time (being as aforesaid, the Log. of the Rate multiplied by the Term) cannot well be had. And because the Price is to be multiplied by the Rate figurate according to the Term, or their Logarithms added together; it is a most difficult Proposition for a young Scholar to resolve such a Question by the former Rules. But it is to be considered, seeing 5 is the first Term of one *Progression*, and 27,9119, *&c.* the first Term of another, which shall have the *Ratio* and Number of Terms alike to the first *Progression* that beginneth with 5, and the Number of Terms in both must be 7, and the Arrerages sought must be the Sum of all the Terms beginning with 5, and the last Term only of the other; by necessary Consequence 27,9119, *&c.* must be multiplied by the seventh Power of some Root, which shall make the seventh Term of that Progression equal to the Total of all the 7 Terms of the Progression that beginneth with 5: And because in this Example no other Number can do this but 1,50363025899136, which is

By Logarithms. the 7*th* Power of the Root 1,06, therefore 27,91190, *&c.* multiplied thereby, or their Logarithms added together, the Product of their Multiplications answering to the Sum of their Logarithms will be 41,9691824928, which Number shall be Z, if 5 be α, and shall be ω if 27,911907198, *&c.* be α the Terms of both 7, and the *Ratio* 1,06. This may a little illustrate the Matter, but the best way is to proceed as above in the second *Variety.*

1,44578,95123	.	Log. of 27,91190, *&c.* Price.
0,17714,10571	.	Log. of 1,50363, *&c.* Time.
1,62293,05694	.	Log. of 41,96918, *&c.* Arrerages.

Prop. 2. To find the Price. *Proposition* 2. To know what an Annuity or Pension for a certain Time or Term, after a certain Rate by Compound Interest is worth, to buy or sell in ready Money.

Data under the second Prop. The *Data* for the Resolution of Questions under this Classis will be either,

1 . 3 . 4 . The Arrerages, Annuity and Term.
1 . 3 . 5 . The Arrerages, Annuity and Rate.
1 . 4 . 5 . The Arrerages, Term and Rate.
Or 3 . 4 . 5 . The Annuity, Term and Rate.

These set as the former. The Varieties here are set as the former in their retrograde Order.

Variety 4. Q. *Of 5 l. per An. for 7 Years, what worth.* *Variety* 4. What is an Annuity of 5 *l.* a Year for 7 Years, worth in ready Money at 6 *l. per Cent. per Annum* by Compound Interest?

By the 5th Theorem. *Answ. l.* 27,91190, *&c.* For the Annuity, Term and Rate being given, and seeing in this Example $\beta\omega$ is 1,5036302589136 and $\beta-1$ is 0,06, according to the 5*th* Theorem it shall be, That,

$\beta-1$ in $\beta\omega$		$\beta\omega-1$		*Ann.*		*Price.*
0,090217815394816	.	0,5036302589136	::	5	.	27,91190, *&c.*

By Progression. And because by the *Data* α . 5 . T . 7 . and R . 1,06 . Z may be found according to the third Proposition for finding the 5*th* Principal of a *Geometrical Progression*, and the 4*th* Variety of the first Proposition here; then Z which will be 41,9691, *&c.* being divided by R. figurate to T. which is 1,50363, *&c.* giveth in the Quotient 27,91190, *&c.* as before.

By Logarithms. But the work is most expeditious by Logarithms thus: Out of the Log. of the Principal Money, which at Simple Interest will in 1 Year raise such an Annuity after the Rate propounded, substract the Log. for the Time, and the absolute Number of the remaining Log. out of that Principal.

1,92081,87539	.	Log. of *l.* 83,33333, *&c.* as before *Prop.* 1.
0,17714,10571	.	Log. for the Time there also.
1,74367,76968	.	Log. of 55,4214, *&c.*
		27,9119, *&c.* Price.

Otherwise. Or if from the Log. of the Annuity, added to the Log. for the Time lacking 1, be taken the Log. of the Rate lacking 1, added to the Log. for the Time; the Remain shall be the Log. of the desired Number, agreeable to the Work of the 5*th* Theorem above-mentioned.

0,69897,00043	.	Log. of 5 . Annuity α.
−1,70211,18155	.	Log. of 0,50363, *&c.* Time−1 . or $\beta\omega-1$.
0,40108,18198	.	Sum . $\beta\omega-1$ in α.
−2,77815,12504	.	Log. of 0,06 . Rate−1 . or $\beta-1$.
0,17714,10571	.	Log. of 1,50363, *&c.* Time or $\beta\omega$.
−2,95529,23075	.	Sum $\beta-1$ in $\beta\omega$.
1,44578,95123	.	Log. of 27,91190, *&c.* Price.

Otherwise. Also if the Arrerages be first found, the Log. for the Time taken out thereof, shall leave the Log. of the Price remaining.

1,62293,05694	.	Log. of 41,9691, *&c.* Arrerages, as before *Prop.* 1.
0,17714,10571	.	Log. for the Time.
1,44578,95123	.	Log. of 27,9119, *&c.* Price.

If the Paiment be Half-yearly. Example. If this Annuity had been payable half Yearly, then the Principal Money, as appeareth under the former Proposition, being *l.* 84,5594, *&c.* the Price will be augmented to *l.* 28,32258, *&c.* as by the first work of Logarithms (most used because the shortest) here appeareth.

1,

1,92716,21254 . Log. of *l.* 84,55944, *&c.* Principal.
0,17714,10571 . Log. for the Time.
1,75002,10683 . Log. of 56,23686, *&c.*
28,32258, *&c.* Price.

And if the Paiment be Quarterly, when the Principal Money is had that will raise such a Quarterly Paiment at Simple Interest, the rest of the Work is alike. *If Quarterly.*

Variety 3. The Arrears of an Annuity detained 7 Years after the Rate of 6 *l. per. Cent. per Annum* by Compound Interest, do amount to 41,969188, *&c.* what is it worth in ready Money? Variety 3. Q. *Of Arrears in 7 Years, what worth.*

Answ. l. 27,91190, *&c.* The *Data* here being the Arrerages, Term and Rate; and because the Arrerages or Z may be accompted ω, or the last Term of a *Geometrical Progression*, when the Price which is here the Number sought shall be α or the first Term; and so by the Rules in the first Proposition, for finding the first Principal of a *Geometrical Progression*, the *Ratio* being figurate to T as aforesaid, may 27,91190, *&c.* be found by ω . 41,969, *&c.* T . 7 . and R . 1,06 . the *Data*: And this is also agreeable to the second Theorem. Answer. *By Progression. By the second Theorem.*

Wherefore by Logarithms, if the Log. for the Time be taken from the Log. of the Arrerages; the Remain shall be the Log. of the ready Money or Price desired. *By Logarithms.*

1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
0,17714,10571 . Log. for the Time.
1,44578,95123 . Log. of 27,9119, *&c.* Price.

Variety 2. If the Arrerages of an Annuity of 5 *l. per. Annum*, after the Rate of 6 *l. per. Cent. per Annum*, come to *l.* 41,96918, *&c.* what ready Money will purchase the same? Variety 2. Q. *Of Arrears of* 5 *l.* per Ann. *what worth.*

Answ. l. 27,9119, *&c.* The *Data* here are the Arrerages, Annuity and Rate; so the Work without Logarithms is thus: Answer.

Get (as before) the Principal Money, that at Simple Interest after the Rate propounded will raise such an Annuity, to this add the Arrerages, and divide the Sum by the Principal, then by the Quotient divide the Arrerages, and this last Quotient is the Price desired. *By Decimals.*

Accordingly the Rule of working by Logarithms is framed for avoiding the tedious Multiplications and Divisions thus: *By Logarithms.*

After the Principal Money, that at Simple Interest after the Rate propounded will raise the Annuity, is found and added to the Arrerages, from the Log. of the Sum take the Log. of that Principal Money, and take the remaining Log. from the Log. of the Arrerages.

1,92081,87539 . Log. of *l.* 83,3333, *&c.* Principal.
41,9691, *&c.* Arrerages.
2,09795,98110 . Log. of 125,3024, *&c.* Sum.
0,17714,10571 . Log. for the Time.
1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
1,44578,95123 . Log. of 27,9119, *&c.* Price.

Variety 1. How much ready Money will purchase an Annuity of 5 Pounds *per Annum* for seven Years, whose Arrerages forborn so long will amount to *l.* 41,96918, *&c.* Variety 1. Q. *Of Arrears of* 5 *l.* per Ann. 7 *Years, what worth.*

Answ. l. 27,91190, *&c.* Here are given the Arrerages, Annuity and Term: And because the Arrerages are as Z in a *Geometrical Progression*, when the Annuity is α, by α . 5 . T . 7 . & Z . 41,9691, *&c.* the Rate may be found according to the third Rule in *Progression Geometrical*, for finding the 4*th* Principal; but with some Variation, because of the Decimal in the *Data* here, (the former being fitted for Integers): For by reason of the Retrogression of Figurate Decimals towards the right Hand, contrary to Integers which increase to the Left, it is very difficult to find out the highest Figurate Decimal (according to that Rule) to be taken out of the Quotient of Z divided by α, seeing greater Numbers must be left than those taken away. Yet inasmuch as the Quotient of Z divided by α, both here and there, is the Sum of a *Progression*, whose first Term or α is 1, and so consequently is 1, and the Sum of all the Terms of the *Ratio*, multiplied into it self figurately according to the Number of Terms lacking 1, the Contrariety lies Answer. *By Progression. A difficult Proposition.*

only in this, That in Integers from the Quotient must be taken the highest Power contained therein, whose Index was T—1: in Decimals to the Quotient must be added such a Number as will make the Quotient a Power according to T, lacking 1, (in the left-hand Place); and instead there of *Addition*, let this Number found in the Quotient be multiplied by one or more of the Digits, till the Power desired be produced; and the Number so multiplying, shall be the Decimal Root to be added to the Antecedent of the *Ratio*, and the Antecedent shall always be the Root of 1, and so many Ciphers as there be Decimals.

Wherefore dividing 41,969188249280 (or Z) by 5 (that is α) the Quotient is 8,393837649856: So shall this Number be the Sum of a Progression beginning at 1, and the Arrears of 1 *l.* Annuity let run 7 Years, and contain therein a Zenzicube Number, whose Index is 6, (or T—1) but must be exalted to T, (that is, to a Power whose Index is 7) lacking only the Left-hand Unit. And because the Decimals in the Quotient are 12, and the Number of Terms of that Power but 6, it is easy to discern the Root was Seconds, and that the 7*th* Power in Decimal Seconds with 1 Integer, cannot have less than 15 Places. Now if 8,393837649856 be multiplied by 2″. 3″. 4″. 5″. or any other Digit except 6″, and 1 prefixed to the left Hand, the Product will be no qqc, or Power of the 7*th* Quantity, but multiplied by 6″, produceth ,503630258991136, which with 1 prefixed, and no other, is the Number sought, that is, 1,503630258991136, the second Sursolid, or 7*th* Power of the Root 1,06, to which the Antecedent is 1,00, the √qqc of 1,00000000000000, or which is all one therewith, the √cc of 1,000000000000.

By Logarithms. Thus then having found the *Ratio* thereby, with the other *Data*, the Price may be found by any of the other 3 Varieties for this second Proposition. And seeing this Figural Power found out, is always the Rate multiplied into it self according to T, if Z be divided thereby, the Quotient will be the Price: And accordingly the Log. thereof taken from the Log. of Z, will leave remaining the Log. of the ready Money, or Price desired. So as upon the whole Matter it is most easy to proceed, as in the second Variety above.

1,62293,05694	. Log. of *l.* 41,969188, *&c.*	Z.
0,17714,10571	. Log. of 1,503630, *&c.*	Time.
1,44578,95123	. Log. of 27,911907, *&c.*	Price.

Prop. 3. To find the Annuity. *Proposition* 3. To know what Annuity or Pension any Sum of ready Money will buy, for a certain Time or Term, after a certain Rate, by Compound Interest?

Data under the third Prop. The *Data* for the Resolution of Questions under this Classis, will be either,

1 . 2 . 4 . The Arrerages, Price and Term.
1 . 2 . 5 . The Arrerages, Price and Rate.
1 . 4 . 5 . The Arrerages, Term and Rate.
Or 2 . 4 . 5 . The Price, Term and Rate.

How set. The Varieties of this Classis being alike easy, are set in their proper Order.

Variety 1. Q. Of Arrears for 7 Years, and Price, what the Annuity. *Variety* 1. An Annuity run in Arrears for 7 Years, will come to *l.* 41,96918, *&c.* and may be bought for *l.* 27,91190, *&c.* what was the Annuity?

Answer. *Answ.* 5 *l.* a Year: For seeing the Arrerages of every Annuity, divided by the Price or ready Money, giveth in the Quotient the *Ratio* figurate according to the Number of Terms; and here being given the Arrerages, Price and Term, from the Quotient may be extracted the Septupled Root: And then by the *Analogy*, according to the 4*th* or 6*th* Theorems, 5 may be found. For the Quotient *By the 4th and 6th Theorems.* of 41,969188, *&c.* divided by 27,91190, *&c.* being 1,503630258991136, and the Septupled Root thereof 1,06, it shall by the

	ϐω—1	ϐ—1	*Arrerages.*	*Annuity.*
4th Theor.	0,503630258991136 .	0,06 ::	41,969188249280 .	5 .
	ϐω—1	ϐ—1 in ϐω	*Price.*	*Annuity.*
6th Theor.	0,503630258991136 .	0,0902178155394816 ::	27,9119, *&c* .	5 .

By Logarithms. And accordingly by Logarithms, take the Log. of the Price from the Log. of the Arrerages, the Remainder shall be the Log. for the Time; that is, the Log. of the Rate multiplied by the Term, which divided by the Term, shall give in the Quotient the Log. of the Rate. And then the Log. of the Rate lacking 1, added to the Log. of the Arrears; and from thence the Log. for the Time lacking 1 substracted, leaves remaining the Log. of the Annuity: Or else the Log. of the

the Rate lacking 1, and the Log. for the Time added to the Log. of the ready Money; and from thence the Log. for the Time lacking 1 substracted, shall leave remaining the Log. of the Annuity.

	1,62293,05694	. Log. of 41,96918, &c. Arrerages.
	1,44578,95123	. Log. of 27,91190, &c. Price.
	0,17714,10571	. Log. for the Time 1,50363, &c.
7)	0,02530,58653	. Log. of the Rate 1,06.
The former way	—1,62293,05694	. Log. of 41,96918, &c. Arrerages.
	—2,77815,12504	. Log. of 0,06 R—1.
	0,40108,18198	. Sum.
	—1,70211,18155	. Log. of 0,50363, &c. Time—1.
	0,69897,00043	. Log. of 5 Annuity.
The latter way	—2,77815,12504	. Log. of 0,06 R—1.
	0,17714,10571	. Log. of 1,50363, &c. Time.
	1,44578,95123	. Log. of 27,91190, &c. Price.
	0,40108,18198	. Sum.
	—1,70211,18155	. Log. of 0,50363, &c. Time—1.
	0,69897,00043	. Log. of 5. Annuity.

Variety 2. The Arrears of an Annuity to be forborn, at the Rate of 6 *l. per Cent. per Annum*, by Compound Interest, amounting to *l.* 41,969188, &c. are sold for *l.* 27,91190, &c. what was the Annuity? Variety 2. *Q. Of Arrears sold, what the Annuity.*

Answ. 5 *l.* a Year as before: Here being given the Arrerages, Price and Rate, the Arrerages 41,9691, &c. divided by 27,9119, &c. as in the last Variety, giveth the Power of the Root figurate according to T, as aforesaid in the Quotient, that is 1,50363, &c. and then Resolution may be had, as in the first Variety of this third Proposition, according to the Theorems, or by Logarithms. Answer. *By the Theorems and Logarithms.*

	1,62293,05694	. Log. of 41,9691, &c. Arrerages.
	1,44578,95123	. Log. of 27,9119, &c. Price.
	0,17714,10571	. Log. of 1,5036, &c. Time.
The former way	1,62293,05694	. Log. of 41,9691, &c. Arrerages.
	—2,77815,12504	. Log. of 0,06 R—1.
	0,40108,18198	. Sum.
	—1,70211,18155	. Log. of 0,5036, &c. Time—1.
	0,69897,00043	. Log. of 5. Annuity.
The latter way	—2,77815,12504	. Log. of 0,06. R—1.
	0,17714,10571	. Log. of 1,5036, &c. Time.
	1,44578,95123	. Log. of 27,9119, &c. Price.
	0,40108,18198	. Sum.
	—1,70211,18155	. Log. of 0,5036, &c. Time—1.
	0,69897,00043	. Log. of 5. Annuity.

Variety 3. An Annuity detained 7 Years, with the Compound Interest, after the Rate of 6 *l. per Cent. per Annum*, did amount to *l.* 41,9691, &c. how much was the Annuity? Variety 3. *Q. Of Arrears in 7 Years, what the Annuity.*

Answ. 5 *l.* a Year. Seeing the *Data* here being the Arrerages, Term and Rate answering to T. R. and Z. of a *Geometrical Progression*, and the Annuity sought, as α the first Term, that is T. 7. R. 1,06, and Z. 41,9691, &c. the Rules in the 4*th* Proposition for finding the first Principal there, will give Resolution here. And seeing the Rate septupled according to T, is 1,503630258991 36, by the 4*th* Theorem; Answer. *By Progression.* *By the 4th Theorem.*

℞ω—1	℞—1	*Arrerages.*	*Annuity.*
0,50363025899136 .	0,06 ::	41,969188249280 .	5

And accordingly by Logarithms: If from the Log. of the Arrerages added to the Log. of the Rate lacking 1, be taken the Log. for the Time lacking 1, the Remainder shall be the Log. of the Annuity. *By Logarithms.*

1,

1,62293,05694 .	Log. of 41,96918, &c.		Arrerages.
—2,77815,12504 .	Log. of 0,06		R—1.
0,40108,18198 .	Sum.		
—1,70211,18155 .	Log. of 0,50363, &c.		Time—1.
0,69897,00043 .	Log. of 5.		Annuity.

Otherwise. And because the Arrears of any Annuity, divided by the Annuity it self, always giveth in the Quotient the Arrears of 1 *l.* Annuity let run according to T: And contrary, the Arrears of 1 *l.* Annuity for the Term propounded, dividing the Arrears given, shall be the Annuity, and the Difference of their Logarithms accordingly: If then by the first Proposition be gotten the Arrears of 1 *l.* Annuity forborn for the Term given, let the Log. thereof be taken from the Log. of the Arrerages propounded, and the Remainder is the Log. of the Annuity desired.

The Principal Money to raise 1 *l.* Annuity at Simple Interest, after the Rate propounded by the 4*th* Variety of the first Proposition, is found to be 16 *l.* 13 *s.* 4 *d.* and in Decimals 16,6666, &c. the Residue of the Work follows.

1,22184,87496 . Log. of *l.* 16,66666, &c. or 16 *l.* 13 *s.* 4 *d.* Principal, to raise 1 *l.* Annuity at Simp. Interest
0,17714,10571 . Log. for the Time.
1,39898,98067 . Log. of 25,06050, &c.
Arrerages 8,39383, &c. of 1 *l.* Annuity 7 Years.

1,62293,05694 . Log. of 41,96918, &c. Arrerages given.
0,92396,05651 . Log. of 8,39383, &c. Arrerages of 1 *l.* Annuit.
0,69897,00043 . Log. of 5. Annuity.

Variety 4. Q. *Of the Price to buy: what Annuity.* Answer. *By the sixth Theorem.*

Variety 4. What yearly Annuity will *l.* 27,91190, &c. in ready Money purchase for 7 Years, after the Rate of 6 *l.* on the 100 for a Year, by Compound Interest?

Answ. 5 *l.* a Year: For here are given the Price, Term and Rate. So by the 6*th* Theorem, If the Price be multiplied by the Product of the Rate lacking 1, led into the Rate figurate according to the Term, and this Product be divided by the said Rate so figurate lacking 1, the Quotient shall be the Annuity desired, as before under the first Variety of this Proposition.

	$\beta\omega$—1	β—1 in $\beta\omega$	*Price.*	*Annuity.*
As	0,503630258991 36 .	0,090217815539481 6 ::	27,9119, &c.	. 5 .

By Logarithms. And by Logarithms, the Log. for the Time lacking 1, shall be taken out of the Log. for the Time added to the Log. of the Rate lacking 1, and the Log. of the Price.

0,17714,10571 .	Log. of 1,50363, &c.	Time or $\beta\omega$.
—2,77815,12504 .	Log. of 0,06	R—1 . or β—1.
1,44578,95123 .	Log. of 27,91190, &c.	Price.
0,40108,18198 .	Sum.	
—1,70211,18155 .	Log. of 0,50363, &c.	Time—1 . or $\beta\omega$—1.
0,69897,00043 .	Log. of 5.	Annuity.

Otherwise. Or if by the second Proposition be gotten the ready Money, that after the Rate propounded will buy 1 *l.* Annuity yearly for the Term given, then substract the Log. thereof out of the Log. of the ready Money given, and you have the Log. of the Annuity; because the Price propounded, divided by the Annuity, will always bring in the Quotient the Price of 1 *l.* Annuity to continue according to T. And contrary, the Price propounded divided by the Price of 1 *l.* Annuity according to T, shall give the Annuity; and accordingly the Difference of their Logarithms.

The Principal Money to raise 1 *l.* Annuity at Simple Interest, after the Rate propounded, being found as before to be 16 *l.* 13 *s.* 4 *d.* the Work appeareth thus:

1,22184,87496 . Log. of 16,66666, &c. or 16 *l.* 13 *s.* 4 *d.* Principal in 1 Year, to raise 1 *l.* Annuity.
0,17714,10571 . Log. for the Time.
1,04470,76925 . Log. of 11,08428, &c.
Ready Money 5,58238, &c. to buy 1 *l.* Annuity 7 Years.

1,44578,95123 . Log. of 27,9119, &c. Price given.
0,74681,95080 . Log. of 5,5823, &c. Price of 1 *l.* Annuity.
0,69897,00043 . Log. of 5, Annuity.

And in all the Varieties of this Classis, the like may be done for Annuities after other Rates, and for Half-yearly and quarterly Paiments if occasion be. *For Half-yearly and quarterly Paiments.*

Proposition 4. To know in what Time or Term a Sum of Money propounded will be paid by an Annuity according to a certain Rate given: or how long an Annuity was detained to increase to a Sum propounded, after a certain Rate, by Compound Interest. *Prop. 4. To find the Time or Term.*

The *Data* for the Resolution of Questions under this Classis, will be either, *Data under the 4th Prop.*

1 . 2 . 3 . The Arrerages, Price and Annuity.
1 . 2 . 5 . The Arrerages, Price and Rate.
1 . 3 . 5 . The Arrerages, Annuity and Rate.
Or, 2 . 3 . 5 . The Price, Annuity and Rate.

The Varieties here, as the last Precedent, follow in their proper order. *How placed.*

Variety 1. An Annuity of 5 *l.* a Year, if the Arrerages by Compound Interest be forborn a certain Time or Term, will amount to *l.* 41,95918, &c. and may be bought for *l.* 27,91190, &c. ready Money: what is the Time or Term of that Annuity? Variety 1. Q. Of 5 *l.* per Ann. such Arrears and Price, what the Time.

Answ. 7 Years. The *Data* here, are the Arrerages, Price and Annuity: And because the Arrerages divided by the Price, give in the Quotient always the Rate multiplied into it self, according to the Term, and the Difference of their Logarithms is the Log. thereof; it follows, that the Index of the Highest Power of the Quotient shall be the Term desired. So the Quotient of their Division being 1,5036302589913 6, is a Power of the 7th Quantity arising from the Root 1,06. Answer. *By Decimals.*

And accordingly the Log. of the Annuity added to the Log. of the said Power lacking 1, and from the Sum the Log. of the Arrerages substracted, leaveth the Log. of the Rate lacking 1: wherefore the Difference between the Log. of the Rate, and the Antecedent thereof, dividing the Difference between the Logarithms of the Arrerages and Price, shall give the Term desired. *By Logarithms.*

1,62293,05694 . Log. of 41,9691, &c. Arrerages.
1,44578,95123 . Log. of 27,9119, &c. Price.
0,17714,10571 . Log. of 1,5036, &c. Time. βω.
—1,70211,18155 . Log. of 0,5036, &c. βω—1.
0,69897,00043 . Log. of 5. Annuity.
0,40108,18198 . Sum.
1,62293,05694 . Log. of 41,9691, &c. Arrerages.
—2,77815,12504 . Log. of 0,06. R—1 or β—1.
0,02530,58653 . Log. of 1,06. R.
0,00000,00000 . Log. of 1,00. Antecedent.

$\frac{,17714,10571}{2530,58653}$ (*Years.* 7 . Term.

Variety 2. The Arrerages of an Annuity, accompted by Compound Interest, after 6 *l. per Cent. per Ann.* were *l.* 41,9691, &c. and purchased for *l.* 27,9119, &c. For what Term of Years was the Annuity to endure? Variety 2. Q. Of Arrears and Price, what the Time at such Rate.

Answ. 7 Years. In this Variety the *Data* being the Arrerages, Price and Rate, and the Price being as α, the Arrerages as ω, and the Rate R of a *Geometrical Progression* to find T, the Question may be resolved by the Rules in the first Propo- Answer. *By Progression.*

Proposition for finding the third Principal of such a Progression, observing only that the Number found out here will be the Rate figurate according to T, but there lack 1.

$$\begin{array}{lllll} & \alpha & 1 & \omega & \text{R figurate to T.} \\ \text{For} & 27{,}9119,\ \&c\ . & 1 :: & 41{,}9691,\ \&c\ . & 1{,}5036302589913 6. \end{array}$$

And so this Number being a Power, (as was noted in the last Variety above) the Index of the highest Power therein shall be the Term desired, and the Rate being the Root thereof, dividing it till the Quotient be an Unit, shall shew the Index by the Number of Divisions, to which it will always be equal.

By Logarithms. But shortning the Work by Logarithms, the Process is thus; take the Log. of the Price from the Log. of the Arrerages, and divide the Log. remaining by the Log. of the Rate.

1,62293,05694	. Log. of	41,9691, &c.	Arrerages.
1,44578,95123	. Log. of	27,9119, &c.	Price.
0,17714,10571	. Log. of	1,5036, &c.	Time.
0,02530,58653	. Log. of	1,06.	Rate.

$$\frac{,17714,10571}{2530,58653}\left(\begin{array}{l} \textit{Years.} \\ 7 \end{array} \right. \ \text{. Term.}$$

Variety 3. *Q. Of* 5 *l.* per Ann. *and Arrerages, in what Time.*

Variety 3. If an Annuity of 5 *l.* yearly detained, with the Compound Interest, after the Rate of 6 *l. per Cent. per Annum*, increase to *l.* 41,9691, *&c.* how long was it detained?

Answer. *By Progression.*

Answ. 7 Years: Here the *Data* are the Arrerages, Annuity and Rate answering to α . R . Z of a *Geometrical Progression* to find T. that is α . 5 . R . 1,06 . and Z. 41,96918, *&c.* So by the Rules in the third Proposition, for finding the third Principal of such a Progression, the Question will be answered.

For 5 . 41,9691, *&c.* :: 0,06 . 0,50363025899136.

And this Number found being the Ratio figurate to T—1, answering to the Log. for the Time lacking an Unit; if 1 be added, it shall be a Power, the Index whereof shall be the desired Term.

By Logarithms. And so to the absolute Number of the remaining Log. when the Log. of the Annuity is taken from the Sum of the Log. of the Arrerages, and the Rate lacking 1, add 1, and divide the Log. thereof by the Log. of the Rate.

	1,62293,05694	. Log. of	41,9691, &c.	Arrerages Z.
	—2,77815,12504	. Log. of	0,06	R—1 or β—1
	0,40108,18198	. Sum.		
	0,69897,00043	. Log. of	5.	Annuity α.
	—1,70211,18155	. Log. of	0,5036, &c.	Time—1, or βω—1.
	0,17714,10571	. Log. of	1,5036, &c.	Time, or βω.
Rate .	0,02530,58653	*Years.* 7	. Term.	

Variety 4. *Q. Of* 5 *l.* per Ann. *bought for l.* 27,9119, &c. *for what Time.*

Variety 4. If *l.* 27,9119, *&c.* were to be paid by 5 *l. per Annum*: in what Time will it be paid, with allowance of 6 *l. per Cent. per Annum*?

Answer. *By Decimals.*

Answ. In 7 Years: For here being given the Price, Annuity and Rate, the Work, without Logarithms, is to get the Principal Money, which at Simple Interest, after the Rate propounded, will raise such an Annuity, and take the Price out thereof, and by the Remainder divide the same Principal, this being a Figural Number, whereof the Rate is the Root, divided by the Root, till the Root be brought in the Quotient, the Number of the several Divisions shall be the desired Term.

B. Logarithms And much sooner by Logarithms, for after finding the Principal Money, which at Simple Interest in one Year, after the Rate propounded, will raise the Annuity; from the same Principal take the given Price, and the Log. of the Remain from the Log. of that Principal, and divide the remaining Log. by the Log. of the Rate.

1,92081,87539 . Log. of 83,3333, &c. Principal to raise 5 *l.* a Year.
27,9119, &c. Price.
1,74367,76968 . Log. of 55,4214, &c.
0,17714,10571 . Log. of 1,5036, &c. Time, or $\beta\omega$.
Years.
Rate. 0,02530,58653 (7 . Term T.

In like manner may the Term be found for Half-yearly Paiments, Quarterly, or others, and at other Rates as the Case may require, under all the Varieties of this Classis. *For other Rates, and half-yearly and quarterly Paiments.*

Proposition 5. To know after what Rate, by Compound Interest, any Annuity did increase to a Sum propounded; or the Rate the Compound Interest is reckoned at by the ready Money the Annuity is sold for to endure a certain Time or Term. Prop. 5. *To find the Rate.*

The *Data* for the Resolution of Questions under this Classis, will be either Data *under the fifth Prop.*

1 . 2 . 3 . The Arrerages, Price and Annuity.
1 . 2 . 4 . The Arrerages, Price and Term.
1 . 3 . 4 . The Arrerages, Annuity and Term.
or 2 . 3 . 4 . The Price, Annuity and Term.

The first two of these Varieties agree in Facility, as the two latter in Difficulty, with some of the former. *Set in order, but the latter 2 harder than the first.*

Variety 1. The Arrears of an Annuity of 5 *l.* a Year to endure a certain Time, with Interest upon Interest, were reckoned to amount to *l.* 41,96918, &c. and may be bought for *l.* 27,9119, &c. after what Rate was the Interest reckoned? Variety 1. Q. *Of* 5 *l.* per Ann. *such Arrears and Price, what the Rate.*

Answ. After 6 *l.* *per Cent. per Annum.* The *Data* here, *viz.* the Arrerages, Price and Annuity, being the same as in the first Variety of the 4*th* Proposition; the Work is the same as there, till the last Division to find the Term. *By Logarithms.*

1,62293,05694 . Log. of 41,96918, &c. Arrerages.
1,44578,95123 . Log. of 27,91190, &c. Price.
0,17714,10571 . Log. of 1,50363, &c. Time. $\beta\omega$.
—1,70211,18155 . Log. of 0,50363, &c. $\beta\omega$—1.
0,69897,00043 . Log. of 5. Annuity.
0,40108,18198 . Sum.
1,62293,05694 . Log. of 41,96918, &c. Arrerages.
—2,77815,12504 . Log. of 0,06. R—1, or β—1.
1,00. Antecedent added.
1,06. Rate, or β.

Variety 2. For *l.* 27,9119, &c. ready Money, one may buy 7 Years Arrerages of an Annuity; which cast up by Compound Interest, amounts to *l.* 41,9691, &c. after what Rate is the Interest accompted? Variety 2. Q. *Of an Annuity for 7 Years, such Arrears and Price, what the Rate.*

Answ. After 6 *l.* on the 100 for a Year. Here the *Data* being the Arrerages, Price and Term, the same as in the first Variety of the third Proposition, the Work shall be as the former Part thereof, seeing the Rate was there found to get the Annuity thereby. *By Logarithms.*

1,62293,05694 . Log. of 41,96918, &c. Arrerages.
1,44578,95123 . Log. of 27,91190, &c. Price.
0,17714,10571 . Log. for the Time 1,50363, &c.
7) 0,02530,58653 . Log. of the Rate 1,06.

And this agreeth with the Rules in the first Proposition for finding the 4*th* Principal of a *Geometrical Progression*, seeing the Price is as α, the Arrerages as ω. So by α . ω . T. may R be found, with this only Difference, that here the Index of the Quotient is T, but there T—1. *By Progression.*

Variety 3. An Annuity of 5 *l.* a Year, with the Compound Interest thereof in 7 Years, amounted to *l.* 41,96918, &c. after what Rate on the 100 did it increase? Variety 3. Q. *Of* 5 *l.* per Annum 7 *Years such Arrears, what the Rate.*

Answ.

Answer. *Answ.* As before, after 6 *l.* on the 100 for a Year: The Arrerages, Annuity and Term here are given, and are the same *Data* as in the first Variety of the second Proposition: And because there the *Ratio* was gotten before the Price could be found, the Work here will be the same for the *Ratio*, and by Logarithms thus after this way.

By Logarithms. Take the Log. of α, the Annuity from the Log. of Z the Arrerages, and the Remainder is the Log. of the Arrerages of 1 *l.* let run according to T; to which a Log. being added, that will make the Sum the Log. of a Power lacking 1, whose Index shall be T, which in this Example will be done by the Log. of 0,06 the *Ratio* lacking 1, and the Sum will be the Log. of 0,50363025899136, which is ρω—1. And then without farther Work, add 1 the Antecedent to the Absolute Number of the Log. so added, and you have the Rate 1,06. See more at the end of the next Variety.

1,62293,05694	Log. of 41,96918, *&c.*	Arrerages.
0,69897,00043	Log. of 5.	Annuity.
0,92396,05651	Log. of 8,39383, *&c.*	Arrerages of 1 *l.* in 7 Years.
—2,77815,12504	Log. of 0,06	R—1, or ρ—1.
—1,70211,18155	Log. of 0,50363, *&c.*	Time—1, or ρω—1.

Ergo, to 0,06
add 1,00 Antecedent.
1,06 Rate.

Variety 4. Q. Of 5 *l.* per Ann. 7 Years, such Price, what the Rate. *Variety* 4. If *l.* 27,91190, *&c.* in ready Money, purchase an Annuity of 5 *l.* a Year, to endure for 7 Years: after what Rate on the 100 is the Compound Interest thereof reckoned?

Answer. *Answ.* After 6 *l.* *per Cent. per Annum.* For seeing the *Data* here, *viz.* the Price, Annuity and Term, agree with the *Data* of the first Variety of the first Proposition, and the *Ratio* there was found thereby before the Arrerages could be found, the Work to get the same here will be the same, and so needs no Repetition, but is a most difficult Proposition to resolve, as there was observed,

By Logarithms. because it is hard to find a Number to multiply the ready Money by, that shall produce the Arrerages of the Annuity forborn to T. which in this Example must be 1,50363025899136, and no other; and accordingly the Log. thereof added to the Log. of the Price, maketh the Sum the Log. of 41,96918, *&c.* the Arrerages, or Z of a Progression, where 5 is α; and the last Term of a Progression or ω, where 27,9119, *&c.* is α. But both in this and the last Variety, it is best to divide by the Time the Log. for the Time, which may be gotten by the first or second Varieties of the first and second Propositions of Annuities.

1,44578,95123	Log. of 27,9119, *&c.*	Price.
0,17714,10571	Log. of 1,5036, *&c.*	Time.
1,62293,05694	Log. of 41,9691, *&c.*	Arrerages.

&c.

Of Interest and Annuities mixt.

Touching Interest and Annuities intermixt.

Because the Principal Propositions about Interest are 4, and about Annuities

Propositions many. are 5, that is 9 in all as aforesaid, such a vast Number of Propositions concerning them, as well amongst themselves as mixed one with another, according to the foregoing Chapter of *Transmutation*, might be thence deduced, as would weary

Those most useful. the Reader: But the most useful are such as follow, and relate more especially to the second and third Propositions of Annuities, or the first or second of Interest already handled.

Prop 1. To put a Value on Wares sold for Time. *Prop.* 1. To put a Value, present or future, on Commodities sold according to the Time given for Paiment.

For the Future. By the first Proposition of Interest, add the Log. for the Time, to the Log. of the Value in ready Money, and the Sum is the Log. of the future Value.

Q. Calico what worth at 9 Months. *Example.* Calico is worth 25 *d.* the Yard ready Money: what is it worth to be paid at 9 Months end, accompting 10 *per Cent. per Annum* by Compound Interest?

Answ.

Answ. 26 ¾ *d.* and somewhat more. Answer.

0,04139,26852	. Log. of 1,10. R.
0,03104,45139	. Log. for the Time ¾, or 9 Months.
1,39794,00087	. Log. of 25.
1,42898,45226	. Log. of 26,85, &c.

On the contrary, by the second Prop. of Interest, take the Log. for the Time from the Log. of the future Value, and the Difference is the Log. of the Present. *For the present.*

Wherefore the Log. 0,03104,45139, taken from the Log. of 26,85, &c. shall leave the Log. of 25.

Prop. 2. To ballance an Accompt present or future, of several Merchants, where divers Sums of Money are paid and received by each of them, to be reckoned with the Compound Interest. *Prop 2. To ballance an Accompt.*

According to the Times of the several Receipts and Paiments by the first Proposition of Interest, the Charge and Discharge of every Person is to be gotten, by adding the Log. for the Time between the Accompt, and such Receipt or Paiment accordingly, to the same Receipt or Paiment for the future Ballance of the Accompt. *For the future.*

Example. A and *B* have paid divers Sums of Money one to and for another; and received of and for one another divers other Sums between the 25*th* Day of *March* 1611, and the 27*th* Day of *March* 1613, according to their particular Accompts following: they agree to clear Accompts, and allow each to other 10 *per Cent. per Annum*, Interest upon Interest; and that their Reckoning shall conclude upon the 27*th* Day of *March* 1613, and demand which of them is indebted to the other, and how much? *Q. Of divers Sums paid and received between A and B, which is debt.*

The Accompt of A. *Particular Accompt of A.*

		l.			l.
Mar. 27. 1611.	Received of *B* ——	200	*June* 27. 1611.	Paid to *B* ——	100
Sept. 27.	Received for Accompt partable in halves between them ——	260		More then paid for Accompt partable in halves between them --	200
Dec. 27.	Received of *B* ——	300	*June* 27. 1612.	Paid to *B* ——	300
Mar. 27. 1612.	Received for Accompt partable in thirds, *viz.* ⅓ for himself, and ⅔ for *B* ——	210	*Sept.* 27.	Paid for Accompt, partable in Thirds, *viz.* ⅓ for himself, and ⅔ for *B* ——	300
Sept. 27.	Received of *B* ——	200	*Dec.* 27.	Paid to *B* ——	300

The Accompt of B. *Particular Accompt of B.*

		l.			l.
June 27. 1611.	Received of *A* ——	100	*Mar.* 27. 1611.	Paid to *A* ——	200
Sept. 27.	Rec. for Accompt partable in Thirds, *viz.* ⅓ for *A* & ⅔ for himself-	450	*June* 27.	Paid for Accompt partable in halves between them ——	300
June 27. 1612.	Received of *A* ——	300	*Dec.* 27.	Paid to *A* ——	300
Sept. 27.	Received for Accompt partable in halves between them ——	300	*June* 27. 1612.	Paid for Accompt partable in Thirds, *viz.* ⅓ for *A* & ⅔ for himself -	300
Dec. 27.	Received of *A* ——	300	*Sept.* 27.	Paid to *A* ——	200

Accompt of A cast up.

The Charge of *A*.	The Discharge of *A*.
0,08278,53704. *Log. of* R *in* 2 *Years.*	0,07243,71991. *Log. of* R *in* 1 $\frac{3}{4}$ *Year.*
2,30102,99957. *Log. of* 200 *received.*	2,00000,00000. *Log. of* 100 *paid.*
2,38381,53661. *Log. of* 242.	2,07243,71991. *Log. of* 118,150, &c.
0,06208,90278. *Log. of* R *in* 1 $\frac{1}{2}$ *Year.*	0,07243,71991. *Log. of* R *in* 1 $\frac{3}{4}$ *Year.*
2,11394,33523. *Log. of* 130 *received.*	2,00000,00000. *Log. of* 100 *paid.*
2,17603,23801. *Log. of* 149,979, &c.	2,07243,71991. *Log. of* 118,150, &c.
0,05174,08565. *Log. of* R *in* 1 $\frac{1}{4}$ *Year.*	0,03104,45139. *Log. of* R *in* $\frac{3}{4}$ *Year.*
2,47712,12547. *Log. of* 300 *received.*	2,47712,12547. *Log. of* 300 *paid.*
2,52886,21112. *Log. of* 337,957, &c.	2,50816,57686. *Log. of* 322,229, &c.
0,04139,26852. *Log. of* R *in* 1 *Year.*	0,02069,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.*
2,14612,80357. *Log. of* 140 *received.*	2,30102,99957. *Log. of* 200 *paid.*
2,18752,07209. *Log. of* 154.	2,32172,63383. *Log. of* 209,761, &c.
0,02069,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.*	0,01034,81713. *Log. of* R *in* $\frac{1}{4}$ *Year.*
2,30102,99957. *Log. of* 200 *received.*	2,47712,12547. *Log. of* 300 *paid.*
2,32172,63383. *Log. of* 209,761, &c.	2,48746,94260. *Log. of* 307,234, &c.
0,07243,71991. *Log. of* R *in* 1 $\frac{3}{4}$ *Year.*	0,06208,90278. *Log. of* R *in* 1 $\frac{1}{2}$ *Year.*
2,17609,12591. *Log. of* 150 *paid by* B.	2,17609,12591. *Log. of* 150 *received by* B.
2,24852,84582. *Log. of* 177,226, &c.	2,23818,02869. *Log. of* 173,053, &c.
0,03104,45139. *Log. of* R *in* $\frac{3}{4}$ *Year.*	0,[illegible]9,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.*
2,00000,00000. *Log. of* 100 *paid by* B.	[illegible],12591. *Log. of* 150 *received by* B.
2,03104,45139. *Log. of* 107,409, &c.	[illegible]78,76017. *Log. of* 157,321, &c.

Collection of the Charge.	*Collection of the Discharge.*
242.	118,1509, &c.
149,9796, &c.	118,1509, &c.
337,9575, &c.	322,2298, &c.
154.	209,7617, &c.
209,7617, &c.	307,2341, &c.
177,2264, &c.	173,0534, &c.
107,4099, &c.	157,3213, &c.
1378,3351	1405,9021

Total Charge of *A*, *l.* 1378,3351.
Rest due to *A*, 27,5670.
1405,9021. Total Discharge of *A*.

Accompt of B cast up.

And by like Examination of the Accompt of *B*, there will be found in his Hands the Sum of *l.* 27,5670, &c. due to *A* as aforesaid, seeing the Charge of the one is the Discharge of the other.

The Charge of B.	*The Discharge of* B.
Dr. 1 $\frac{3}{4}$ Year by 100 R of A. 118,1509	Cr. 2 Years to 200 Pd to A. 242.
1 $\frac{1}{2}$ Year by 150 R for A. 173,0534	1 $\frac{3}{4}$ Year to 150 Pd for A. 177,2264
0 $\frac{3}{4}$ Year by 300 R of A. 322,2298	1 $\frac{1}{4}$ Year to 300 Pd to A. 337,9575
0 $\frac{1}{2}$ Year by 150 R for A. 157,3213	0 $\frac{3}{4}$ Year to 100 Pd for A. 107,4099
0 $\frac{1}{4}$ Year by 300 R of A. 307,2341	0 $\frac{1}{2}$ Year to 200 Pd to A. 209,7617
1 $\frac{3}{4}$ Year by 100 Pd by A. 118,1509	1 $\frac{1}{2}$ Year to 130 R by A. 149,9796
0 $\frac{1}{2}$ Year by 200 Pd by A. 209,7617	1 Year to 140 R by A. 154.
B Debitor 1405,9021	*B* Creditor 1378,3351

Total Discharge of *B*, *l.* 1378,3351.
Rest in the Hands of *B*, 27,5670
1405,9021 Total Charge of *B*.

So to ballance this Accompt, it appears that *B* is indebted to *A* *l.* 27,5670, or 27 *l.* 11 *s.* 4 *d.* &c. due on the Foot of his Accompt, to be paid *March* 27. 1613.

Ballance B indebted to A.

On the contrary, to ballance an Accompt at present, when the Paiments are to be made afterward, by the second Prop. of Interest take the Log. for the Time gotten, as aforesaid, from the Log. of the Monies so to be paid or received.

For the present:

Example. *A* owes to *B* 800 *l.* to be paid at 4 Years end: And *B* is indebted to *A* 900 *l.* to be paid in 6 Years, that is to say, at every 2 Years end 300 *l.* Now they agree to clear their Debts presently, allowing each other 8 *per Cent. per Annum*, Interest upon Interest: The Question is, which of them must pay Money to the other, and how much?

Q. Of the Debts of A and B to be paid in Time, how to clear at present.

The Charge of A.

2,90308,99870 . Log. of 800.
0,13369,50220 . Log. of R in 4 Years.

2,76939,49650 . Log. of 588,023.

Collection of the Discharge.

257,201, &c.
220,509, &c.
189,051, &c.

666,761, &c.

The Discharge of A.

2,47712,12547 . Log. of 300.
0,06684,75110 . Log. of R in 2 Years.

2,41027,37437 . Log. of 257,201.

2,47712,12547 . Log. of 300.
0,13369,50220 . Log. of R in 4 Years.

2,34342,62327 . Log. of 220,509.

2,47712,12547 . Log. of 300.
0,20054,25330 . Log. of R in 6 Years.

2,27657,87217 . Log. of 189,051.

Total Charge of *A* *l.* 588,023, &c.
Rest due to *A* 78,738, &c.
666,761, &c. Total Discharge of *A*.

And by like examination of *B* his Accompt, so much over-ballance will be found in his Hands as is due to *A* on the Foot of his Accompt; and therefore *B* must pay to *A*, to ballance the Accompt, *l.* 78,738, &c. or 78 *l.* 14 *s.* 9 *d.* &c. in ready Money.

Ballance B indebted to A.

Prop. 3. To discover what ready Money is to be paid for a Lease, when certain Years of the Term are of greater Value than the Rest, and so consequently to bring several Annuities issuing out of the same Lands into one Paiment.

Prop. 3. *To find the ready Money paid for a Lease, when part of the Term of more Value than the rest: Or to bring divers Annuities into one.*

Suppose the greatest Annuity or Annual Profit, to begin and continue for the whole Term or Number of Years propounded: Then according to the 4*th Variety* of the Second *Proposition* of Annuities, find what such an Annuity or Rent is worth in ready Money. And also in like manner having substracted the lesser Rent or Annuity from the Greater, find what the Difference of the Rents or Annuities to continue for the Term of the least Annuity is worth in ready Money: Then abate this ready Money from the former, and the remainder is the ready Money that ought to be paid for the Lease, or both Annuities; and with this ready Money, according to the 4*th* Variety of the third Proposition of Annuities, is to be found the Annuity or Rent to continue for the whole Term in lieu of the other.

Example. Certain Lands stand charged with the Paiment of 40 *s. per Annum* for 13 Years, and afterwards for paiment of 10 *l. per Annum* for 17 Years: Or suppose a Lease for 30 Years to come, whereof the present yearly Profit all Out-Rents paid be 2 *l.* but after 13 Years expired, it will be worth 10 *l.* Yearly, all out-Rents paid. If this Lease be offered to sale, what may be given for it, reckoning 6¼ *per Cent. per Annum*, Interest upon Interest? Or if the Owner of the Lands agree with the Party to whom the aforesaid Annuities are to be paid, to reduce both into one for the whole 30 Years; the Question is, what that new Annuity ought to be at the Rate aforesaid by Compound Interest?

Q. Of Lands charged with 2 Annuities, what worth to be sold for the Term of a Lease, &c.

Answ. The Purchase of the Lease will be found worth in ready Money, *l.* 64,245, &c. which will buy an Annuity of 4 *l.* 15 *s.* 10 *d.* and something above, for 30 Years.

Answer.

0,02632,53434 . Log. of R. $6\frac{1}{4}$, or 1,0625.
30 . T.
0,78976,03020 . Log. for the whole Time.
2,20411,99827 . Log. of 160 Principal, to raise 10 *l.* the greatest Annu.
1,41435,96807 . Log. of 25,963, *&c.*
Ready Money 134,037—to buy 10 *l. per Annum* 30 Years.

Now seeing for 13 Years of these 30, the Annuity or Annual Profit is but 2 *l. per Annum*, this is to be taken from 10 *l. per Annum*, and there remaineth 8 *l. per Annum*; and the 134,037—before found being too much, so much as 8 *l. per Annum* is worth in ready Money for 13 Years, therefore in like manner the Price thereof is found.

0,02632,53434 . Log. of R as aforesaid.
13 . T.
0,07897,60302
0,26325,3434
0,34222,94642 . Log. for 13 Years.
2,10720,99696 . Log. of 128 Principal. to raise 8 *l.* the Difference of Ann.
1,76498,05054 . Log. of 58,208, *&c.*
Ready Money 69,792— to buy 8 *l. per Annum* 13 Years.

This *l.* 69,792—taken from *l.* 134,037—leaves *l.* 64,245, *&c.* for the Value of the Lease, which will buy an Annuity of *l.* 4,7931, *&c.* or 4 *l.* 15 *s.* 10 *d*, &c. *per Annum* for 30 Years.

1,20411,99827 . Log. of 16 Principal, to raise 1 *l.* Annuity at Simple Interest, after the Rate of $6\frac{1}{4}$ *per Cent.*
0,78976,03020 . Log. for the Time of 30 Years.
0,41435,96807 . Log. of 2,5963, *&c.*
Ready Money 13,4037—to buy 1 *l.* Annuity for 30 Years.

1,80783,93335 . Log. of 64,245, *&c.* Price given.
1,12722,46987 . Log. of 13,4037, *&c.* Price of 1 *l.* Annuity.
0,68061,46348 . Log. of 4,7931, *&c.* New Annuity to continue 30 Years.

When the several Annuities are payable together.

If the several Annuities be issuant together, then get the Price of them severally, and add them together for the Sum to buy the Annuity desired, and the Total taken from the Price of the Whole, shews the Price of the Purchase so charged with the Annuities.

Q. Of a Farm to be sold, charged with 2 Annuities, what the Purchaser may keep in hand to satisfy for them.

Example. A Farm is to be sold for 680 *l.* which stands charged with the Paiment of 30 *l. per Annum*, viz. 7 *l.* 10 *s. per* Quarter for 2 Years: And also 9 *l. per Annum*, viz. 2 *l.* 5 *s. per* Quarter for 7 Years: how much Money may the Purchaser retain in his Hands of the 680 *l.* to satisfy the Paiments abovementioned, reckoning after $6\frac{1}{4}$ *per Cent. per Annum* by Compound Interest?

Answer.

Answ. *l.* 107,0339, *&c.* that is, for the first Annuity *l.* 56,0804, *&c.* and for the other *l.* 50,9535, *&c.* So this *l.* 107,0339 taken from 680 *l.* leaves *l.* 572,9661 to be paid for the Purchase as charged with the said Annuities. And if the Question were, what new Annuity might have been granted in lieu of the other two, this *l.* 107,0339 being found to be the ready Money both the said Annuities are worth, the Work will be resolved as before by the *4th* Variety of the third Prop. of Annuities.

0,02632,53434 . Log. of 1,0625 . R . as before.
2 . T.
0,05265,06868 . Log. for the Time 2 Years or 8 Quarters.
2,69125,06683 . Log. of 491,1913, *&c.* Principal to raise 7 *l.* 10 *s.* in 1 Quart.
2,63859,99815 . Log. of 435,1109, *&c.*
Ready Money 56,0804, *&c.* to buy 30 *l. per Annum* for 2 Years payable quarterly.

0,

0,02632,53434 . Log. of 1,0625 . R . as before.
7 . T.

0,18427,74038 . Log. for Time 7 Years, or 28 Quarters.
2,16837,19230 . Log. of 147,3573, &c. Principal to raise 2 *l.* 5 *s.* in 1 Quarter.

1,98409,45192 . Log. of 96,4038, &c.
Ready Money 50,9535, &c. to buy 9 *l.* *per Annum* for 7 Years, payable quarterly.

Prop. 4. To exchange one Annuity for another, with or without Money to boot. *Prop 4. To exchange one Annuity for another.*

With Money is, when the Terms of both Annuities are equal, but the Annuities are one greater than the other, then abate the lesser Annuity from the Greater; and as aforesaid, by the 4*th* Variety of the second Proposition of Annuities, find what the Remainder esteemed as an Annuity for the Time given is worth in ready Money, and so much is to be given with the lesser Annuity for the Purchase of the Greater. *With Money to boot.*

Example. *A* hath an Annuity of 70 *l.* *per Annum*, and *B* another of 150 *l.* both to endure 15 Years: They agree to exchange, and that *A* shall pay to *B* so much ready Money as will countervail the Difference of the Annuities: how much ready Money shall *B* receive of *A*, after the Rate of 6 *per Cent.* *per Annum* by Compound Interest? Q. *Of* 70 *l.* per Ann. *exchanged for* 150 *l.* per Ann. *what Money to be given.*

Answ. *l.* 776,9799, &c. For 70 taken from 150, leaves 80; and 80 *l.* *per Annum* for 15 Years is worth so much ready Money. Answer.

0,02530,58653 . Log. of 1,06 . R.
15 . T.

0,37958,79795 . Log. for the Time.
3,12493,87366 . Log. of 1333,3333, &c. Principal to raise 80 *l.* *per An.*

2,74535,07571 . Log. of 556,3534, &c.
Ready Money 776,9799, &c. to buy 80 *l.* *per Annum* for 15 Years.

Exchange of one Annuity for another without Money, must ballance the Inequality of the Annuities with more or less Time: Wherefore as before, get the ready Money that will buy each Annuity, and then by the 4*th* Variety of the 4*th* Proposition of Annuities, see how many yearly, half-yearly, or quarterly Paiments accordingly the ready Money of one Annuity will buy of the other. *Without Money to boot.*

Example. *A* hath an Annuity of 20 *l.* *per Annum* for 21 Years, yearly to be paid, and will exchange with *B* for an Annuity of 30 *l.* *per Annum*: how long shall the Annuity of 30 *l.* *per Annum* be paid to *A*, that neither of them lose, reckoning 6 *l.* *per Cent.* *per Annum*, Interest upon Interest? Q. *Of* 20 *l.* per Ann. *exchanged for* 30 *l.* per Ann. *what Term*

Answ. 10 Years *A* shall receive 30 *l.* *per Annum*, but for the 11*th* Year no more than 27 *l.* 7 *s.* 4¼ *d.* Answer.

0,02530,58653 . Log. of 1,06. R.
21 . T.

0,53142,31713 . Log. for the Time.
2,52287,87453 . Log. of 333,3333, &c. Principal to raise 20 *l.* *per Ann.*

1,99145,55740 . Log. of 98,0518, &c.
Ready Money 235,2815, &c. to buy 20 *l.* *per Ann.* for 21 Years.

2,69897,00043 . Log. of 500 Principal, to raise 30 *l.* *per Annum.*
235,2815, &c.

2,42278,42932 . Log. of 264,7185, &c.
0,27618,57111

(23127058
27618,57111 | 1 (Years. 10
2530,58653

Then if 2530,58653 answer to 1 Year, or 365 Days, 2312,70581 the Remain shall answer to 333 Days, and part of a Day. And if 365 Days pay 30 *l.* then 333 Days shall pay *l.* 27,3698, &c. or 27 *l.* 7 *s.* 4¼ *d.*

In like sorts Debts by a new Agreement to be paid at other Times.

Hence also after the manner of Annuities, a Debt to be paid at certain Times, and afterwards agreed to be paid at other Times, with Allowance or Discount of Compound Interest, the Paiments are found.

Q. Of 5000 l. payable in 5 Years, what to be paid in every 3 Months.

Example. One oweth 5000 *l.* to be paid in 5 Years, *viz.* every Year 1000 *l.* but afterwards agreeth with his Creditor to pay the Whole in 20 equal Paiments, *viz.* every 3 Months one Paiment: The Question is, what each Paiment shall be, with Allowance of Interest upon Interest, after the Rate of 10 *l. per Cent. per Annum?*

Answer.

Answ. l. 241,2561, *&c.* every Quarter during the 5 Years: For 1000 *l.* a Year for 5 Years being found to be worth, in ready Money, *l.* 3790,7868, *&c.* that ready Money will buy such a quarterly Paiment as aforesaid, by the 4*th* Variety of the third Proposition of Annuities.

1,61752,36773 . Log. of 41,4499, *&c.* Principal to raise 1 *l. per* Quarter.
0,20696,34260 . Log. for the Time 5 Years, or 20 Quarters.
1,41056,02513 . Log. of 25,7371, *&c.*
Ready Money 15,7127, *&c.* to buy 1 *l.* Annuity for 5 Years.

3,57872,93597 . Log. of 3790,7868, *&c.* Price given.
1,19625,10297 . Log. of 15,7127, *&c.* Price of 1 *l.* Annuity.
2,38247,83300 . Log. of 241,2561, *&c.* New Annuity.

If the Debt be payable at once.

But supposing by the new Agreement, the whole Debt had been agreed to be discharged in one Paiment, as at the end of 3 Years, with Allowance of Interest as aforesaid: Then after the ready Money found, as aforesaid, by the 4*th* Variety of the second Proposition of Annuities, the rest of the Resolution is wrought by the first Proposition of Interest, and that one Paiment found to be *l.* 5045,5372, *&c.*

Example.

0,04139,26852 . Log. of 1,10 . R.
3 . T.
0,12417,80556 . Log. for the Time.
3,57872,93597 . Log. of 3790,7868, *&c.* ready Money.
3,70290,74153 . Log. of 5045,5372, *&c.* ω.

Prop. 5. *To lessen the Rent or Fine.*

Prop. 5. To lessen the yearly Rent of an House or Land by increasing the Fine, or to lessen the Fine by increasing the Rent.

The Rent.

For lessening the Rent by the 4*th* Variety of the second Proposition of Annuities, find what the Sum diminished, esteemed as an Annuity, is worth ready Money, and add this ready Money to the Fine, and the Aggregate is the new Fine.

Q. *Of* 10 *l.* per Ann. *lessened to* 4 *l. what Fine.*

Example. A Lease is to be sold for 30 Years, whereof the Fine is 100 *l.* and the Rent 10 *l.* yearly. The Lessee is desirous to pay less Rent, and to increase the Fine or Income: If therefore the yearly Rent be decreased to 4 *l. per Annum*, what Fine shall he pay after 10 *l. per Cent. per Annum* by Compound Interest.

Answer.

Answ. l. 156,5614, *&c.* For seeing the 10 *l.* yearly Rent decreased to 4 *l.* is diminished 6 *l. per Annum*, the ready Money that will buy an Annuity of 6 *l. per Annum*, at the Rate and for the Time aforesaid, is found to be *l.* 56,5614, *&c.* which added to 100 *l.* makes *l.* 156,5614, *&c.* as before.

0,04139,26852 . Log. of 1,10. R.
30 . T.
1,24178,05560 . Log. for the Time.
1,77815,12504 . Log. of 60 Principal to raise 6 *l. per Annum.*
0,53637,06944 . Log. of 3,4385, *&c.*
Ready Money 56,5614, *&c.* to buy 6 *l.* Annuity for 30 Years.
Fine 100.
Total 156,5614, *&c.* Fine to be paid.

For

For lessening the Fine by the 4*th* Variety of the third Proposition of Annuities, find what Annuity may be bought for that Quantity of the Fine desired to be abated, and add that Annuity to the Rent, and the Aggregate is the new Rent. *The Fine.*

Example. If a Lease of an House for 21 Years be worth 100 *l.* Fine, and 10 *l. per Annum* Rent: of how much yearly Rent ought it to be, to bring the Fine down to 60 *l.* reckoning 10 *l. per Cent. per Annum,* Interest upon Interest? *Q. Of 100 l. Fine lessened to 60 l. what Rent.*

Answ. l. 14,6250, *&c.* For seeing the Fine of 100 *l.* decreased to 60 *l.* the Lessee abateth 40 *l.* of his Fine. And this 40 *l.* at the Rate, and for the Term aforesaid, will buy an Annuity of *l.* 4,6250, *&c.* which added to 10 *l.* makes the Rent *l.* 14,6250, *&c.* as before. Answer.

0,04139,26852 . Log. of 1,10 . R.
21 . T.

0,86924,63892 . Log. for the Time.
1,00000,00000 . Log. of 10 Principal to raise 1 *l.* Annuity.
0,13075,36108 . Log. of 1,3513, *&c.*

Ready Money 8,6486, *&c.* to buy 1 *l.* Annuity for 21 Years.

1,60205,99913 . Log. of 40. Price given.
0,93694,58113 . Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
0,66511,41800 . Log. of 4,6250, *&c.* New Annuity.

Rent 10.

Total 14,6250, *&c.* Rent to be paid yearly.

In like manner Exchanges of Fines for Rents, or Rents for Fines, or Fines and Rents for Rents or Fines, are performed: And the Annuity for the one, and the ready Money for the other found out accordingly.

Prop. 6. To find what Annuity for any Term one may buy for a Sum of Money, when the Annuity beginneth presently, and the Buyer hath Time for the Payment of his Money. Prop. 6. *To find what Annuity to begin presently, may be bought for a Sum paid afterward.*

This *Prop.* by the second *Prop.* of Interest, first findeth what the Purchaser's Money is worth to be paid presently; and then by the 4*th Variety* of the third *Prop.* of *Annuities,* findeth what Annuity to endure for the Time propounded may be bought for that ready Money.

Example. What Annuity to endure for 10 Years, and begin presently, may I grant for 500 *l.* to be received at 4 Years end, reckoning Interest upon Interest, after the Rate of 10 *l. per Cent. per Annum?* *Q. Of an Annuity for 10 Years bought for 500 l. paid 4 Years after.*

Answ. l. 55,5786, *&c.* For 500 *l.* due at 4 Years end, being worth in ready Money *l.* 341,5067, *&c.* will buy at the Rate aforesaid an Annuity of *l.* 55,5786, *&c.* for 10 Years. Answer.

0,04139,26852 . Log. of 1,10 . R.
4 . T.

0,16557,07408 . Log. for the Time 4 Years.
2,69897,00043 . Log. of 500 . ω.
2,53339,92635 . Log. of 341,5067, *&c.* α.

0,41392,68520 . Log. for the Time 10 Years.
1,00000,00000 . Log. of 10 Principal to raise 1 *l.* Annuity.
0,58607,31480 . Log. of 3,8554, *&c.*

Ready Money 6,1445, *&c.* to buy 1 *l.* Annuity for 10 Years.

2,53339,92635 . Log. of 341,5067, *&c.* Price above found.
0,78849,14961 . Log. of 6,1445, *&c.* Price of 1 *l.* Annuity.
1,74490,77674 . Log. of 55,5786, *&c.* Annuity desired.

But if the present Value of a Lease be given, and the Term begin afterward, the present Worth is only to be valued according to the future beginning of the Term. *If the Term begin after.*

Example.

Q. *Of the Reversion of a Lease, what worth.*

Example. If a Lease for 30 Years to begin presently, be worth 1000 *l.* what is it worth in Reversion, to begin at 7 Years end, after the Rate of 10 *l. per Cent. per Annum*, Interest upon Interest?

Answer.

Answ. l. 513,1581, *&c.* Because the Purchaser pays his Money 7 Years before he enters upon the Land, the Drift is to know what 1000 *l.* due at 7 Years end, after the Rate aforesaid, is worth at present; which by the second *Prop.* of Interest, is discovered to be *l.* 513,1581, *&c.* and no more is the present Value of that Lease.

0,04139,26852	. Log. of 1,10 . R .
7	. T .
0,28974,87964	. Log. for the Time.
3,00000,00000	. Log. of 1000 . ω .
2,71025,12036	. Log. of 513,1581, *&c.* α .

Prop. 7. *To countervail Fines and Rents, and the Lease in Reversion.*

Prop. 7. To countervail a present Fine, with an Annual Rent, beginning and continuing with a Lease, when the Lease in Reversion is offered to sale for ready Money.

Here is to be found what the Fine will amount to, being put out at Interest for so many Years as are to come between the Bargain and the beginning of the new Lease, according to the first Proposition of Interest: and then find what Annuity to endure for the like Term, with the Lease, may be bought for that Fine so increased by the 4*th Variety* of the third Proposition of Annuities; and that same Annuity is the Annual Rent desired.

Q. *Of the Rent upon renewing a Lease instead of a Fine.*

Example. Suppose I have 10 Years to come of an old Lease, and would renew it for 21 Years after the Expiration of the Old; for which I am demanded 100 *l.* to be paid presently, and instead thereof I offer such a Rent during the new Lease, as shall countervail the said present Fine, after the Rate of 10 *per Cent. per Annum* by Compound Interest: what shall that Rent be?

Answer.

Answ. l. 29,9903, *&c.* For the 100 *l.* paid 10 Years before-hand at the Rate propounded, will produce *l.* 259,3742, *&c.* which Sum will at the same Rate, purchase a Rent of *l.* 29,9903, *&c.* for 21 Years.

2,00000,00000	. Log. of 100. α.
0,41392,68520	. Log. for the Time 10 Years, as before.
2,41392,68520	. Log. of 259,3742, *&c.* ω.
1,00000,00000	. Log. of 10 Principal to raise 1 *l.* Annuity.
0,86924,63892	. Log. for the Time 21 Years, as before.
0,13075,36108	. Log. of 1,3513, *&c.* as before.
Ready Money	8,6486, *&c.* to buy 1 *l.* Annuity for 21 Years.
2,41392,68520	. Log. of 259,3742, *&c.* Price as before.
0,93694,58113	. Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
1,47698,10407	. Log. of 29,9903, *&c.* Annuity or Rent required.

Prop. 8. *To find an Annuity answerable to another, and Money to begin presently.*

Prop. 8. To get the Annuity for a certain Term, which another Annuity for a lesser or greater Term with some Money will buy to begin presently.

This Proposition is almost like the 4*th* before: For first must be found, what the Buyer's Annuity is worth ready Money, and thereto add the Money to be paid with the lesser Annuity, or the greater Annuity with the lesser Time, as the Case requires; and then find what Annuity that Sum will buy for the Term propounded, the one by the 4*th* Variety of the second Proposition of Annuities, and the other by the 4*th* of the Third.

Q. *Of an Annuity for 21 Years, bought for 200 l. and 50 l. per Annum.*

Example. What Annuity to endure for 21 Years is worth 200 *l.* ready Money, and an Annuity of 50 *l. per Annum* for 7 Years, reckoning 10 *l. per Cent. per Annum*, Interest upon Interest?

Answer.

Answ. l. 51,2708, *&c. per Annum.* For such an Annuity for 21 Years, at the Rate of 10 on the 100, will be purchased for the Sum of *l.* 443,4209, *&c.* that is, 200 *l.* in present Money, and *l.* 243,4209, *&c.* the Value of the Annuity of 50 *l. per Annum* for 7 Years.

2,

2,69897,00043 . Log. of 500 Principal to raise 50 *l.* Annuity.
0,28974,87964 . Log. for the Time 7 Years as before.
2,40922,12079 . Log. of 256,5790, *&c.*

Ready Money 243,4209, *&c.* to buy 50 *l. per Ann.* for 7 Years.

2,64681,62080 . Log. of 443,4209, *&c.* Price as before.
0,93694,58113 . Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
1,70987,03967 . Log. of 51,2708, *&c.* Annuity desired.

Prop. 9. To know by the Price or Fine of a Lease to endure for a certain Term, what a greater or lesser Term of the same is worth, not altering the Rent.

Prop. 9. *To find what a longer or shorter Term is worth, not altering the Rent.*

When the Fine given is the Price of the lesser Term, and the Value of the Greater required; then first by the 4*th* Variety of the third Proposition of Annuities, find what Annuity to endure with the first Lease may be bought for the Fine: And then by the 4*th* Variety of the second Proposition of Annuities, find what this last found Annuity esteemed with the second Lease is worth in ready Money, and this ready Money is the Value of the Lease for the greater Term.

If the Fine be the Price of the lesser Term, and the greater be sought.

Example. If a Lease of an House for 10 Years be worth 100 *l.* Fine, and 10 *l.* a Year Rent: What Fine is a Lease of the same House worth for 20 Years, not altering the Rent, after the Rate of 10 *l. per Cent. per Annum* by Compound Interest?

Q. *Of a Lease, with* 100 *l. Fine for* 10 *Years, what Fine for* 20 *Years.*

Answ. *l.* 138,5504, *&c.* because at the Rate aforesaid 100 *l.* will buy an Annuity of *l.* 16,2741, *&c.* for 10 Years; and this Annuity in 20 Years is worth as before *l.* 138,5504, *&c.* ready Money.

Answer.

1,00000,00000 . Log. of 10 Principal to raise 1 *l.* Annuity.
0,41392,68520 . Log. for the Time 10 Years.
0,58607,31480 . Log. of 3,8554, *&c.*

Ready Money 6,1445, *&c.* to buy 1 *l.* Annuity for 10 Years.

2,00000,00000 . Log. of 100 Price given.
0,78849,14961 . Log. of 6,1445, *&c.* Price of 1 *l.* Annuity.
1,21150,85039 . Log. of 16,2741, *&c.* Annuity for 10 Years.

2,21150,85039 . Log. of 162,7415, *&c.* Principal to raise *l.* 16,2741, *&c.* Annuity.
0,82785,37040 . Log. for the Time 20 Years.
1,38365,47999 . Log. of 24,1910, *&c.*

Ready Money 138,5504, *&c.* to buy *l.* 16,2741, *&c.* for 20 Years.

On the contrary, when the Fine given is the Price of the greater Term, and the Value of the Lesser is required: Then first find, as above, what Annuity to endure with the greater Term may be bought for the Fine; and afterward find what this last found Annuity is worth in ready Money, according to the lesser Term, and this ready Money is the Value of the Lease for that lesser Term.

If the Fine be the Price of the greater Term, and the Lesser be sought.

Example. *A* for a Lease of 50 Years paid 360 *l.* and after 11 Years agreed with his Landlord to surrender his Lease, conditionally, that he might receive so much as the Residue of the Years in the Lease will come to, after the Rate of 5 *l. per Cent. per Annum*, and reckoning the Rent half-yearly: how much therefore shall the Landlord pay back of the 360 *l.*?

Q. *Of* 360 *l. paid for a Lease of* 50 *Years, surrendred at* 11 *Years, how much to be paid back.*

Answ. *l.* 335,5696, *&c.* For after finding the Principal Money, that at 5 *l. per Cent. per Annum* Simple Interest, will raise 1 *l.* Annuity in half a Year, to be *l.* 40,4740, *&c.* the Fine 360 *l.* working as before, is found to buy an Annuity of *l.* 9,74433, *&c.* half-yearly, which for 39 Years (that is the Residue of 50 after expiration of 11) is worth in ready Money *l.* 335,5696, *&c.* as abovesaid, and plainly appeareth by the Operations following.

Answer.

$\frac{1}{2}$) 0,02118,92991 . Log. of 1,05. R. 2,00000,00000 . Log. of 100.
0,01059,46495$\frac{1}{2}$. Half. 0,39282,35307 . Log. of 2,4707, &c.
1,60717,64693 . Log. of 40,4740, &c.

1,60717,64693 . Log. of 40,4740, &c. Principal to raise 1 *l.* Annuity at Simple Interest in half a Year.
1,05946,49550 . Log. for the Time 50 Years.
0,54771,15143 . Log. of 3,5294, &c.

Ready Money 36,9445, &c. to buy 1 *l.* Annuity for 50 Years, payable half-yearly.

2,55630,25008 . Log. of 360 Price given.
1,56755,03222 . Log. of 36,9445, &c. Price of 1 *l.* Annuity.
0,98875,21786 . Log. of 9,7443, &c. Half-yearly Annuity for 50 Years.
2,59592,86479 . Log. of 394,3925, &c. Principal to raise *l.* 9,7443, &c. Half-yearly.
0,82638,26649 . Log. for the Time 39 Years.
1,76954,59830 . Log. of 58,8228, &c.

Ready Money 335,5696, &c. to buy *l.* 9,7443, &c. Half-yearly for 39 Years.

Prop. 10. *To turn a Rate on the Hundred, to a Purchase by the Year, &c.*

Prop. 10. To reduce a Rate on the Hundred to a Purchase by the Year, or the contrary.

Analogies.

To the Resolution of these serve the following Analogies, *viz.*

For the former; As the Rate, to an Unit: so is an Hundred, to the Number of Years desired.

For the Rate on the Hundred.

For the latter; As the Number of Years propounded, to an Unit: so is an Hundred, to the Rate desired.

As if I would know what Rates on the Hundred Lands bought at 16 or 20 Years Purchase is equivalent to:

Then as 16 . 1 :: 100 . 6$\frac{1}{4}$ } by the Latter.
or 20 . 1 :: 100 . 5 }

For the Years Purchase.

Or on the contrary; How many Years Purchase the Lands are bought for, when the Money is reckoned at 6$\frac{1}{4}$ *l.* or 5 *l.* *per Cent. per Annum*?

Then as 6,25 . 1 :: 100 . 16 } by the Former.
or 5 . 1 :: 100 . 20 }

Whereby it appears that Land bought for 16 Years Purchase, is all one with 6$\frac{1}{4}$ *per Cent. per Annum*; and Land bought for 20 Years Purchase, is alike as if the Purchase-Money were reckoned at 5 *l.* on the 100.

If the Enquiry be which is most.

When the Enquiry is, Which is most, so many Years Purchase, or such a Rate, both are to be gotten and compared together.

Q. Of 1000 *l. paid for* 70 *l. per Ann.* 7 *Years before any Profit, if more or less than* 20 *Years purchase.*

Example. September the 29*th*, 1643, A Merchant paid 1000 *l.* for certain Lands, which was out in Lease till *September* the 29*th*, 1650; during which Lease the Merchant shall receive but a Pepper-corn a Year, but afterward he will receive 70 *l.* *per Annum*: whether now doth this Merchant pay more or less than 20 Years Purchase?

Answer.

Answ. More than 20 Years Purchase, by 7 *l.* 2 *s.* +: For 20 Years Rent at 70 *l.* *per Annum*, is but 1400 *l.* and 1000 *l.* paid 7 Years before any Rent received, makes the Arrerages, with Interest upon Interest, after the Rate of 5 *l.* *per Cent. per Annum*, (that is all one with 20 Years Purchase) amount to the Sum of 1407 *l.* 2 *s.* + by the first Proposition of Interest.

0,02118,92991 . Log. of 1,05 . R. 70 *l.*
7 . T. 20
0,14832,50937 . Log. for the Time. 1400
3,00000,00000 . Log. of 1000. α.
3,14832,50937 . Log. of 1407,1004, &c. ω.

When

When the Enquiry is, Whether more or less be taken than a Rate propounded, or than so many Years Purchase, both being gotten and compared together, as last above-mentioned, the Difference will appear on which Side it is. *If the Enquiry be which is most of a Rate, or Year's Purchase.*

Example. If a Man disburse 500 *l.* and at 12 Months end receive in part 80 *l.* and at the end of every Half-year afterward 40 *l.* till 10 Years in all be expired: doth he take more or less than 10 in the 100 Interest upon Interest? *Q. Of 500 l. received first, 80 l. and 40 l. half-yearly, &c. whether 10 l. in the 100 Interest.*

Answ. He takes more than 10 in the 100 by *l.* 4,53977, *&c.* For seeing he is a Year before he receive any, the Year's Interest of 500 *l.* at 10 *l.* *per Cent. per Annum*, is 50 *l.* which added to 500 *l.* makes 550 *l.* out of which 80 *l.* taken, (which was then paid in) leaves 470 *l.* the Compound Interest whereof, at the Rate propounded for the other 9 Years, amounts with the Principal by the first Proposition of Interest, but to *l.* 1108,23567, *&c.* And the Rent of 40 *l.* every Half-year at the same Rate for nine Years, or eighteen Half-years, amounts to *l.* 1112,77544, *&c.* by the first Proposition of Annuities. Answer.

0,04139,26852	. Log. of 1,10 R.	0,37253,41668	. Log. for the Time.
9	. T.	2,91352,44915	. Log. of 819,4538, *&c.*
0,37253,41668	. Log. for the Time.	3,28605,86583	. Log. of 1932,2292, *&c.*
2,67209,78579	. Log. of 470.		1112,7754, *&c.*
3,04463,20247	. Log. of 1108,2356, *&c.*		

When the Enquiry is, which is best, such a Sum or such an Annuity, the ready Money of the Annuity gotten is to be compared with the Sum propounded. *If the Enquiry be, whether a Sum or an Annuity be best.*

Example. *A* offers *B* 50 *l.* for an Annuity of 10 *l.* *per Annum* for 8 Years: whether were *B* best to accept thereof, accompting the Compound Interest at 10 *l.* *per Cent. per Annum*? *Q. Of 10 l. per Ann. for 8 Years, or 50 l. be best.*

Answ. No; because an Annuity of 10 *l.* yearly for 8 Years, at the Rate propounded, is worth in ready Money *l.* 53,349, *&c.* by the first Proposition of Annuities, which is *l.* 3,349, *&c.* more than 50 *l.* Answer.

0,04139,26852	. Log. of 1,10. R.
8	. T.
0,33114,14816	. Log. for the Time.
2,00000,00000	. Log. of 100 Principal to raise 10 *l.* Annuity.
1,66885,85184	. Log. of 46,6507, *&c.*
	53,3492, *&c.*

In like manner when the Difference between so many Years Purchase, and the Money paid, or to be paid for the Purchase, is sought; both are gotten, and the one substracted from the other. *If the Difference of the Years Purchase & the Price be sought.*

Example. Upon *May*-Day, *Anno Regni Elizabethæ* 30, *A* sold to *B* certain Lands, that yielded 33 *l.* 16 *s.* 8 *d.* *per Annum* Rent: At which Sale *A* and *B* agreed thus, That *A* should pay to *B* 50 *l.* upon the first Day of *November* next following: And that then *B* should surrender the Lands back again to *A* (in nature of a Mortgage) for the Paiment of 100 Marks *per Annum* for eight Years, *viz.* at the end of every Year, still upon the first Day of *November*, 66 *l.* 13 *s.* 4 *d.* *Example in fulfilling an Order of the High Court of Chancery.*

Before the Day appointed for the first Paiment *B* died, so as the Surrender was not made, nor any of the abovesaid Paiments paid: Wherefore upon Suit in *Chancery*, it was ordered by the Lord Keeper, that there should be an Accompt made, and the Questions in the Accompt to be, what the Land was worth at the Sale, after the Rate of 20 Years Purchase; and what the Paiments above-mentioned were worth in ready Money at the same Time, reckoning the Interest at 10 *per Centum per Annum*; And then look how much the Land was worth more than the Paiments, so much should the Heirs of *B* pay to *A*: how much therefore by this Order ought to be paid to *A*? *The Case.*

Answ. *l.* 289,8833, *&c.* For such is the Difference between *l.* 676,6666, *&c.* the Worth of the Land at 20 Years Purchase, and *l.* 386,7833, *&c.* the Sum of the ready Money of the Annuity added to the 50 *l.* and taking from the Total one *What to be paid according to the Order*

one half Year's Intereſt, becauſe by the Agreement the 50 *l.* was not to be paid, nor the Annuity to begin till one half Year after the Bargain was made.

l. : *s.* : *d.*		
33 : 16 : 08	.	Yearly Rent of the Land.
20	.	Years.
676 : 13 : 04	.	Sum of 20 Years Purchaſe.

0,04139,26852 . Log. of 1,10. R.
$8\frac{1}{2}$.

0,33114,14816
0,02069,63426

Marks
2,82390,87366 . Log.of 666,6666, &c. Principal to raiſe 100 Annu.
0,33114,14816 . Log. for the Time 8 Years.
2,49276,72550 . Log. of 311,0049, &c.

Marks
Ready Money 355,6617, &c. to buy an Annuity of 100 for 8 Years.
50 added.

2,60816,40598 . Log. of 405,6617, &c.
0,02069,63426 . Log. for the Time ½ Year.
2,58746,77172 . Log. of 386,7833, &c.
676,6666, &c.—386,7833, &c. = 289,8833, &c.

Proof of Anatociſm. The Varieties of Queſtions are even numberleſs; but what hath been ſaid in this Chapter, is ſufficient for a through underſtanding of all *Anatociſm:* And ſince both the way of Work by *Decimals*, and alſo by *Logarithms* hath been ſeen, and ſeveral of the Operations and Propoſitions converſe one to another; the Truth of the Concluſions is ſufficiently proved thereby, and to give farther Example here, will be altogether ſuperfluous.

Partis tertiæ Libri quarti

FINIS.

The

The Fourth PART of the Fourth BOOK.

CHAP. I.

Of EQUATIONS.

Equations close the Work. They are Compound Proportions, or Proportions of Equality.

NOthing remains now to finish the whole Work, but to overlook *Equations*, which in the Chapter of *Ratio's* before are called *Compound Proportions*, to distinguish them from others; and sometimes *Proportions of Equality*, because the Number or Numbers, Magnitude or Magnitudes, between which Comparison is made, are equal one to the other in Value, though of different Denominations.

Wherein their Computation consists. The several Names.

The whole Computation of *Equations*, consisteth in the Invention, Reduction and Resolution of an *Equation*; and is called (though by some, with reference only to the latter) the *Art of Equation*, *Rule of Equation*, *Rule of Cos*, *Rule of Quantity*, *Almucabula*, (signifying an hidden or secret Tradition) *Algeber*, or *Algeber*'s *Rule*, another Arabick word taken by some for the Name of the Inventor, from the Article *Al* and *Geber* a Man; and by some that it was *Geber* the old Arabian: But others will have it only for a Name of singular Excellence, *quasi Ars magistralis*, it being of such Perfection, that it performs not only what may be done by other Rules of Proportion, *Alligation*, *False Position*, and the Rules of *Archindus*, and six Quantities of *Cataym*, (as *Digges* in his *Stratioticos* tells us) but with much Facility, Evidence and Demonstration. *Excellency thereof.*

Algebra from whence. How to be wrote.

From this *Algeber* came the Name *Algebra*, now most common thereto: Neverthelefs neither the one nor the other of them is the right Word, but *Algiebar*, as Dr. *Dee* in his Math. Preface, &c. proves from *Avicen*, translated by *Andreus Alpagus*, (one very skilful in the Arabick Tongue) who calls The Science of working *Algiebar* and *Almachabel*, The Science of finding an unknown Number, by adding of a Number, *Division* and *Equation*: Nor could *Geber* (however skilled therein, and other profound Knowledg) be the Inventor, since as that Learned Doctor saith, *Diophantus*, a Greek Philosopher and Mathematician, before the Time of *Geber*, wrote 13 Books thereof. *Geber not the Inventor.*

Parallels of what use. How called. What to be understood thereby.

The Numbers or Quantities compared one to another, are set one on the one Side, and the other on the other, of a pair of Parallels or Gemowe Lines, called the *Sign of Equation*, and sometime Metonymically called *Equation* it self, there being no two things more alike or equal, than such two right Lines ═══ equally extended, whereby is saved the oft repetition of the words *equal to*: As if I would say, 1 *s.* is equal to 12 *d.* or 3 Crowns are equal to 15 *s.* they are set thus:

Example.

s.		*d.*	∧		*s.*
1	═══	12	3	═══	15

So shall one Shilling be understood to be equal in Value to twelve Pence, and 3 Crowns equivalent to 15 Shillings, though their Denominations differ, and neither 1 nor 12, nor yet 3 and 15 are alike.

The Sorts of Equations.

Equations, in the Chapter of *Ratio's* before-mentioned, were there divided into two principal Sorts, *viz.* *Pure* and *Mixt*, as one Number or Magnitude compared to another, or many others to one or to many, and both Sorts there observed principally to converse with Contract Numbers: Not but that sometime Abstract Numbers are used, as well as and with those that are Contract; but if the Denominations in Contract, or the Abstract Numbers themselves on both Sides of the *Equation*, be alike, the *Equation* is either Nugatory or Impossible. For when the *Equation* shall be identical in Numbers and Denominations, it is nugatory or trifling; as to say, 3 ʒ ═ 3 ʒ, is all one as to say, 3 Squares is as much as 3 Squares. And when the Denominations are identical, and the Numbers annexed different, the *Equation* is impossible, for to say 2 ʒ ═ 3 ʒ, is as much as to say 2 ═ 3, *Abstract Numbers used how. Nugatory and Impossible Equations, what.*

which is impoſſible. So as if one Number or Magnitude be valued with another, their Denominations muſt be different; but one greater Abſtract Number may be compared to divers leſſer Numbers, and one Contract Number of higher Power or Demonination with divers lower.

One ſort of Equation reduced to another.

Oftentimes by *Reduction*, (as afterwards in the Chapter thereof may be ſeen) *Equations* of the latter Sort are reduced from one Form to another: For ſome of the Numbers on the one Side may be abated, or elſe carried over to the other Side of the *Equation*; ſo as one Number may be left alone at one Side, which is therefore called, the *Solitary Side of the Equation.*

Solitary Side, what.

Equality of a Number to its Parts abſtract, what helped thereby.

Although *Equations* are moſt frequent with Contract Numbers, yet the Equality conſidered between Abſtract Numbers, (as leſſer Numbers may be Parts of the Greater) hath not been without ſome Effect, and among others in giving Light to the Way of parting equally an even Magnitude by 2 odd Parts thereof: And hence this Problem gained Solution, *viz.*

To part one Veſſel of Liquor by unequal Veſſels.

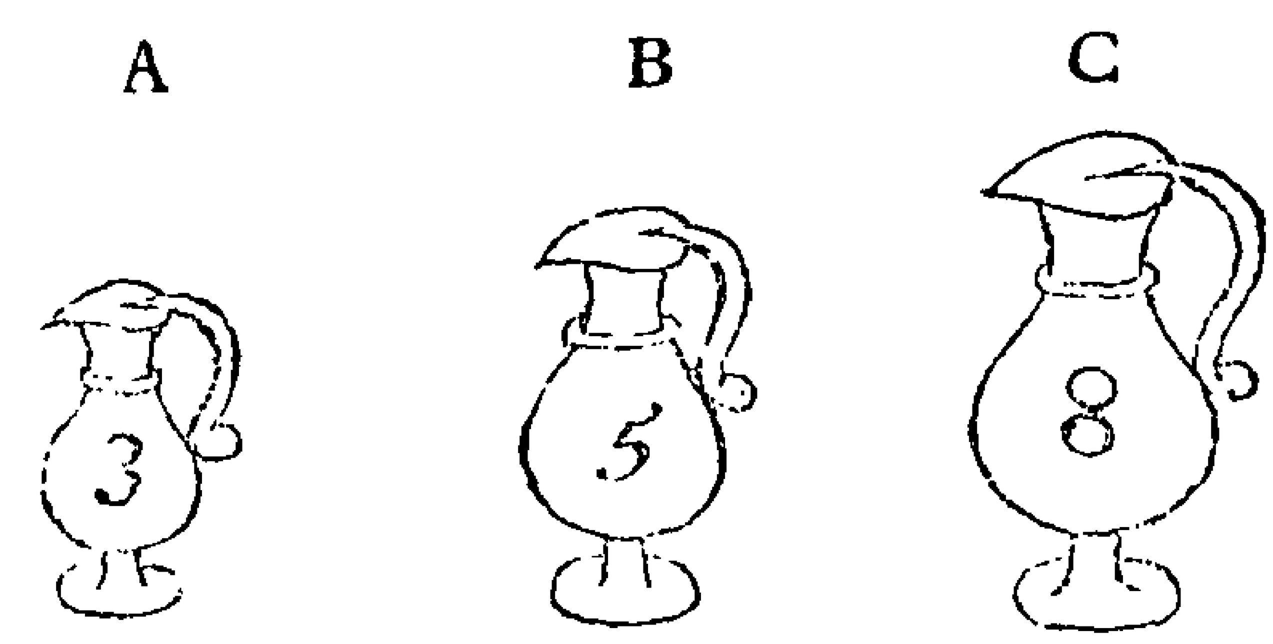
To part a Veſſel full of Wine, or other Liquor, into two equal Parts, by other Veſſels, each of them unequal Parts of the greater Veſſel, and both together equal thereto.

What Parts to be taken.

Nevertheleſs all Parts of a Number which together are equal thereto, are not taken to make ſuch a Diviſion, leſt the *Equation* ſhould be nugatory: But thoſe moſt apt for the *Equation*, and which have a ſeeming Difficulty, as one an Unit below, and the other an Unit above the Half: For ſuppoſe a Veſſel full of Wine contain 8 Pints or Gallons, &c. propounded to be equally divided by two other Veſſels unequal, containing together as much as the other, theſe two Veſſels muſt be one 5, and the other 3 Pints or Gallons, &c. which are one above 4, the Half of 8, and the other 3, one below 4: For if the Parts were 1 and 7, or 2 and 6, there were no difficulty therein.

Example.

Let then the 3 Veſſels be repreſented by *A*, *B*, *C*; of which *C* being full, and the other 2 empty, pour out of *C* into *B* till it be full, ſo will there be left 3 Pints in *C*; then fill *A* out of *B*, ſo will there be in *A* 3 Pints, in *B* 2, and in *C* 3: Then emptying *A* into *C*, it will have 6 Pints, and *B* 2, but *A* none. Furthermore, emptying the 2 Pints of *B* into *A*, and filling *B* out of *C*, there will be in *A* 2 Pints, in *B* 5, and in *C* but 1: But now it is evident, that if from *B* you fill *A*, yet will *B* retain 4 Pints; and if *A* be emptied into *C*, there will be 4 Pints alſo.



Equations chiefly are uſed in Contract Numbers, but not to be confounded therewith.

However the chief exerciſe of *Equations* be among Contract Numbers, and of them eſpecially the 3 latter, *viz.* *Coſſicks*, *Surds*, and *Species*; yet I ſee little reaſon to crowd all the Simple Elements thereof into the Accompt of *Equations*, and leſs to ſubject all Figural Numbers, and their Operations, under the Title of *Algebra*, as ſome have done, but confuſedly.

Nor becauſe *Equations* frequently deal with *Coſſicks*, (whence the Rule for reſolving an *Equation* came to be called the *Rule of Coſs*) will it be reaſonable to ſubjugate the whole Proceſs of *Equations* with *Coſſicks*, as is done by others; ſince *Equations* are as familiar to *Species* as to them.

How ſome ſort Equations.

The late uſe of *Species* hath occaſioned ſome to make three Sorts of *Equations.*

1. *Pure.*

The firſt Pure or Abſolute, wherein is no Power on either Side of the *Equation.*

2. *Powerful.*

2. A powerful *Equation*, wherein is ſome Figural Number or other.

3. *Affected.* *Reckoned here only of 2 ſorts, Pure and Affected.*

3. A mixt or affected *Equation*, made up of both, and of many Numbers: But ſeeing all *Equations* by Coſſical Reduction may be brought to two Numbers or more, thoſe brought thereby to 2 only, are here reckoned for pure *Equations*, though the one thereof be a Power, and all others are to be counted for mixt *Equations.*

As

As well pure *Equations* as those mixt, (now commonly known by the Name of *Affected*, and sometime *Adfected*) when made up with Powers, whether Cossical or others, come under farther observation. *Both sorts, both considered.*

1. When the Quantities are orderly different one from the other, without omission of any Quantity between them. *1. Quantities orderly distant, none omitted.*

Examples.	*Pure.*	√	*Affected.*	√
	1 ℨ = 3 N	3	1 ʒ = 2 ℨ + 3 N	3
	2 ʒ = 4 ℨ	2	1 φ = 6 ʒ − 5 ℨ	5
	4 φ = 12 ʒ	3	3 φ = 2 ʒ + 10 ℨ − 4 N	2

In all these Examples, one Denomination immediately succeeds the other; for between φ and ʒ. ʒ and ℨ. ℨ and N. no Quantity intervenes.

2. When the Quantities are orderly different one from the other, but between them are 1, 2, 3, or more Quantities omitted. *2. Quantities orderly distant, some omitted.*

Examples.	*Pure.*	√	*Affected.*	√
	1 ʒʒ = 4 ʒ	2	1 ʒʒ = 12 ʒ + 64 N	4
	1 ſs = 16 ℨ	2	1 ʒφ = 108 ʒ − 3 ʒʒ	3
	1 ʒφ = 32 ℨ	2	1 φφ = 7 ʒφ + 9 φ − 8 N	2

In the first Pure, between the Quantities ʒʒ and ʒ, is one Quantity, *viz.* φ omitted. In the next, between ſs and ℨ, three Quantities are omitted, *viz.* ʒʒ φ. and ʒ. And in the other Instance are omitted four Quantities, ſs. ʒʒ. φ. and ʒ. between ʒφ and ℨ.

In the first Affected is omitted only one Quantity, that is φ, between ʒʒ and ʒ, and ℨ between ʒ and N. In the next also one Quantity only is omitted, *viz.* ſs between ʒφ and ʒʒ, and φ between ʒʒ and ʒ. But in the last Instance two Quantities are omitted, that is ʒʒʒ and Bſs, between φφ and ʒφ, and ſs and ʒʒ between ʒφ and φ, and ʒ and ℨ between φ and N.

3. When the Quantities omitted in an *Affected Equation*, are between the Sides or Parts of the *Equation*, but on either Side none. *3. Quantities in Affected between the Parts omitted.*

Examples.

	√
1 ʒφ = 30 ℨ + 4 N	2
2 ʒʒ + 3 φ = 82 ℨ − 3 N	3

Here between ʒφ on the solitary Side, and ℨ on the other Side in the first Example, are four Quantities omitted, but between ℨ and N none. And in the second Instance, between φ and ℨ is but one Quantity omitted, and none on either Side between ʒʒ and φ, or between ℨ and N.

4. When the Quantities omitted in an *Affected Equation* are on either Side, but none between the Sides or Parts. *4. Quantities in an Affected between the Parts none, but on either side omitted.*

Examples.

	√
1 ʒφ = 1 ſs + 32 N	2
4 φφ − 3 Bſs = 10 ʒφ + 1024 N	2

Between ʒφ and ſs in the first Example, no Quantity is wanting, but ʒʒ. φ. ʒ. and ℨ. are omitted between ſs and N. And in the other Example Bſs at one Side the 7*th* Quantity, and ʒφ the 6*th* Quantity at the other Side, shew none omitted; but between φφ and Bſs one Quantity is omitted, and between ʒφ and N five Quantities are wanting.

5. When the Quantities omitted are disorderly between, and on both Sides of the *Equation*. *5. Quantities omitted disorderly.*

Examples.

	√
1 φφ + 2 ʒ = 20 ſs − 4 ℨ − 112 N	2
3 ʒφ − 100 ℨ = 16 ʒʒ + 618 N − 1 φ	3

The first Example wants 6 Quantities between φφ and ʒ of the one Side, and 3 Quantities between ſs and ℨ of the other Side. And ʒφ is distant from ℨ in the latter Example 4 Quantities on the one Side, and φ from N 2 Quantities on the other Side.

The Truth of these, and other *Equations*, may be tried, having the Root as *Cossicks* before noted, by taking Abstract Numbers. For in the last Example, seeing the Root is 3, the ʒφ thereof shall be 729, the ʒʒ 81, and the Cube 27: So shall 3 ʒφ be 2187; from which 100 ℨ, that is 300 taken, there shall remain *Proof of Equations.*

for

for the left Side of the *Equation* 1887. And so 1655 is 1296, to which 618 added, makes 1914; from whence 27 the Sum of 1 ȹ abated, leaves 1887 for the right Side of the *Equation* equal to the other.

$$2187 - 300 = 1296 + 618 - 27 = 1887.$$

CHAP. II. *Invention of Equations.*

Invention of Equations first to be done. WHEN a Question is propounded to be resolved by an *Equation*, the *Equation* is first to be found, and such an one too as is apt and pertinent to the Question. This *Equation* found, (because many times altered by *Reduction*, as in the next Chapter) is sometime called the *First Equation* in resolving the Question; and the finding thereof, here called *Invention*, is by some called *Composition*.

First Equation what, and why so called. *Invention, &c. how called.*

What the Steps thereto. He that would arrive at the end of *Equations*, will find it convenient, for the *Invention* of an *Equation*, to proceed gradually by these two following Steps.

1. To be stored with Analytical Provision. 1. Let him be well stored with Analytical Provision, to use as occasion serves: For as in examining a Proposition of 4 Proportionals, it is necessary to know, That the Rectangle of the Means is equal to that of the Extreams: So in a Work of *Equations*, it is as well beneficial as necessary, to know both how to deduce and draw an *Equation* out of a Proposition, and out of the first and most easy *Equations*, (which are nothing but the simple Affections or Expositions of the Terms) to draw forth, by meet and proper Consequences, other *Equations* equal to the First, but in other Terms.

Example by two Numbers given, what may be found thereby. For suppose of two Numbers given, A be the Greater, E the Lesser, Z the Sum, X the Difference, &c. as before shewed in *Species*, it will follow by *Oughtred*, Chap. 11. of his *Clavis*:

1. By A and E, what found. 1. That by the *Species* for any two Numbers given, (whereof one the Greater, and the other the Lesser) may be found the 13 following Conclusions.

Data.	*Quesita.*	*Resolution.*
Greater A.	Sum.	Z=A+E.
	Difference.	X=A-E.
	Rectangle.	P=Æ, or AE.
	Sum of the Squares.	Z=Aq+Eq.
	Difference of the Squares.	X=Aq-Eq.
	Sum of the Sum and Difference.	Z+X=2A.
	Difference of the Sum and Difference.	Z-X=2E.
	Rectangle of the Sum and Difference.	ZX=Aq-Eq, or X.
	Square of the Sum.	Zq=Aq+2Æ+Eq, or Z+2Æ
	Square of the Difference.	Xq=Aq-2Æ+Eq, or Z-2Æ
	Sum of the Squares of the Sum and Difference.	Zq+Xq=2Aq+2Eq, or 2Z.
	Difference of the Squares of the Sum & Difference.	Zq-Xq=4Æ.
Lesser E.	Square of the Rectangle.	Pq=Æq, or AqEq.

2. By Z and A or E, what found. 2. That by the Species for the Sum of any two Numbers, (whereof one the Greater, and the other the Lesser) and one of those Numbers, may be found the other, the Difference, the Rectangle, the Sum of the Squares, and the Difference of the Squares.

Data.	*Quesita.*	*Resolution.*
Greater A	Lesser.	E = Z—A.
	Difference.	X = 2A—Z.
	Rectangle.	P = ZA—Aq.
	Sum of the Squares.	Z = Zq—2ZA+2Aq.
	Difference of the Squares.	X = 2ZA—Zq.
Sum Z		
	Greater.	A = Z—E.
	Difference.	X = Z—2E.
	Rectangle.	P = ZE—Eq.
	Sum of the Squares.	Z = Zq—2ZE+2Eq.
Lesser E	Difference of the Squares.	X = Zq—2ZE.

3. That

3. That by the Species for the Difference of any 2 Numbers, (whereof one the Greater and the other the Lesser) and one of those Numbers, may be found the other, the Sum, the Rectangle, the Sum of the Squares, and the Difference of the Squares. *3. By X & A or E, what found.*

Data.	*Quæsita.*	*Resolution.*
Greater A	Lesser.	$E = A - X$.
	Sum.	$Z = 2A - X$.
	Rectangle.	$P = Aq - XA$.
	Sum of the Squares.	$Z = 2Aq - 2XA + Xq$.
	Difference of the Squares.	$X = 2XA - Xq$.
Difference X.		
	Greater.	$A = E + X$.
	Sum.	$Z = 2E + X$.
	Rectangle.	$P = Eq + XE$.
	Sum of the Squares.	$Z = 2Eq + 2XE + Xq$.
Lesser E	Difference of the Squares.	$X = 2XE + Xq$.

4. That by the Species for the Rectangle of any 2 Numbers (whereof one the Greater and the other the Lesser) and one of those Numbers, may be found the other, the Sum, the Difference, the Sum of the Squares, and the Difference of the Squares. *4. By P & A or E, what found.*

Data.	*Quæsita.*	*Resolution.*
Greater A.	Lesser.	$E = \frac{P}{A}$.
	Sum.	$Z = \frac{Aq + P}{A}$.
	Difference.	$X = \frac{Aq - P}{A}$.
	Sum of the Squares.	$Z = \frac{Aqq + Pq}{Aq}$.
	Difference of the Squares.	$X = \frac{Aqq - Pq}{Aq}$.
Rectangle P.		
	Greater.	$A = \frac{P}{E}$.
	Sum.	$Z = \frac{P + Eq}{E}$.
	Difference.	$X = \frac{P - Eq}{E}$.
	Sum of the Squares.	$Z = \frac{Pq + Eqq}{Eq}$.
Lesser E.	Difference of the Squares.	$X = \frac{Pq - Eqq}{Eq}$.

5. That by the Species for the *Ratio* of any 2 Numbers (whereof one the Greater, and the other the Lesser) and one of those Numbers, may be found the other, the Sum, the Difference, the Rectangle, the Sum of the Squares, and the Difference of the Squares. *5. By RS & A or E, what found.*

Data.	*Quæsita.*	*Resolution.*
Greater A, Ratio R. S	Lesser.	$E = \frac{SA}{R}$.
	Sum.	$Z = \frac{RA + SA}{R}$.
	Difference.	$X = \frac{RA - SA}{R}$.
	Rectangle.	$P = \frac{SAq}{R}$.
	Sum of the Squares.	$Z = \frac{RqAq + SqAq}{Rq}$.
	Difference of the Squares.	$X = \frac{RqAq - SqAq}{Rq}$.
Ratio R. S, Lesser E	Greater.	$A = \frac{RE}{S}$.
	Sum.	$Z = \frac{RE + SE}{S}$.
	Difference.	$X = \frac{RE - SE}{S}$.
	Rectangle.	$P = \frac{REq}{S}$.
	Sum of the Squares.	$Z = \frac{RqEq + SqEq}{Sq}$.
	Difference of the Squares.	$X = \frac{RqEq - SqEq}{Sq}$.

Many more may be deduced thence. Value not altered by different Expressions.

And not only these, but from these a Multitude of other Propositions may be deduced by Consequence, and yet the different Terms of Expression alter nothing of the equal Value of such Conclusions; as supposing according to the first of these Propositions A be 3, and E 2. then,

Examples.

$$\text{Because}\left\{\begin{array}{l} Z + X = 2A \\ 5 + 1 = 6 \\ Z - X = 2E \\ 5 - 1 = 4 \\ Zq - Xq = 4Æ \\ 25 - 1 = 24 \end{array}\right\}\text{shall}\left\{\begin{array}{l} \frac12 Z + \frac12 X = A \\ 2\frac12 + \frac12 = 3 \\ \frac12 Z - \frac12 X = E \\ 2\frac12 - \frac12 = 2 \\ \frac14 Zq - \frac14 Xq = Æ \\ \frac{25}{4} - \frac14 = 6 \end{array}\right\}$$

$$\text{And}\left\{\begin{array}{l} Z = A + E = 2A - X = 2E + X = \frac{Aq + P}{A} = \frac{P + Eq}{E}, \ \&c. \\ 5 = 3 + 2 = 6 - 1 = 4 + 1 = \frac{9 + 6}{3} = \frac{6 + 4}{2}, \ \&c. \\ X = A - E = 2A - Z = Z - 2E = \frac{Aq - P}{A} = \frac{P - Eq}{E}, \ \&c. \\ 1 = 3 - 2 = 6 - 5 = 5 - 4 = \frac{9 - 6}{3} = \frac{6 \quad 4}{2}, \ \&c. \end{array}\right.$$

Authors where Store of such Provision is.

Deductions of this sort being endless, to endeavour their recital here were an *Herculean* Task: He that desires to see more, let him read *Bachetus* before *Diophantus*; *Billy* before his Continual Proportionals, Mr. *Thomas Harriot* his *Artis Analyticæ Praxis*, Mr. *Jonas Moore* in the *9th* and *10th* Chapters of his Second Book, Mr. *Richard Balam* the *16th* Chapter of his *Algebra*; and above all that profound Analyst Mr. *William Oughtred* in the *18th* Chapter of his *Clavis Mathematicæ limata*, whom none hath outdone.

2. Step to find the Equation to suppose in Cossicks 1 ꝛ, *in Species A for that sought. When a to be taken.*

2. Being well provided of such Store as aforesaid, and grounded in the common Principles of *Arithmetick* and *Geometry*, (because sometime the Use of some known Problem will much expedite and ease the Work) the next Step of the Artist is to suppose the Number or Magnitude sought in any Question to be 1 ꝛ working in *Cossicks*, or in *Species* A, and the given Magnitudes Consonants. But if A be any of the *Data*, let the Capital Letter be changed for *a*, or some other not given, for distinction sake.

And

How to proceed with the Supposition.

And with this Supposition, proceed to examine the Question according to the Tenor thereof (as though this were given, and you were proving the Truth of it) till you bring it to an *Equation.*

Q. Of a Wager, how much offered to be laid.

Example. Two Men in a Controversy are upon a Wager, one offers to lay 20 *l.* the other to lay as much Money as will make the Sum of both when added together as much as the Product, if one Number be multiplied by the other: how much then did he offer to lay on the Wager against the other 20 *l?*

Resolution.

Here supposing he offered to lay 1 ℨ; this according to the Question must be added to 20, and multiplied by 20; when added, the Total is 1 ℨ + 20, when multiplied, the Product is 20 ℨ; and so I have gotten an *Equation* that is 1 ℨ + 20 = 20 ℨ. And this being the first *Equation*, and capable of *Reduction*, as in the next Chapter, by taking 1 ℨ at the one side from 20 ℨ at the other; so will there be left 20 = 19 ℨ another *Equation*, and be resolved by Consequence, if 19 ℨ be 20, that 1 ℨ shall be $1\frac{1}{19}$ *l.*

What to be done if more be sought than one thing.

If more than one Supposition.

Sometime for ease in working the Question, A may be joined with some Number, and sometime the Numbers or Magnitudes sought are more than one; and then oftentimes one being found, the other is known by Consequence. But if it be necessary to suppose more than once, let the Suppositions be different Letters if the Work be in *Species*; and as many Letters as are used for Roots, so many several *Equations* to find out the Value of those Letters in Relation to A the first Supposition, which is to be kept distinct from others. And sometime A being supposed, those other sought Numbers or Magnitudes shall by Consequence be drawn from A before the first *Equation* be found: but till *Reduction* be learned, one Example more may suffice here.

Q. Of the Number of Bottles of Wine.

Example. A Vintner sold 30 Bottles of Wine for 210 Pence, whereof some were White, others Claret; but he sold one Bottle of White for 5 *d.* and a Bottle of Claret for 8 *d:* how many Bottles of each sort were there?

Resolution in Species.

By supposing for the White.

Here the Numbers given being thus noted, *viz.* 30 *B*, 5 *C*, 8 *D*, 210 *F*, as in divers other Questions where 2 Things are sought, the Supposition may be for either. And if A be supposed for the White, the Value of A when found, shall be the Number of those Bottles: But if A be supposed for the Claret, the Number of those Bottles shall be denoted thereby; wherefore supposing A to be the Number of the Bottles of White-wine, then by consequence because there were 30 Bottles in all, the Number of the Bottels of Claret must be B—A; then each part multiplied by their Rates respectively, it will follow the White to be CA, and the Claret to be DB—DA, and both together to be equal to F. So will the first *Equation* be found, and stand thus, CA + DB — DA = F, which by *Reduction* as in the next Chapter may be brought to $A = \frac{DB-F}{D-C}$. So if from DB 240 be taken F 210, and the residue 30 divided by 3, that is, D—C, the Value of A is found to be 10 for the Number of Bottles of White-wine; and then by consequence there must be 20 Bottles of Claret, because the whole were 30; and 10 at 5 *d.* a Bottle, and 20 at 8 *d.* a Bottle make together 210 *d.*

By supposing for the Claret.

In like manner, If A had been supposed for the Claret, then should the White be B—A, which severally multiplied by their Roots, makes the Product of the Claret to be DA, and the Product of the White to be CB—CA, and both together the first Equation DA + CB — CA = F: And by the Reduction of the next Chapter brought to $A = \frac{F-CB}{D-C}$. So from F 210, shall 150 the Product of C into B be taken, and the Remainder 60 divided by 3, that is, D—C, or 8—5, and the Quotient 20 shall be the Bottles of Claret.

Resolution in Cossicks.

And those less accustomed to Species, may work with Cossicks thus: If 1 ℨ be supposed for the Number sought, either of White or Claret, then the other by Consequence shall be 30 — 1 ℨ; and accordingly each multiplied by the Prices respectively shall produce, if 1 ℨ be supposed for the White, 5 ℨ + 240 — 8 ℨ; but if 1 ℨ be supposed for the Claret, 8 ℨ + 150 — 5 ℨ; and either of them must be equal to 210: And when reduced, as taught in the next Chapter, the one will be 3 ℨ = 30, or 1 ℨ = 10 of the White; and the other will be 3 ℨ = 60, or 1 ℨ = 20 of the Claret, as before.

Supposition is always the true Number sought.

From hence it appears, that this Supposition 1 ℨ or A, is always the true Number or Magnitude sought, and no such false Position, as before in the *Rule of Falshood* was taken: But it is observed, that as to those that know not the Cause, it was

Difference herein from Falshood. was somewhat strange there, out of False Positions to procure True Conclusions, (seeing commonly Erroneous Premisses have such Conclusions); so here it seems much more strange and admirable, that a right Number shall be taken at first touch before any absolute Knowledg can be had what it is: And on this Accompt *Record* says, it may be called the *Rule of Dark* or *Strange Position*. *Rule of dark or strange Position.*

Proof of the Invention of Equations. The Different work by Cossicks and Species, seeing both concur in the Conclusion, may serve for a Proof of the Truth of both Operations.

CHAP. III. *Reduction of Equations.*

Reduction *of* Equations. THE first *Equation* found as suggested, either by the Sense of the Question, or by some known Theorem pertinent thereto, according to the former Chapter; the next Work is to reduce or order this *Equation*, so that it may be fit for Resolution.

How called by Balam. *Balam* calls this *Reduction*, *Equative Inference*, Chap. 13. But seeing, according to the Nature of all *Reduction*, the Terms of the *Equation* are only altered thereby, but the same Equality always kept, it may very properly be called *Reduction*.

The sorts of Reduction. *Equations* are reduced several ways.

1. *By the Simple Elements of Numbers; as* First, By some of the Simple Elements of Numbers, as *Addition*, *Substraction*, *Multiplication* and *Division*: For according to the common Axiom, equal things added to or taken from, or multiplying or dividing equal things, shall make the Totals, Remains, Products or Quotients, accordingly equal.

Addition *to take off Fractions, or increase the* Data. *Addition* is used on both sides of an *Equation*, to clear the Species or Quantities of Fractions, or increase the *Data* with New Quantities.

Examples. *Examples.* $5ʒ + 2\tfrac{1}{2}N = 6ȥ + 10\tfrac{1}{2}N$ $\sqrt{2}$.

$5Aq + 2\tfrac{1}{2}B = 6A + 10\tfrac{1}{2}B$

In both, if $\tfrac{1}{2}$ be added on either side, they stand thus:

$5ʒ + 3N = 6ȥ + 11N$. $5Aq + 3B = 6A + 11B$.

So because $A + E = Z$, if B be added, it shall be that $A + E + B = Z + B$.

Substraction *to take away Fractions, and double Denominations.* *Substraction* is used as well to clear the *Equation* of double Denominations, as of Fractions: And so not only $\tfrac{1}{2}$ may be taken from either Side of the former Example, and the *Equations* stand thus:

$5ʒ + 2N = 6ȥ + 10N$. $5Aq + 2B = 6A + 10B$.

Examples. But because the $2\tfrac{1}{2}$ and $10\tfrac{1}{2}$, on both Sides of the *Equation* are of one Denomination, *viz.* Absolute Numbers, the lesser of them (seeing their Signs are alike, that is, both +) must be taken from the greater; and so $2\tfrac{1}{2}$ taken from $10\tfrac{1}{2}$, leave the *Equations* thus:

$5ʒ = 6ȥ + 8N$. $5Aq = 6A + 8B$.

If doubled at one Side. If any Denominations on the one Side of the *Equation* be doubled, they must be added or substracted to lessen the Terms.

Example. *Example.* $13ʒ = 2φ + 1ʒ + 2ȥ + 4N - 3ʒ$ $\sqrt{2}$.

Because ʒ is found with + and − on one Side of the *Equation*, $+1ʒ$ shall be taken from $-3ʒ$, and the Remain $-2ʒ$, set down in the *Equation* thus:

$13ʒ = 2φ + 2ȥ + 4N - 2ʒ$.

To lessen the Value. *Substraction* is also used to lessen the Value of the first given Species by new Ones. And so if $A + E = Z$ and B be taken away, then shall $A + E - B = Z - B$.

Multiplication *to turn Fractions into Integers &c Thus called* Conversion *and Isomeria.* *Multiplication* is used to turn Fractions, or Heterogeneal Numbers, into Integers or Homogeneal; this is sometime called *Conversion*; and by *Vieta*, *Isomeria*, and is thus done: Multiply cross-wise after the manner of Fractions, all the other Species or Magnitudes, into the Denominator of each other's Fraction except his own, and these Products shall make the New *Equation*. And seeing the Work is as *Reduction* of common Fractions, as occasion serves, the greatest Common Divisor may be used.

If

If one Side be an Integer, and the other a Fraction; then multiply the Integer by the Denominator of the Fraction, and set this Product against the Numerator of the Fraction for the new *Equation*.

Examples.

Examples. $\frac{3\phi+3\text{ʒ}}{3\mathfrak{z}}=6\text{N}.$ $\qquad A+B=\frac{D}{C}.$

These converted by *Multiplication*, shall be thus:

$3\phi+3\text{ʒ}=18\mathfrak{z}.$ $\qquad AC+BC=D.$

Thus $A-C=\frac{Aq+Bq}{D}+B+C$ converted, is $DA-DC=Aq+Bq+DB+DC.$

So where the Fractions are on both Sides of the *Equation*.

Examples. $\frac{1\mathfrak{z}+2\text{N}}{3\phi}=\frac{1\text{ʒ}-2\mathfrak{z}+3\text{N}}{3\text{ʒ}+6\text{N}}.$ $\qquad A+\frac{B}{DC}=B+\frac{5A}{G}.$

Converted by *Multiplication*, shew themselves thus:

$3\text{ſs}+9\phi-6\text{ʒʒ}=3\phi+6\text{ʒ}+6\mathfrak{z}+12\text{N}.$ $\qquad DCGA+BG=BGDC+5DCA.$

Division is used, either to depress an *Equation* in Numbers or Quantities, or both, and so reduce them to their least Terms; and therefore is this often called *Depression*, and is the *Hypobibasmus* of *Vieta*: Or else when the Magnitude sought, or his highest Power in the *Equation* is joined with some other Species, then must the rest of the Species be divided by that Species so adjoined, and the higher Power cleared thereof: And this latter Sort is *Vieta* his *Parabolismus*.

Division *to reduce them to their least Terms,* &c. *This called* Depression, Hypobibasmus, *and* Parabolismus.

Examples of the first sort are performed, in all respects, like *Reduction* of Compound Cossical Fractions and Species.

Examples of Hypobibasmus.

As $3\text{ſs}=6\text{ʒʒ}+6\text{ʒ}+12\mathfrak{z}-6\phi$ $\qquad \sqrt{2}.$

Depressed in Numbers. $\qquad$ *Depressed in Quantities.*

$1\text{ſs}=2\text{ʒʒ}+2\text{ʒ}+4\mathfrak{z}-2\phi.$ $\qquad 1\text{ʒʒ}=2\phi+2\mathfrak{z}+4\text{N}-2\text{ʒ}.$

And thus the Cossical Examples last above-mentioned, after Conversion, may be also depressed; that is,

$3\phi+3\text{ʒ}=18\mathfrak{z}.$ to $1\text{ʒ}+1\mathfrak{z}=6\text{N}.$ $\qquad \sqrt{2}.$

And the other in Numbers only to

$1\text{ſs}+3\phi-2\text{ʒʒ}=1\phi+2\text{ʒ}+2\mathfrak{z}+4\text{N}.$

Examples in Species. $3Ac+3Aq=18A.$

Depressed in Numbers. $\qquad$ *Depressed in Quantities.*

$Ac+Aq=6A.$ $\qquad Aq+A=6.$

And depressed in the Quantities only as,

$Ac+BAq=DA.$ to $Aq+BA=D.$

Examples of the latter sort, or Parabolism.

Examples of Parabolismus.

$AqB=D+C$, divided by B is, $Aq=\frac{D+C}{B}.$

And so $AqB-DA=R$, divided by B is, $Aq-\frac{D}{B}\times A=\frac{R}{B}.$

And $DAq+DqA=Zc$, divided by D is, $Aq+DA=\frac{Zc}{D}.$

Secondly, Equations are reduced by Transposition or Translation, that is, removing some Species or Quantities from one Side of the *Equation* to the other; in which Removal the Signs shall be changed, for + at one Side of the *Equation* shall be − at the other. This sort of *Reduction* is used, when the same Denominations or Quantities are on both Sides of the *Equation*, to lessen the Terms, and when one Side is to be left solitary.

2. *Reduction by Transposition or Translation.*

Thus in the first Example of the former Chapter, where the first *Equation* found was $1\mathfrak{z}+20=20\mathfrak{z}$, seeing $\mathfrak{z}$ on both Sides the *Equation* with +, the lesser is abated out of the greater; so $1\mathfrak{z}$ taken from $20\mathfrak{z}$, the Remain $19\mathfrak{z}$ answered at one Side to the 20N at the other Side.

Example to lessen Terms.

But if it had been 22N — 1ℨ = 20ℨ, then the Signs being contrary, — 1ℨ at one Side shall be added to + 20ℨ at the other Side; and the *Equation* would have been by this Transposition, adding 1ℨ to each Side, 22N = 21ℨ.

Example to leave one Side solitary. So to leave one Side solitary in Transposition of the next *Equation* there, the Signs are accordingly changed, for +C is made —C, and —D is made +D; and because +DB was transposed with the same Sign he had before, the Sign of F remaining on that Side he was before, is changed from + to — .

$$CA + DB - DA = F \text{ transposed is, } A = \frac{DB - F}{D - C}.$$

In like manner all other Transpositions are performed.

As 1ſs + 3φ — 2ʒʒ = 1φ + 2ʒ + 2ℨ + 4N transposed, shall be
1ſs = 2ʒʒ + ʒ + 2ℨ + 4N — 2φ.

And DA—DC=Aq+Bq+DB+DC shall be DA=Aq+Bq+DB+2DC.

If one Side be left empty, what to be done. Sometime in *Transposition*, one side of the *Equation* is left empty, and then the Quantities with + shall be equal with them that are —; wherefore the Defect is easily supplied, for it never happeneth but when the Quantities are joined with contrary Signs.

Example. *Example.* 4ʒ=5ʒ+10N—7ℨ; here transposing because ʒ is on both sides, and taking 4ʒ from 5ʒ, the *Equation* will be left thus, 0=1ʒ+10N—7ℨ: but now because 7ℨ are—, they are equal to 1ʒ+10N, and the *Equation* may be set thus, 7ℨ = 1ʒ + 10N, or by placing the highest Power therein Solitary, 1ʒ=7ℨ—10N.

3. Reduction by Exaltation, when one Side is a Power. *Thirdly.* An *Equation* is sometime reduced by *Exaltation*, which is used when some *Cossick* or *Species* on the one side of the *Equation* is referred to the Power of a Number on the other side; for then must the plain Part of the *Equation* be exalted to that Power by squaring, cubing, &c. that plain Part, and cancelling the Cossick Character on the other Part.

Example. *Example.* If 1ℨ+4N=√ʒ36, then shall 1ℨ+4N be squared on the one side and set against 36 on the other Side, and √ʒ cancelled. And so 1ℨ+4N=√ʒ36 reduced, shall be 1ʒ+8ℨ+16N=36N, and translated thus, 1ʒ+8ℨ=20N, or thus, 1ʒ=20N—8ℨ.

Examples in Species.

B + C = √A exalted, shall be Bq + 2BC + Cq = A.
And B = √AB — D transposed, is B + D = √AB.
And then exalted shall be Bq + 2BD + Dq = AB.

$$\text{And if A be left Solitary } \frac{Bq + 2BD + Dq}{B} = A.$$

Proof of Reduction *of* Equations. That all these sorts of *Reduction* keep the Numbers and Magnitudes still in equality, may be proved by reducing the *Cossicks* and *Species* into *Integers*; and then comparing them before *Reduction* with those reduced, the Truth of the several Operations will appear.

CHAP. IV. *Resolution of Equations.*

Resolution *of* Equations. AFTER the Invention and Reduction of the *Equation*, as in the two Chapters last before, the *Equation* is to be resolved, being so found and ordered that *What to be set by themselves.* the known Magnitudes may be on the one side, and the unknown on the other, if *Species* be used, but if *Cossicks*, the highest Quantity set Solitary.

Resolution, what. The Resolution of an *Equation* is to find out the Value of the Root supposed, often called, but improperly, the Root it self, because the Root of every *Cossick* *Root of an* Equation *improperly called so.* Number must be a *Cossick* Number, and such as by Multiplication will make that Rooted Number, which this so called Root will not.

How gotten. The Value of this Supposititious Root is to be procured or extracted as the *Equation* is Pure or Affected.

Touching

Touching Pure Equations.

Of pure Equations.

1. To get the Value of 1ℨ, *when the Quantities are orderly distant, and none omitted.*

1. When the Quantities or Magnitudes compared one to another, are orderly different each from other without omission of any Quantity between them, then divide the Number of the lowest Denomination by the Number of the highest Denomination, and the Quotient shall be the Value of 1ℨ. But oftentimes the Reduction of the first found *Equation* brings the Numbers so low, that Division is needless, because 1ℨ is one part of the *Equation*.

Q. Of the Value of a Cloak.

Example 1. A Gentleman hireth a Servant to serve him a Year for 24 *s.* and a Cloak; but at 8 Months end they fall at variance, and the Gentleman dischargeth his Servant, and giveth him 13 *s.* and the Cloak: what was the Cloak rated at?

Resolution.

Answ. 9 *s.* Here supposing the Value of the Cloak sought to be 1ℨ or A, or keeping the *Species* for Cloak; if 8 Months give 1 Cloak and 13 *s.* then shall 12 Months give $1\frac{1}{2}$ Cloak and $19\frac{1}{2}$ *s.* and this shall be equal to the whole Year's Wages, and being set thus $1\frac{1}{2}$Cl + $19\frac{1}{2}$ *s.* = 1 Cl + 24 *s.* by Transposition of both Species will be reduced to $\frac{1}{2}$Cl = $4\frac{1}{2}$ *s.* And so by consequence without Division it is easily perceived, if half a Cloak be worth 4 *s.* 6 *d.* the whole Cloak shall be valued at 9 *s.* and such will be the Quotient if $4\frac{1}{2}$ be divided by $\frac{1}{2}$.

Q. Of the Age of Alexander, &c.

Example 2. *Alexander* being asked how old he was, answered, he was 2 Years elder than *Ephestio*; yea said *Ephestio*, and my Father is as old as we both and 4 Years more; and my Father, said *Alexander*, having all those Years, was 96 Years of Age: the Question is, how old each of them was?

Answer.

Here supposing the Age of *Ephestio* (who was the Youngest) to be 1ℨ, then *Alexander* being 2 Years Elder, must be 1ℨ + 2N. and then must the Father of *Ephestio* be 2ℨ + 6N (that is, 4 Years more than them both) all which added together make up the Age of *Alexander*'s Father, that is, 4ℨ + 8N, and is to be equal to 96.

This *Equation* 4ℨ + 8N = 96N reduced by *Transposition* to 4ℨ = 88N, if depressed, or the 88 divided by 4, makes 1ℨ = 22N for the Age of *Ephestio*.

Supposition.		*Proof.*
1ℨ	Age of *Ephestio*	22 Years.
1ℨ + 2	Age of *Alexander*	24
2ℨ + 6	Age of *Ephestio*'s Father	50
4ℨ + 8	Age of *Alexander*'s Father	96

2. To get the Value of 1ℨ *when the Quantities are omitted.*

2*ly*. When the Quantities or Magnitudes compared one to another are distant one from another by 1, 2, 3, or more Quantities or Powers, after *Reduction* of the *Equation* first found as before, and Division thereof (if occasion be) from the Quotient, extract a Root according to the Quantities omitted; that is, if one Quantity be omitted, extract the Square Root, if 2 the Cube Root, if 3 the squared Square Root, &c.

Q. Of the Length and Breadth of a Pavement.

Example 1. A Floor paved with square Bricks, is longer than it is broad by $\frac{1}{7}$; and the whole Pavement contained 3584 Bricks: what was the Length and Breadth thereof?

Resolution.

Answ. 56 Broad, and 64 Long: For supposing the Breadth which is the lesser Quantity to be 1ℨ, then shall the Length be $1\frac{1}{7}$ℨ: now multiplying the Breadth by the Length, (as in all such Forms to find the *Area*) there is produced $\frac{8}{7}$ʒ, which according to the Proposition must be equal to 3584, and by *Reduction* 8ʒ = 25088, and again, 1ʒ = 3136: and so if in this *Equation* 3584 be divided by $\frac{8}{7}$, will the Quotient be 3136; and because between ʒ and N, one Denomination is omitted, that is ℨ, the Square Root of 3136 is taken, which is 56 for the Breadth, to which 8 that is $\frac{1}{7}$ of 56 added, makes 64 for the Length.

And if in this Instance 1ℨ had been supposed for the Length, then shall the Breadth be $\frac{7}{8}$ℨ, and the Square $\frac{7}{8}$ʒ, and $\frac{7}{8}$ʒ = 3584, which divided by $\frac{7}{8}$ is 4096, whose Square Root is 64, as before.

Q. Of the Length, Breadth and Height of a Pile of Brick.

Example 2. A Pile of Brick, whose Length was to the Breadth as 7 to 2, that is, *Tripla-sesquialter*, and the Height was 5 times as much as the Length; being sold for 980 Crowns, the Owner received for every Yard so many Crowns as the Pile had Yards in Breadth: what then was the Length, Breadth, and Height of this Pile?

Resolution. *Supposition for the Breadth.*

Answ. Two Yards broad, 7 Yards long, and 35 Yards high. For here supposing the Breadth 1ℨ, then was the Length $3\frac{1}{2}$ℨ, and the Height $17\frac{1}{2}$ℨ; all which multiplied one into another, make $\frac{245}{4}$φ, and this shall be equal to all the Yards

in

in the whole Pile: And feeing every Yard coſt as many Crowns as the Breadth contained Yards, if the Breadth be 1 ℨ, then did every Yard coſt 1 ℨ of Crowns; and then by the *Golden Rule*,

	Yard.		*Crowns.*		*Whole Yards.*		*Crowns.*
As	1	.	1 ℨ	::	$\frac{245}{4}$ Φ	.	$\frac{245}{4}$ ʒʒ.

So muſt $\frac{245}{4}$ ʒʒ $=$ 980 Crowns, and conſequently 245 ʒʒ $=$ 3920 Crowns; wherefore dividing 3920 by 245, the Quotient is 16; from whence the *Zenzizenzike* Root taken, becauſe three Quantities are omitted in this *Equation*, the Root 2 is the Breadth, therefore ſhall 7 be the Length, and 35 the Height, as before; each of which will be gotten in like manner, if Suppoſition be made for them.

For the Length and Height.

Suppoſition for the Length.

1 ℨ Length.
$\frac{2}{7}$ ℨ Breadth.
5 ℨ Height.
$\frac{10}{7}$ Φ Pile.

y △ y △
1 . $\frac{2}{7}$ ℨ :: $\frac{10}{7}$ Φ . $\frac{20}{49}$ ʒʒ.

$\frac{20}{49}$ ʒʒ $=$ 980 △

20 ʒʒ $=$ 48020 △

1 ʒʒ $=$ 2401 △ √ʒʒ 7. *Length.*

Suppoſition for the Height.

1 ℨ Height.
$\frac{1}{5}$ ℨ Length.
$\frac{2}{35}$ ℨ Breadth.
$\frac{2}{175}$ Φ Pile.

y △ y △
1 . $\frac{2}{35}$ ℨ :: $\frac{2}{175}$ Φ . $\frac{4}{6125}$ ʒʒ.

$\frac{4}{6125}$ ʒʒ $=$ 980 △.

4 ʒʒ $=$ 6002500 △.

1 ʒʒ $=$ 1500625 △ √ʒʒ 35. *Height.*

Proof thereof. And that all theſe Varieties are true, appears, becauſe if every Yard of the whole Pile coſt 2 Crowns, that is as many as there were Yards in the Breadth, then ſhall the whole Pile coſt 980 Crowns, becauſe it contained 490 ſolid Yards.

	Breadth.		*Length.*		*Height.*	*Solid Yards.*		△		△
For	2	×	7	×	35 $=$	490	×	2	$=$	980

Falſhood *cannot do what* **E-quations** *can.* By the Work of theſe two laſt Examples of the Pavement and Pile it appeareth, That though all the Operations of *Falſhood* may be performed by *Equations*; yet ſuch Queſtions as theſe, which concern Figural Forms, will not be wrought right by *Falſhood*, as at leiſure the Artiſt may make experiment.

Several other Queſtions reſolved. Under theſe two Sorts of *Reſolution* fall very many of the Examples, Queſtions and Propoſitions propounded in moſt Books of *Arithmetick*: Some of which follow, reſolved by way of *Coſſicks* as well as *Species*, and may ſerve to ripen the Apprehenſions of the Ingenious, as well in the Invention (the moſt difficult Part) of the *Equation*, as in the *Reſolution* of ſuch kind of Propoſals.

Q. Of a Number multiplied by 6, and added to 8. 1. What Number is that which multiplied by 6, and the Product added to 8, will be 48?

Reſolution. *Anſw.* $6\frac{2}{3}$: For if multiplied by 6, and 8 be added, the Total is 48.

B 6. C 8. D 48. Then $A \times B + C = D$, or $AB + C = D$.

$$\text{And } A = \frac{D-C}{B}, \text{ or } \frac{48-8}{6} = \frac{40}{6} = 6\tfrac{2}{3}.$$

1 ℨ × 6 $=$ 6 ℨ. And 6 ℨ + 8 $=$ 48. Then will 6 ℨ $=$ 40
And 1 ℨ $=$ $6\frac{2}{3}$.

Q. Of 2 Parts of 100, &c. 2. Which are the two Parts of 100, that the Quotient of the greater Part divided by 3, added to the Quotient of the leſſer Part divided by 5, ſhall together make 30?

Reſolution. *Anſw.* 75 and 25: For 75 divided by 3 gives 25, and 25 divided by 5 gives 5, and 25 and 5 make together 30.

B 100. D 30. Greater Part $A = a$ Suppoſition.
Leſſer Part $E = B - a$ Conſequence.

$$\text{Then } \frac{a}{3} + \frac{B-a}{5} = D. \quad \text{And } 5a + 3B - 3a = 15D.$$

$$\text{And } 2a + 3B = 15D. \quad \text{And } a = \frac{15D - 3B}{2} \text{ or } \frac{450-300}{2} = \frac{150}{2} = 75.$$

If

If A be 1ᵹ, then E 100 — 1ᵹ.

$\frac{1ᵹ}{3} + \frac{100-1ᵹ}{5} = 30$. Then $5ᵹ + 300 - 3ᵹ = 450$.

And $2ᵹ + 300 = 450$. And $2ᵹ = 150$. And $1ᵹ = 75$.

3. Two Numbers, one Greater and the other Lesser, or A and E, the Greater is as much as the Lesser, and 4 more; and the Square of the Greater is equal to the Square of the Lesser, and 32 more: what are those Numbers? *Q. Of 2 Numbers, what they are.*

Answ. Greater 6, Lesser 2: For $6 = 2 + 4$; and $36 = 4 + 32$. Resolution.

B 4. D 32. Greater suppose *a*. Lesser consequently *a*—B.

Then $aq = aq - 2aB + Bq + D$. And $2aB = Bq + D$.

And $a = \frac{Bq+D}{2B}$. Or $\frac{16+32}{8} = \frac{48}{8} = 6$.

A E B

1ᵹ 1ᵹ — 4 D

1ʒ = 1ʒ — 8ᵹ + 16 + 32. Or 1ʒ = 1ʒ — 8ᵹ + 48.

And 8ᵹ = 48. And 1ᵹ = 6.

And thus both in this and the next precedent Example, may the lesser Number be found if the Supposition be therefore.

4. Two Numbers, or A and E, the Square of A lacking the Square of E is 45, and A lacking E is 5: what are those Numbers? *Q. Of 2 Numb. what they are.*

Answ. A 7, and E 2: For $49 - 4 = 45$. And $7 - 2 = 5$. Resolution.

B 45. X 5. Greater supposed *a*. Lesser consequently *a*—X.

Then $aq - aq - 2aX + Xq = B$. And $2aX = B + Xq$.

And $a = \frac{B+Xq}{2X}$, Or $\frac{45+25}{10} = \frac{70}{10} = \overset{E}{7}$

A E

1ᵹ 1ᵹ — 5

1ʒ — 1ʒ — 10ᵹ + 25 = 45. And 10ᵹ — 25 = 45.

And 10ᵹ = 70. And 1ᵹ = 7.

5. A Number joined with 18, and taken from 100, the Sum and Remain are found to be in a *Subtriple Ratio*, that is, as 1 to 3: what is that Number? *Q. Of the Ratio of a Sum and Remain, what the Number.*

Answ. $11\frac{1}{2}$: For $29\frac{1}{2}$ to $88\frac{1}{2}$, is as 1 to 3. Resolution.

B 18. C 100. R 1. S 3. Supposition A.

Then B + A . C — A :: R . S. And because the Product of the Extreams is equal to the Product of the Means, it shall be that $RC - RA = BS + SA$.

And $A = \frac{RC-BS}{R+S}$, Or $\frac{100-54}{1+3} = \frac{46}{4} = 11\frac{1}{2}$

1ᵹ. Then 1ᵹ + 18 . 100 — 1ᵹ :: 1 . 3.

And 100 — 1ᵹ = 3ᵹ + 54. And 4ᵹ + 54 = 100.

And 4ᵹ = 46. And 1ᵹ = $11\frac{1}{2}$.

6. There is a Number from which if $\frac{2}{5}$ be taken, the Number will be left as much under 100, as it was at first above: what is that Number? *Q. Of a Number, &c.*

Answ. 125, of which $\frac{2}{5}$ is 50. And $125 - 50 = 100 - 25$. Resolution.

$\frac{R}{S}$ $\frac{2}{5}$. B 100. Supposition A.

Then $A - \frac{R}{S} = \frac{SA-RA}{S}$. And $B - \frac{SA-RA}{S} = A - B$.

And $2B = \frac{2SA-RA}{S}$. And $2BS = 2SA - RA$.

And $A = \frac{2BS}{2S-R}$. Or $\frac{200\times5}{2\times5-2} = \frac{1000}{8} = 125$.

1ᵹ greater than 100, so shall 1ᵹ — $\frac{2}{5}$ᵹ be lesser than 100.

And 100 being the Mean between them, 100 = 1ᵹ — $\frac{1}{5}$ᵹ.

And $\frac{4}{5}$ᵹ = 100. And 1ᵹ = 125.

Q. *Of the Stock of two Merchants.*

7. Two Merchants trade till they gain 150*l.* whereof *A* having 200*l.* in Stock more than *B*, taketh 100*l.* what Money had each in Stock?

Resolution.

Answ. *A* 400*l.* *B* 200*l.* For seeing *A* of the 150 took 100, that is 50*l.* more than the 50*l.* left for *B*; it must follow, That if 50*l.* Gain require 200*l.* Stock, then 100*l.* Gain shall require 400*l.* Stock.

C 150. D 200. F 100. Suppose Stock of *B* to be a. Then *A* is $a+\mathrm{D}$.

And as $2a+\mathrm{D}\ .\ \mathrm{C}\ ::\ a\ .\ \frac{\mathrm{C}a}{2a+\mathrm{D}}$. And $\frac{\mathrm{C}a}{2a+\mathrm{D}}+\mathrm{F}=\mathrm{C}$.

And $\mathrm{C}a+2a\mathrm{F}+\mathrm{DF}=2a\mathrm{C}+\mathrm{DC}$. And $2a\mathrm{F}-\mathrm{C}a=\mathrm{DC}-\mathrm{DF}$.

And $a=\frac{\mathrm{DC}-\mathrm{DF}}{2\mathrm{F}-\mathrm{C}}$. Or $\frac{30000-20000}{200-150}=\frac{10000}{50}=200$.

And by *Cossicks*, supposing the Stock of *B* to be 1℥: Then *A* is 1℥ + 200.

And as 2℥ + 200 . 150 :: 1℥ . $\frac{150℥}{2℥+200}$.

And $\frac{150℥}{2℥+200}+100=150$. And 150℥ + 200℥ + 20000 = 300℥ + 30000.

And 50℥ = 10000. And 1℥ = 200.

Q. *Of Travel.*

8. A Man travelling 9 Miles a Day, is followed by another from the same Place, that sets forth 10 Days after, and went 14 Miles a Day: in how many Days will the latter overtake the former?

Resolution.

Answ. In 18 Days: For 5 Miles (that is 14 — 9) Gain in a Day will reach 90 Miles (that is 10x9) in 18 Days.

B 9. C 90. D 14. Supposition A.

Then $\mathrm{BA}+\mathrm{C}=\mathrm{DA}$. And $\mathrm{C}=\mathrm{DA}-\mathrm{BA}$.

And $\mathrm{A}=\frac{\mathrm{C}}{\mathrm{D}-\mathrm{B}}$. Or $\frac{90}{14-9}=\frac{90}{5}=18$.

1℥. Then shall 9℥ + 90 = 14℥. And 5℥ = 90.
And 1℥ = 18.

Q. *Of the Days a Servant worked and played.*

9. A Gentleman hired a Servant for a Year, upon Condition that for every Day he laboured, he should have 1 *s.* and for every Day he was idle, he should lose or discount 8 *d.* Now at the Year's end the Master was by this Agreement to give the Servant nothing, nor was the Servant in the Master's Debt: what Days did the Servant work, and how many Days did he play?

Resolution.

Answ. 146 he wrought, and 219 he plaid: For 146 Shillings and 219 Eightpences are equal.

B 365. C 12. D 8. Greater supposed A. Lesser then B—A.

Then $\mathrm{CA}=\mathrm{BD}-\mathrm{DA}$. And $\mathrm{CA}+\mathrm{DA}=\mathrm{BD}$.

And $\mathrm{A}=\frac{\mathrm{BD}}{\mathrm{C}+\mathrm{D}}$. Or $\frac{365\times 8}{12+8}=\frac{2920}{20}=146$.

1℥ Greater, 365 — 1℥ Lesser. Then 12℥ = 2920 — 8℥.
And 20℥ = 2920. And 1℥ = 146.

Q. *Of two Sorts of Monies.*

10. There are two Sorts of Monies, in Number 1000 Pieces, worth 80*l.* whereof 10 of the first Kind, and 20 of the other are worth 1*l.* what Number were there of both these Sorts severally?

Resolution.

Answ. 600 of the one, and 400 of the other: For the Quotients of 600 divided by 10, and 400 by 20, make together 80.

B 1000. C 10. D 20. F 80. Greater supposed A. Lesser then B—A.

Then $\frac{\mathrm{A}}{\mathrm{C}}+\frac{\mathrm{B}-\mathrm{A}}{\mathrm{D}}=\mathrm{F}$. And $\mathrm{DA}+\mathrm{BC}-\mathrm{CA}=\mathrm{DCF}$.

And $\mathrm{A}=\frac{\mathrm{DCF}-\mathrm{BC}}{\mathrm{D}-\mathrm{C}}$. Or $\frac{20\times 10\times 80-1000\times 10}{20-10}=\frac{16000-10000}{10}$

And $\frac{6000}{10}=600$ of the Greater.

1 z Greater. Then 1000 — 1 z the Lesser.

Then $\frac{1z}{10} + \frac{1000-1z}{20} = 80$. And $20z + 10000 - 10z = 16000$.

And $10z + 10000 = 16000$. And $10z = 6000$. And $1z = 600$.

11. Suppose *Berwick* and *London* are distant 228 Miles, out of which 2 Foot-posts take their Journeys, and meet the 12*th* Day; but the one went each day one Mile farther than the other: how many Miles did each of them go every day? *Q. Of two Posts, how many Miles they travel.*

Answ. 10 the one, and 9 the other. For the difference between them being 1, yet each of them severally multiplied by 12, will make together 228. Resolution.

X 1. B 12. C 228. Greater supposed A. Lesser consequently A — X.

Then $BA + BA - BX = C$. And $A = \frac{C + BX}{2B}$.

Or $\frac{228+12}{24} = \frac{240}{24} = 10$ Greater. 9 Lesser.

1 z Greater. 1 z — 1 Lesser.

Then as 1 Day . 1 z :: 12 Days . 12 z.

And as 1 . 1 z — 1 :: 12 . 12 z — 12.

And $24z - 12 = 228$.

And $24z = 240$. And $1z = 10$ Greater. 9 Lesser.

12. Suppose the same Distance as above, and the one Post travel 10 Miles a Day from the one Place, and the other 9 from the other: when shall they meet? *Q. Of two Posts, when they meet.*

Answ. The 12*th* Day: For 10 multiplied by 12, and 9 by 12, make together 228. Resolution.

B 10. C 9. D 228. Supposition A.

Then $BA + CA = D$. And $A = \frac{D}{B+C}$. Or $\frac{228}{19} = 12$.

1 z. Then $10z + 9z = 228$. And $19z = 228$. And $1z = 12$.

13. Three are in Company: The Stock of *A* and *B* together is 238 *l.* of *B* and *C* 470 *l.* of *A* and *C* 568 *l.* what Stock had each of them severally? *Q. Of the Stock of 3 Merchants.*

Answ. *A* 168 *l.* *B* 70 *l.* *C* 400 *l.* For $168 + 70 = 238$. And $70 + 400 = 470$. And $168 + 400 = 568$. Resolution.

Persons *A*, *B*, *C*. Stocks supposed *a*, *b*, *c*. D 238. F 470. G 568.

Then $a = D - b$. And $a = G - c$. Therefore $D - b = G - c$.

And $c = G + b - D$. And because by the *Data* it appears that $c = F - b$. Therefore $G + b - D = F - b$. And $2b + G = F + D$.

And $2b = F + D - G$. And $b = \frac{F + D - G}{2}$. Or $\frac{470 + 238 - 568}{2} =$

$\frac{140}{2} = 70$.

In *Cossicks*: If *b* be supposed 1 z, then *a* is 238 — 1 z. And *c* is 330 + 1 z.

And $330 + 2z = 470$. And $2z = 140$. And $1z = 70$.

14. One buying 100 Yards of Cloth, being demanded what a Yard cost, answered, For how much less than 80 *s.* I bought 40 Yards, by so much less than 95 *s.* I bought 50 Yards: what was the Price of a Yard? *Q. Of the Price of a Yard of Cloth.*

Answ. 1 *s.* 6 *d.* For 40 Yards at that Rate is 60 *s.* which is 20 less than 80; and 50 Yards at that Rate is 75 *s.* less than 95 by 20 also. Resolution.

B 40. C 50. D 80. F 95. Supposition A.

Then $BA - D = CA - F$. And $F - D = CA - BA$.

And $A = \frac{F - D}{C - B}$. Or $\frac{95 - 80}{50 - 40} = \frac{15}{10} = 1\frac{1}{2}$.

1 z. Then $40z - 80 = 50z - 95$. And $40z = 50z - 15$.

And $10z = 15$. And $1z = 1\frac{1}{2}$.

15. A General marshaling his Men in *Battalia*, intended a Square Form; and to that purpose, when he had placed a Number at random in the Front, found *Q. Of the Number of Soldiers, and the Fronts.*

100

100 Men over; and thinking to amend it by placing one more in the Front, found 201 Men wanting: how many Souldiers had he, and what were the Fronts?

Resolution. *Answ.* The whole Number of Souldiers were 22600, the first Front 150, the other 151; for the Square of 150 is 22500, to which 100 added, makes 22600; and the Square of 151 is 22801, which is 201 above 22600.

B 100. C 201. First Front supposed A. Then the Second A+1.
Square thereof Aq. Square thereof Aq+2A+1.
Then $Aq+B=Aq+2A+1-C$. And $B=2A+1-C$.

And $2A=B+C-1$. And $A=\frac{B+C-1}{2}$. Or $\frac{100+201-1}{2}=\frac{300}{2}=150$.

1 ℨ First Front. Then the Second shall be $1ℨ+1$.
Square of the First 1 ʒ. Square of the Second $1ʒ+2ℨ+1$.
Then $1ʒ+100=1ʒ+2ℨ-200$. And $100=2ℨ-200$.
And $2ℨ=300$. And $1ℨ=150$.

Likewise if Supposition be made for the second Front 1 ℨ; Then the first Front shall be $1ℨ-1$, and their Squares 1 ʒ, and $1ʒ+1-2ℨ$; and the Body of Men by the one shall be $1ʒ-201$, and by the other $1ʒ+101-2ℨ$; which *Equation* transposed $2ℨ=302$; and $1ℨ=151$.

Q. *Of Legacies to 4 Children.* 16. A poor Man dying which had 4 Children, bequeathed 72 Crowns to his 4 Children; so that the Second and Third should have together 7 times as much as the First; and the Portions of the Third and Fourth should be five Times so much as the Second's Part, and the First and Fourth should have twice so much as the Third: how were the 72 Crowns to be divided?

Resolution. *Answ.* A $4\frac{1}{2}$. B $11\frac{1}{4}$. C $20\frac{1}{4}$. D 36. which together make 72. And $11\frac{1}{4}+20\frac{1}{4}=31\frac{1}{2}$, that is 7 times $4\frac{1}{2}$. And $20\frac{1}{4}+36=56\frac{1}{4}$, that is 5 times $11\frac{1}{4}$. And $4\frac{1}{2}+36=40\frac{1}{2}$, that is twice $20\frac{1}{4}$.

Persons, A 1. B 2. C 3. D 4. Crowns b. Supposition for A if a. Then by Consequence must B+C be $7a$; and D the Residue be $b-8a$. And because $A+D=2C$, therefore $C=\frac{b}{2}-3\frac{1}{2}a$; and because $C+D=5B$, therefore $5B=1\frac{1}{2}b-11\frac{1}{2}a$; or $5B=\frac{3}{2}b-\frac{23}{2}a$. And $B=\frac{3}{10}b-\frac{23}{10}a$, and the Total shall be equal to b.

$$
\begin{aligned}
\text{That is } A &= a \\
B &= \tfrac{3}{10}b - 2\tfrac{3}{10}a. \\
C &= \tfrac{b}{2} - 3\tfrac{1}{2}a. \\
D &= b - 8a. \\
\text{Total } & 1\tfrac{4}{5}b - 12\tfrac{4}{5}a = b.
\end{aligned}
$$

And $\frac{4}{5}b=\frac{64}{5}a$. And $4b=64a$.

And $b=16a$. And $a=\frac{b}{16}$, or $\frac{72}{16}=4\frac{1}{2}$.
1 ℨ supposed for A. Then $B+C=7ℨ$. And D the Rest $72-8ℨ$.
And the half of A and D is $36-3\frac{1}{2}ℨ$ for C. So shall the Part of B be $\frac{1}{5}$ of $108-11\frac{1}{2}ℨ$, that is $21\frac{3}{5}-2\frac{3}{10}ℨ$. And the Total collected thus,

$$
\begin{aligned}
A &= 1ℨ \\
B &= 21\tfrac{3}{5} - 2\tfrac{3}{10}ℨ \\
C &= 36 - 3\tfrac{1}{2}ℨ \\
D &= 72 - 8ℨ \\
\text{Total } & 129\tfrac{3}{5} - 12\tfrac{4}{5}ℨ = 72.
\end{aligned}
$$

And $12\frac{4}{5}ℨ=57\frac{3}{5}$.

And $64ℨ=288$. And $1ℨ=4\frac{1}{2}$.

Q. *Of keeping Sheep.* 17. A Farmer agreeth with a Shepherd to keep 80 Sheep for a Year, and at 3 Months end delivereth the Shepherd 30 Sheep more; and 4 Months after that 3 Months bringeth to the Shepherd 30 Sheep more, saying to him, Keep me all these Sheep so long, till such time as the Money I promised you at first be all earned: how long must he keep these 140 Sheep?

Resolution. *Answ.* Nine Months the 80 Sheep, six Months the first 30, and two Months the last 30; which together are equivalent to the keeping of 80 Sheep twelve Months.

B 80. C 12. D 30. F 3. G 7. Supposing the Time of keeping 80 be A, then must the Time of keeping the first 30 be A—F; and of the latter be A—G.

And

And then $BA+DA-DF+DA-DG=BC$. And by transposition,

$$BA+2DA=BC+DF+DG. \quad \text{And } A=\frac{BC+DF+DG}{B+2D}$$

$$\text{Or } \frac{960+90+210}{80+60}=\frac{1260}{140}=9.$$

For the 80 suppose the Time $1z$. Then must $1z-3$ be for the first 30.
And $1z-7$ for the last.
Then $1z\times80$ is $80z$; and $1z-3\times30$ is $30z-90$; and $1z-7\times30$ is $30z-210$.
And these 3 Products together make $140z-300$, which are equal to 80×12,
that is $140z-300=960$; and $140z=1260$; and $1z=9$.

18. A Merchant hath laden two Ships with Wine, in the one 20 Tuns, and in the other 30 Tuns; and coming to the *Custom-House*, payeth for the Custom of the former Cargo a Tun of Wine, and receiveth of the Officers in Money 60 *s.* and for the latter Cargo a Tun of Wine lacking 15 *s.* The Question is, how the Wine was rated a Tun, and so consequently how much he paid for the Custom of a Tun of Wine? **Q. *Of the Rate and Custom of a Tun of Wine.***

Answ. The Tun of Wine was valued at 7 *l.* 10 *s.* so must two Tuns be 15 *l.* From which 3 *l.* 15 *s.* abated, leaves 11 *l.* 5 *s.* to be paid for Custom of the whole 50 Tuns; and so for 1 Tun is 4 *s.* 6 *d.* **Resolution.**

B 20. C 30. D 60. F 15. Supposition *A* for the Value of 1 Tun.

Then if B cost $A-D$, shall C cost $\frac{CA-CD}{B}$. And this shall be equal to $A-F$.

And $CA-CD=BA-BF$. And then shall $BA-CA=CD-BF$.

$$\text{And } A=\frac{CD-BF}{C-B}. \quad \text{Or } \frac{1800-300}{30-20}=\frac{1500}{10}=150\,s. \text{ or } 7\,l.\ 10\,s.$$

$1z$. Then as $20 \,.\, 1z-60 :: 30 \,.\, 1\frac{1}{2}z-90$. And
$1\frac{1}{2}z-90=1z-15$; and $\frac{1}{2}z=75$; and $1z=150$.

19. A Post sets out from *Amsterdam* to carry Letters to *Danske*; and at the same time the Merchant to whom the Letters were sent, taketh Horse in *Danske* to ride to *Amsterdam*, and they meet together on the Way. The Post delivereth his Letters, and the Merchant asking what he must have? he answereth, I have gone as much one Day as another, and should have made my Journey in 20 Days, for which my Hire was 30 *s.* conditionally, that if I met you in the Way, that then proportionally for the Way you had travelled, I should be cut off from my Wages: To which the Merchant replied, that he had rode every Day alike, and should have been at his Journey's End in 16 Days: how much then shall the Merchant pay the Post? **Q. *Of a Post carrying Letters.***

Answ. $13\frac{1}{3}$ *s.* For having found the Days of the Post's Travel to be $8\frac{8}{9}$, such a Proportion of 30 *s.* will belong to him. **Resolution.**

B 20. C 16. Supposition for the Post's Days A. The whole Way D. F. 30.

$$\text{Then as } C \,.\, D :: A \,.\, \frac{DA}{C}. \quad \text{And as } B \,.\, D :: A \,.\, \frac{DA}{B}$$

$$\text{And } \frac{DA}{C}+\frac{DA}{B}=D. \quad \text{And } \frac{BDA+CDA}{BC}=D.$$

$$\text{And } BDA+CDA=BCD. \quad \text{And } A=\frac{BC}{B+C}. \quad \text{Or } \frac{320}{36}=8\frac{8}{9}\ \text{Days.}$$

$$\text{Then as } B \,.\, F :: A \,.\, \frac{FA}{B}. \quad \text{Or as } 20 \,.\, 30 :: 8\tfrac{8}{9} \,.\, 13\tfrac{1}{3}\ \text{Shillings.}$$

$1z$ supposed, for the Post's Days.
Then as $16 \,.\, 1 :: 1z \,.\, \frac{1}{16}z$.
And as $20 \,.\, 1 :: 1z \,.\, \frac{1}{20}z$.

$$\text{Total } \frac{36}{320}z. = 1 \text{ whole Journey.}$$

And $36z=320$. And $1z=8\frac{8}{9}$.
And if 20 Days Hire be 30 *s.* what $8\frac{8}{9}$ Days? *Answ.* $13\frac{1}{3}$ *s.*

Q. Of two Ships sailing. 20. A Ship saileth out of the *Texel* to *Spain*, with such a Wind that he might perform his Voyage in 15 days; but when 6 days were past, the Wind changing, the Ship sailed backwards as much in 4 days, as it had done forward in one day. At the beginning of the second Wind there departed another Ship from *Spain* towards the *Texel*, (being light loaden) sailed forwards to the *Texel* as often 7 Leagues, as the other Ship backward from his Port 2 Leagues: in how much Time after the first 6 days, and how far from the *Texel* shall the Ships meet? and in what Time may the Ship from *Spain* arrive at the *Texel*, supposing the Parts 300 Leagues asunder?

Resolution. *Answ.* In $14\frac{2}{5}$ days they meet; and the Ship from *Spain* had then sailed 252 Leagues, which is 48 from the *Texel*; and in 17 days $\frac{1}{7}$ may arrive there: discovered thus,

B 15. C 6. D 1. F 4. G 2. H 7. K 300. Supposition for the meeting A.

Then as B . K :: C . $\frac{KC}{B}$.	Leagues that the Ship coming from the *Texel* had made toward *Spain* with the first Wind.
And $K - \frac{KC}{B}$. ———	Leagues which that Ship had to sail when the Wind changed.
Then as B . K :: $\frac{D}{F}$. $\frac{KD}{BF}$.	Leagues which that Ship sailed backward every day.
Then as G . H :: $\frac{KD}{BF}$. $\frac{HKD}{GBF}$.	Leagues that the Ship coming from *Spain* made every day towards the *Texel*.
Then as D . $\frac{KD}{BF}$ :: A . $\frac{KA}{BF}$.	Leagues that the first Ship had gone backward at the day of meeting.
Then as D . $\frac{HKD}{GBF}$ :: A . $\frac{HKA}{GBF}$.	Leagues that the second Ship had sailed forward at the day of meeting.

And $\frac{HKA}{GBF} - \frac{KA}{BF} = K - \frac{KC}{B}$. And $\frac{HKA}{GF} - \frac{KA}{F} = KB - KC$.

And $\frac{HA}{GF} - \frac{A}{F} = B - C$. And $\frac{HA}{F} - \frac{AG}{F} = BG - CG$.

And $A = \frac{BFG - CFG}{H - G}$. Or $\frac{120 - 48}{7 - 2} = \frac{72}{5} = 14\frac{2}{5}$ days.

1 z Supposition. As 15 . 300 :: 6 . 120. $\frac{KC}{B}$ as before. And $300 - 120 = 180$.

And as 15 . 300 :: $\frac{1}{4}$. 5 . $\frac{KD}{BF}$. And as 2 . 7 :: 5 . $17\frac{1}{2}$. $\frac{HKD}{GBF}$, as before.

Then as 1 . 5 :: 1 z . 5 z. So $5z + 180$ distance of the first Ship from *Spain*.
And 1 . $17\frac{1}{2}$ :: 1 z . $17\frac{1}{2}z$, Run of the second Ship towards the *Texel*.
Then $5z + 180 = 17\frac{1}{2}z$; and $12\frac{1}{2}z = 180$; and $1z = 14\frac{2}{5}$.

Wherefore by Consequence if they meet in $14\frac{2}{5}$ days, then had the Ship from *Spain* sailed 252 Leagues, because she sailed $17\frac{1}{2}$ Leagues in a day; and such a Course will run 300 Leagues in $17\frac{1}{7}$ days.

Q. Of digging a Well. 21. A Gentleman hired a Workman to dig a Well 12 Feet deep for 12 *s.* When the Workman had digged 8 Feet, they fall at Variance, and the Gentleman will pay him off: what must he give the Workman for his Work, considering the Labour at the Bottom is worth more than that at the Top?

Resolution. *Answ.* $5\frac{7}{13}$ *s.* For such a Proportion of the 12 *s.* is the Sum of an *Arithmetical Progression*, beginning with $\frac{2}{13}$, and continued to 8 Terms; which by an increase of $\frac{2}{13}$ to every Term, and continued to 12 Terms, will make the Sum thereof 12 *s.*

Here working in Numbers without Species, a *Progression Arithmetical* is framed of 12 Terms from an Unit, whose Sum being 78, and the Sum of 8 Terms thereof but 36, the *Analogy* is, As the Sum of the greater Number of Terms, is to the whole Price: So is the Sum of the lesser Number of Terms, to the Price desired.

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 == 78$
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 == 36$

Where-

Wherefore as $\overset{Z}{78} . \overset{s.}{12} :: \overset{Z}{36} . \overset{s.}{5\frac{7}{13}}$ price of the 8 Feet.

Admit then in *Species* alike *Progression* be framed, whose first Term be α: Then shall the Sum of 12 Terms be $78\,\alpha$, and the Sum of 8 Terms $36\,\alpha$: And let 12 *s.* the whole Price be B, and A supposed for the Number sought, or Sum of 8 Terms. Then because the Product of the Means is equal to the Product of the Extreams;

$$78\,\alpha\,A = 36\,\alpha\,B. \qquad \text{And } 78\,A = 36\,B.$$

And $13\,A = 6\,B$. Or $\frac{6\times 12}{13} = \frac{72}{13} = 5\frac{7}{13}$.

So by *Cossicks* supposing 1 ℨ for the Sum of 8 Terms:

78 ℨ $= 36\times 12$. Or 78 ℨ $= 432$. And 1 ℨ $= 5\frac{7}{13}$.

22. A Workman is agreed with to make a Well of 12 Feet deep for 12 *s.* and after digging 8 Feet thereof, fell sick, and desired Money for what he had done: but because the digging the lower Part is more labour than the upper, Allowance was agreed on, that every Foot should be a Penny more than the other: what then shall the Workman receive for digging the first 8 Feet? Q. *Of digging a Well.*

Answ. 6 *s.* 8 *d.* For such is the Sum of 8 Terms of a *Progression*, the Sum of 12 Terms whereof is 12 *s.* and the Excess 1 *d.* Resolution.

The *Data* here being T 12. X 1. Z 12 *s.* or 144 *d.* by the former Rules in *Progression* may be found α the first Term, or ω the last Term; and thereby the Sum of 8 Terms of such a *Progression* to be 6 *s.* 8 *d.* as before.

$$\text{For } \frac{2Z}{2T} - \frac{TX}{2} + \frac{X}{2} = \alpha. \qquad \text{And } \frac{2Z}{2T} + \frac{TX}{2} - \frac{X}{2} = \omega.$$

$$6\tfrac{1}{2}\,d. \qquad\qquad 17\tfrac{1}{2}\,d.$$

Otherwise let a *Progression* be framed to 12 Terms, whose first Term may be supposed α, and the Excess 1. So shall the second Term be $\alpha+1$, and the Third $\alpha+2$, *&c.* Then will the eighth Term be $\alpha+7$, and the 12*th* Term $\alpha+11$; and the Sum of 12 Terms $12\alpha+66$, and the Sum of 8 Terms thereof $8\alpha+28$. Now because $12\alpha+66=144$. And $12\alpha=78$. And $1\alpha=6\frac{1}{2}$; therefore $8\alpha+28=80$, that is $52+28$.

Likewise by *Cossicks* 12 ℨ $+66=144$. And 1 ℨ $=6\frac{1}{2}$.
Wherefore 8 ℨ $+28=80$. For $8\times 6\frac{1}{2}=52$. And $52+28=80$.

23. Four Men drink together, and play at Tables for the Wine, every Game 1 *d.* and after they had plaid a while, they found *A* had lost most; whereupon he payeth a Pint of Wine of 5 *d.* upon the Reckoning; and besides found his Loss three times as much more as *B* who had lost least of all; and *C* had lost 2 *d.* more than *B*, and *D* was 4 *d.* less indebted than *A*. Whereupon at last they plaid again who should pay all, and it happened upon *B*: So besides the 5 *d.* *A* had paid, *B* paid 27 Pence: how much should each of them have paid at the first? Q. *Of paying a Reckoning.*

Answ. *A* 14 *d.* *B* 3 *d.* *C* 5 *d.* *D* 10 *d.* For taking 5 from 14, the Remain 9 is 3 times 3; and $3+2=5$; and $14-4=10$. Resolution.

b 5. *d* 2. *e* 4. *f* 27. Suppose the Debt of *A* in all to be *a*.

Then substracting the 5 *d.* paid, the Remain shall be $a-b$. And this being 3 times so much as *B*, he should therefore pay $\frac{a-b}{3}$, to which *d* added, makes the Debt of *C* to $\frac{a-b}{3}+d$. And because *D* was 4 *d.* less indebted than *A*, therefore the Score of *D* was $a-e$. And then these added together must be equal to 27.

$$\left.\begin{array}{l} A\ .\ a-b \\ B\ .\ \dfrac{a-b}{3}. \\ C\ .\ \dfrac{a-b}{3}+d. \\ D\ .\ a-e \end{array}\right\} \begin{array}{l}\textit{Total of their Debts.} \\ 2\tfrac{2}{3}\,a - 1\tfrac{2}{3}\,b + d - e = f.\end{array}$$

Then

Then $8a - 5b + 3d - 3e = 3f$. And $8a = 3f + 5b + 3e - 3d$.

And $a = \frac{3f + 5b + 3e - 3d}{8}$. Or $\frac{81 + 25 + 12 - 6}{8} = \frac{112}{8} = 14$.

Also let the Debt of *A* be $1\mathfrak{z}$. Then taking 5 *d.* thence, the rest $1\mathfrak{z} - 5$ must be divided by 3. So is the Debt of *B* $\frac{1\mathfrak{z} - 5}{3}$. And *C* being 2 *d.* more, must be $\frac{1\mathfrak{z} - 5}{3} + 2$. And *D* is indebted $1\mathfrak{z} - 4$. The Total of which 4 Sums is $2\frac{2}{3}\mathfrak{z} - 10\frac{1}{3}$; and equal to 27.

Then if $2\frac{2}{3}\mathfrak{z} - 10\frac{1}{3} = 27$, shall $2\frac{2}{3}\mathfrak{z} = 37\frac{1}{3}$.

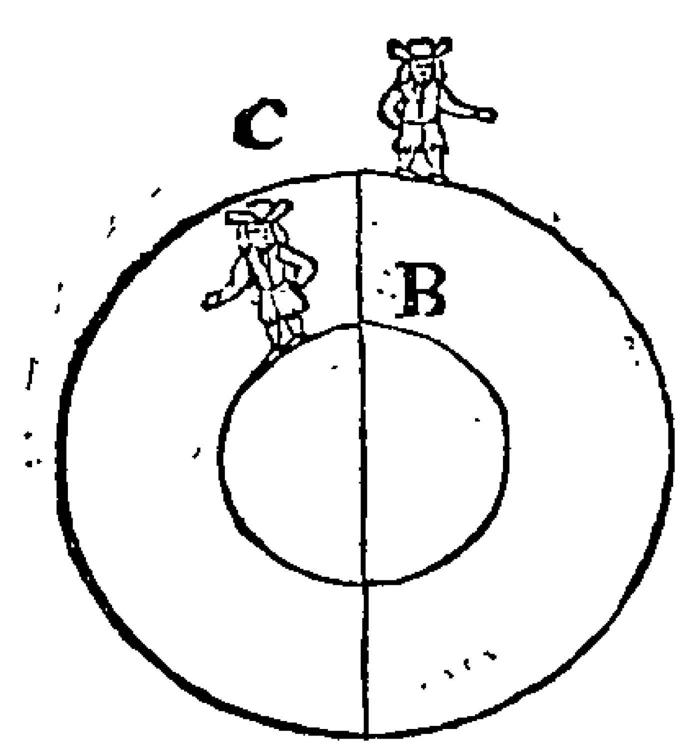
And $8\mathfrak{z} = 112$; and $1\mathfrak{z} = 14$, the whole Loss of *A*.

Q. Of compassing Circles.

24. Let there be two Circles one within another, described upon the same Center (as below); and suppose one upon the outermost Circle in C goeth on the right Hand, and compasseth it about in 12 Hours: And another within upon the innermost Circle in B, at the same Time and in the same Proportion as the first, goeth on the left Hand, and compasseth it about in 3 Hours: when shall they come to be one under the other, or as near together as when they first began their Journies?

Resolution.

Answ. In $2\frac{2}{5}$ Hours: For if the greatest be in compass 8 Miles, then shall he at C in that time go $1\frac{3}{5}$ Mile. And so shall the smallest Circle at B be 2 Miles, (because if he in the greatest Circle go in 12 Hours 8 Miles, then he in the smallest must go in 3 Hours 2 Miles): and then if in 3 Hours he goeth within a Compass of 8 Miles, in $2\frac{2}{5}$ Hours he shall go within the Compass of $6\frac{2}{5}$ Miles; which added to $1\frac{3}{5}$ Mile, shall make together 8 Miles.



b 12. *d* 3. Supposition for the Time sought A.

Then as $b . 1 :: A . \frac{A}{b}$. And as $d . 1 :: A . \frac{A}{d}$.

Both which Quotients added together, make $\frac{A}{b} + \frac{A}{d}$.

Equal to 1 Hour. And then reduced

$\frac{Ad + Ab}{bd} = 1$. And $Ad + Ab = bd$. And $A = \frac{bd}{d + b}$.

Or $\frac{36}{12 + 3} = \frac{36}{15} = 2\frac{2}{5}$.

The Time desired $1\mathfrak{z}$. Then as $12 . 1 :: 1\mathfrak{z} . \frac{1}{12}\mathfrak{z}$.

And as $3 . 1 :: 1\mathfrak{z} . \frac{1}{3}\mathfrak{z}$. And $\frac{1}{12}\mathfrak{z} + \frac{1}{3}\mathfrak{z} = \frac{5}{12}\mathfrak{z}$.

And $\frac{5}{12}\mathfrak{z} = 1$. And then $5\mathfrak{z} = 12$. And $1\mathfrak{z} = 2\frac{2}{5}$.

Q. Of compassing Circles.

25. If in the Circles aforesaid the two Men go both one Way, and keep going in like Pace and Proportion: when shall they come to the same Posture they were in at first?

Resolution.

Answ. In 4 Hours: For if the greatest Circle be 8 Miles, then shall the least be 2 Miles: and if 12 Hours give 8 Miles, then 4 Hours shall give $2\frac{2}{3}$ Miles for him in C: and if 3 Hours give 2 Miles, then 4 Hours shall give $2\frac{2}{3}$ Miles for B, thereby shewing the one in his Circle hath gone as many Miles as the other in his Circle.

Here one of the Quotients is to be taken from the other, that is,

$\frac{A}{b}$ from $\frac{A}{d}$; and then $\frac{A}{d} - \frac{A}{b}$ reduced, shall be $\frac{Ab - Ad}{db} = 1$.

And $Ab - Ad = db$. And $A = \frac{db}{b - d}$. Or $\frac{36}{12 - 3} = \frac{36}{9} = 4$.

So by *Cossicks* $\frac{1}{3}\mathfrak{z} - \frac{1}{12}\mathfrak{z} = \frac{1}{4}\mathfrak{z}$. And $\frac{1}{4}\mathfrak{z} = 1$. And $1\mathfrak{z} = 4$.

Q. Of two Servants, what Cloth they received of their Master.

26. A Merchant hath 2 Servants, to whom he delivereth together the Value of 300 *l.* in Linen Cloth; and one of them selleth his Part, and loseth therein with his Charges $\frac{1}{4}$ Part of that he received of his Master, and with the rest he buyeth Spices, and gaineth by them 42 *l.* The Second gaineth by his Cloth $\frac{1}{5}$ Part as much as he had received of his Master, out of which he spendeth 12 *l.* And

And when they came home, they both pay their Master 330 *l.* how much Money in Cloth did each of them receive of their Master?

Answ. *A* 200 *l.* and *B* 100 *l.* For 200 with 25 (that is $\frac{1}{8}$) loss, is 175, to which 42 (Gain) added, is 217. And 100 with 25 (that is $\frac{1}{4}$) Gain, is 125; from which 12 (spent) being taken away, the Remain is 113. And $217+113=330$. Resolution.

C 300. D 42. F 12. G 330. Suppose *A* received a.

Then taking away $\frac{1}{8}$ leaves $\frac{7}{8}a$; to which adding D, makes $D+\frac{7}{8}a$, that *A* must bring his Master. Now if *A* received of his Master a, then must *B* receive $C-a$; to which $\frac{1}{4}$ added, and his Expences deducted, makes him bring his Master home $1\frac{1}{4}C-1\frac{1}{4}a-F$, both which must be equal to G; wherefore shall $1\frac{1}{4}C+D-\frac{3}{8}a-F=G$; and $1\frac{1}{4}C-\frac{3}{8}a=G+F-D$; and $\frac{10C-3a}{8}=G+F-D$. And $10C-3a=8G+8F-8D$; and $3a=10C+8D-8G-8F$. And consequently $a=\frac{10C+8D-8G-8F}{3}$. Or $\frac{3000+336-2640-96}{3}=\frac{3336-2736}{3}=\frac{600}{3}=200$.

In like sort by *Cossicks*, if *A* receive $1\mathfrak{z}$, then $\frac{1}{8}$ taken away, and 42 added, makes him bring home $\frac{7}{8}\mathfrak{z}+42$. And if *A* receive $1\mathfrak{z}$, then *B* receives $300-1\mathfrak{z}$; to which $\frac{1}{4}$ added, and 12 taken away, makes him bring home $363-1\frac{1}{4}\mathfrak{z}$: both Sums of *A* and *B* added, makes $405-\frac{3}{8}\mathfrak{z}=330$. And $\frac{3}{8}\mathfrak{z}=75$; and $3\mathfrak{z}=600$; and $1\mathfrak{z}=200$.

27. If a Merchant buy Rie for 36 *s.* the Quarter: for how much shall he sell it again that he may gain by 120 *s.* imployed therein, as much as he received for a Quarter? *Q. Of Rie, how sold by the Quarter.*

Answ. For $51\frac{3}{7}$ *s.* the Quarter: Seeing if 36 gain $15\frac{3}{7}$, 120 shall gain $51\frac{3}{7}$. Resolution.

B 36. C 120. Supposition A.

Then as $B . A :: C . \frac{AC}{B}$. And $\frac{AC}{B}=C+A$.

And $AC=BC+BA$. And $AC-BA=BC$.

And $A=\frac{BC}{C-B}$. Or $\frac{4320}{120-36}=\frac{4320}{84}=51\frac{3}{7}$.

By *Cossicks*, supposing $1\mathfrak{z}$: Then as $36 . 1\mathfrak{z} :: 120 . 3\frac{1}{3}\mathfrak{z}$.
And $3\frac{1}{3}\mathfrak{z}=120+1\mathfrak{z}$. And $2\frac{1}{3}\mathfrak{z}=120$; and $7\mathfrak{z}=360$.
And $1\mathfrak{z}=51\frac{3}{7}$.

28. A Merchant hath bought 7 Yards of Cloth, 8 Ells of Damask, and 9 Yards of Sattin together, for 74 *l.* Flemish: And of the same selleth 5 Yards of Cloth, 4 Ells of Damask, and 6 Yards of Sattin for 47 *l.* The Yard of Cloth cost 4 *l.* what cost the Ell of Damask, and Yard of Sattin severally? *Q. Of Damask, Sattin & Cloth, the Price.*

Answ. The Ell of Damask $2\frac{3}{4}$ *l.* and the Yard of Sattin $2\frac{2}{3}$ *l.* For $7\times4=28$. And $8\times2\frac{3}{4}=22$; and $9\times2\frac{2}{3}=24$; and $28+22+24=74$. Resolution.

B 7. C 8. D 9. F 74. G 5. H 4. K 6. M 47.

Suppose the Ell of Damask cost A.

Then Cloth cost BH.
Damask CA.
Sattin F−BH−CA.

Sold G for GH.
H for HA.
K for $\frac{KF-KBH-KCA}{D}$.

Because as $D . F-BH-CA :: K . \frac{KF-KBH-KCA}{D}$.

And then $GH+HA\frac{KF-KBH-KCA}{D}=M$.

And by Reduction $GHD+HAD+KF-KBH-KCA=MD$.
And likewise $HAD-KCA=MD+KBH-KF-GHD$.

And then $A=\frac{MD+KBH-KF-GHD}{HD-KC}$. Or $\frac{423+168-444-180}{36-48}=\frac{-33}{-12}=2\frac{3}{4}$.

In *Cossicks*, supposing the Ell of Damask cost $1\mathfrak{z}$.

Then Cloth cost 28 *l.*
Damask $8\mathfrak{z}$.
Sattin the rest 46 *l.* $-8\mathfrak{z}$.

Sold 5 Yards for 20 *l.*
4 Ells for $4\mathfrak{z}$.
6 Yards for $30\frac{2}{3}$ *l.* $-5\frac{1}{3}\mathfrak{z}$.

 Because

Because as $9 \,.\, 46\,l. - 8z :: 6 \,.\, 30\frac{2}{3}\,l. - 5\frac{1}{3}z$.
Then $20 + 4z + 30\frac{2}{3} - 5\frac{1}{3}z$, that is $50\frac{2}{3} - 1\frac{1}{3}z = 47$.
And $1\frac{1}{3}z = 3\frac{2}{3}$; and $4z = 11$; and $1z = 2\frac{3}{4}$.

Q. *Of Sugar the Hundred* Netto. 29. A Grocer buyeth 21 C. 98 ℔ of Sugar for 600 *l.* Flemish; and being allowed Tare upon the Hundred 10 ℔, the Hundred *Netto* costeth him 30 *l.* Flemish: how many Pound is the Hundred *Netto* accompted?

Resolution. *Answ.* 102 ℔: For if 30 *l.* buy 112 ℔, then will 600 *l.* buy 2240 ℔; which at 102 for 1 C. makes 21 C. 98 ℔.

B 21. D 98. F 600. G 10. H 30. Supposition A.

Then as $H \,.\, A+G :: F \,.\, \frac{AF+GF}{H}$. And $\frac{AF+GF}{H} = B+D$.

And $AF+GF = BH+DH$. And $AF = BH+DH-GF$.

And $A = \frac{BH+DH}{F} - G$. Or $\frac{630}{600}$C. + $\frac{2940}{600}$℔ − 10℔ = $1\frac{1}{20}$C. + $4\frac{18}{20}$℔ − 10℔.

Or $1\frac{1}{20}$C − $5\frac{2}{20}$℔. And $\frac{1}{20}$C = $\frac{102}{20}$℔; and 1C = 102℔.

Supposing $1z$ in *Cossicks*; Then as 30 *l* . 1C + 10℔ :: 600 *l* . 20C + 200℔.

And 21C + 98℔ = 20C + 200℔; and 21C = 20C + 102℔; and 1C = 102℔.

Q. *Of Pitch, how much sold.* 30. Two Merchants sold Pitch for 504 *l.* whereof *B* sold 3 Tuns more than *A.* Whereupon *A* said to *B*, I would have sold all your Pitch for 288 *l.* And *B* said to *A*, I would have sold all your Pitch for 216 *l.* how many Tuns of Pitch did each of them sell?

Resolution. *Answ.* *A* sold 9 Tuns, and *B* 12 Tuns; which at the Rate of 504 *l.* for the whole 21 Tuns, is 24 *l.* a Tun, and makes 9 amount to 216 *l.* for *A*, and 288 *l.* for *B*.

C 504. D 3. F 288. G 216. Suppose *A* sold a; then *B* sells $a+3$.

And as $a \,.\, G :: a+D \,.\, \frac{Ga+DG}{a}$. And as $a+D \,.\, F :: a \,.\, \frac{Fa}{a+D}$.

Then both Quotients added are $\frac{Faq+Gaq+2DGa+DqG}{aq+Da} = C$.

And $Faq+Gaq+2DGa+DqG = Caq+CDa$. And because by the *Data* it appears C was equal to F and G; therefore their Squares are also equal, and so omitted. And then $2DGa+DqG = CDa$; and $CDa - 2DGa = DqG$.

And $a = \frac{DqG}{CD-2DG}$. Or $\frac{1944}{1512-1296} = \frac{1944}{216} = 9$.

In *Cossicks*, supposing $1z$ for *A*; Then must *B* sell $1z+3$.

And $1z \,.\, 216 :: 1z+3 \,.\, \frac{216z+648}{1z}$. And as $1z+3 \,.\, 288 :: 1z \,.\, \frac{288z}{1z+3}$.

Total $\frac{504ʒ+1296z+1944}{1ʒ+3z} = 504$. And by Reduction

$504ʒ+1296z+1944 = 504ʒ+1512z$. And $216z = 1944$. And $1z = 9$.

Q. *Of Cloth shrunk.* 31. Eight Ells of Cloth being $2\frac{1}{4}$ broad, after shrinking $3\frac{3}{4}$ Ells thereof make but $3\frac{3}{5}$ in length, and $2\frac{1}{2}$ Ells in breadth, make but $2\frac{1}{3}$. A second sort of Cloth being $1\frac{3}{4}$ Ell broad; when it is wet is no broader than $1\frac{1}{2}$, and the length of 6 Ells but $5\frac{1}{4}$: how much of this second sort of Cloth will serve to line the first 8 Ells after the shrinking.

Resolution. *Answ.* $12\frac{756}{2625}$ Ells; which will appear, if the length and breadth of each dry, or the length and breadth of each wet and shrunk, be multiplied one by the other, because the Product of the one will be equal to the Product of the other.

As $\frac{8}{1} \times \frac{9}{4}$. Or $10\frac{2}{7} \times 1\frac{3}{4} = \frac{72}{4} = 18$.

Omitting the *Species*, for brevity-sake, the Work by *Cossicks* is thus:

As $3\frac{3}{4}$ dry . $3\frac{3}{5}$ wet :: 8 dry . $7\frac{17}{25}$ wet and shrunk in length.

As $2\frac{1}{2}$ dry . $2\frac{1}{3}$ wet :: $2\frac{1}{4}$ dry . $2\frac{1}{10}$ wet and shrunk in breadth.

$7\frac{17}{25}$ and $2\frac{1}{10}$, or $\frac{192}{25} \times \frac{21}{10} = \frac{4032}{250}$ Square Content.

Then

Then for the length of the ſecond Cloth, ſuppoſe $1\mathfrak{z}$.

As 6 dry . $5\frac{1}{4}$ wet :: $1\mathfrak{z}$ dry . $\frac{7}{8}\mathfrak{z}$, wet and ſhrunk in length.

The breadth given $1\frac{1}{2}$ multiplied by $\frac{7}{8}\mathfrak{z} = \frac{21}{16}\mathfrak{z}$, Square Content.

Then $\frac{21}{16}\mathfrak{z} = \frac{4032}{250}$. And $5250\mathfrak{z} = 64512$.

And $1\mathfrak{z} = 12\frac{756}{2625}$.

32. If 6 Ells of Black, 5 Ells of Red and 7 *l.* be worth as much as 9 Ells of Black, and $3\frac{1}{3}$ Ells of Red: And if at the ſame Price 8 Ells of Black, and 7 of Red lacking 5 *l.* coſt as much as 6 Ells of Black, 5 Ells of Red and 9 *l.* how much ſhall 1 Ell of Black and 1 Ell of Red coſt ſeverally? Q. *Of the Price of Black and Red.*

Anſw. One Red 3 *l.* and 1 Black 4 *l.* Reſolution.

Here being two *Equations* given, there needs no Suppoſition.

Equations given.	*Reduced.*
$6B + 5R + 7 = 9B + 3\frac{1}{3}R$	$1\frac{2}{3}R + 7 = 3B.$
$8B + 7R - 5 = 6B + 5R + 9$	$1B + 1R = 7.$
Both reduced *Equations* added	$1B + 2\frac{2}{3}R + 7 = 3B + 7.$
This laſt *Equation* reduced is	$1\frac{2}{3}R = 1B.$
The ſecond reduced *Equation* added	$1B + 1R = 7.$
Total	$1B + 2\frac{1}{3}R = 1B + 7.$

Then $2\frac{1}{3}R = 7$. And $7R = 21$. And $1R = 3$.

And if $1R$ be 3, then $1B$ muſt be 4, becauſe by the ſecond reduced *Equation* both were equal to 7.

33. A King lying with a great Army before an Enemy, the Adverſary endeavoureth to corrupt one of the Heralds to declare the Strength of the Army: who willing to receive the Reward, dealeth ſubtilly, and deviſeth this Anſwer, (nevertheleſs true) Look, ſaith he, how many Dukes there are, and for each of them there are twice ſo many Earls, and under every Earl there are 4 times ſo many Souldiers as there be Dukes in the Field: And when the Muſter of the Souldiers was taken, the 200 Part of them was 9 times ſo many as the Number of the Dukes: how many then were there of each Sort? Q. *Of an Army, how many therein.*

Anſw. 15 Dukes, 450 Earls, (that is 15 times 15 twice); and the Number of Souldiers (being 4 times 15 multiplied by 450) is 27000, the 200 Part whereof (being 135) is 9 times 15 the Number of the Dukes. Reſolution.

B 200. Suppoſition for the Dukes A.

Then the Earls 2Aq; that is $A \times A + A \times A$.

And the Souldiers 8Ac, that is $2Aq \times 4A$.

Then $\frac{8Ac}{B} = 9A$. And $8Ac = 9BA$; and $\frac{Ac}{A} = \frac{9B}{8}$.

And $Aq = \frac{9B}{8}$. Or $\sqrt{q}\,\frac{1800}{8} = \sqrt{q}\,225(15$.

Dukes ſuppoſe $1\mathfrak{z}$. Then Earls 2ʒ. And Souldiers 8φ.

Then $\frac{8\phi}{200} = 9\mathfrak{z}$. And $8\phi = 1800\mathfrak{z}$. And 8ʒ = 1800N.

And 1ʒ = 225N; whoſe Square Root is 15.

34. Among 4 Walls, the 2 longeſt are in proportion to the ſhorteſt, as 5 to 3; and to the Height they be double Seſquialter: now multiplying the longeſt by the ſhorteſt, and the Total by the Height, there will ariſe 39930 Feet: what then is the Length and Height of each Wall? Q. *Of the length and height of four Walls.*

Anſw. The Height of the ſhorteſt is 22 Feet, and the longeſt being double *Seſquialter* (that is $2\frac{1}{2}$ as much) muſt be 55, and the Length of the ſhorteſt 33, that is as 5 to 3. Reſolution.

B 39930. Suppoſition for the Height of the ſhorteſt A.

Then the longeſt muſt be $2\frac{1}{2}$ A. And becauſe the longeſt to the ſhorteſt are as 5 to 3, the ſhorteſt muſt be $1\frac{1}{2}$ A. For $5 . 3 :: 2\frac{1}{2} . 1\frac{1}{2}$.

And $A \times 2\frac{1}{2}A \times 1\frac{1}{2}A = \frac{15}{4}Ac$. And $\frac{15}{4}Ac = B$.

Wherefore $Ac = \frac{4B}{15}$. or $\frac{159720}{15} = 10648$. And $\sqrt{c}\,10648 = 22$.

In

Because as 9 . 46 *l.*—8 ℨ :: 6 . $30\frac{2}{3}$ *l.*—$5\frac{1}{3}$ ℨ.
Then 20+4 ℨ+$30\frac{2}{3}$—$5\frac{1}{3}$ ℨ, that is $50\frac{2}{3}$—$1\frac{1}{3}$ ℨ=47.
And $1\frac{1}{3}$ ℨ=$3\frac{2}{3}$; and 4 ℨ=11; and 1 ℨ=$2\frac{3}{4}$.

Q. Of Sugar the Hundred Netto.

29. A Grocer buyeth 21 C. 98 ℔ of Sugar for 600 *l.* Flemish; and being allowed Tare upon the Hundred 10 ℔, the Hundred *Netto* costeth him 30 *l.* Flemish: how many Pound is the Hundred *Netto* accompted?

Resolution.

Answ. 102 ℔: For if 30 *l.* buy 112 ℔, then will 600 *l.* buy 2240 ℔; which at 102 for 1 C. makes 21 C. 98 ℔.

B 21. D 98. F 600. G 10. H 30. Supposition A.

Then as H . A+G :: F . $\frac{AF+GF}{H}$. And $\frac{AF+GF}{H}$=B+D.

And AF+GF=BH+DH. And AF=BH+DH—GF.

And A=$\frac{BH+DH}{F}$—G. Or $\frac{630}{600}$C.+$\frac{2940}{600}$℔−10℔=$1\frac{1}{20}$C.+$4\frac{18}{20}$℔—10℔.

Or $1\frac{1}{20}$C—$5\frac{2}{20}$℔. And $\frac{1}{20}$C=$\frac{102}{20}$℔; and 1C=102℔.

Supposing 1 ℨ in *Cossicks*; Then as 30 *l* . 1C+10℔ :: 600 *l* . 20C+200℔.

And 21C+98℔=20C+200℔; and 21C=20C+102℔; and 1C=102℔.

Q. Of Pitch, how much sold.

30. Two Merchants sold Pitch for 504 *l.* whereof *B* sold 3 Tuns more than *A*. Whereupon *A* said to *B*, I would have sold all your Pitch for 288 *l.* And *B* said to *A*, I would have sold all your Pitch for 216 *l.* how many Tuns of Pitch did each of them sell?

Resolution.

Answ. *A* sold 9 Tuns, and *B* 12 Tuns; which at the Rate of 504 *l.* for the whole 21 Tuns, is 24 *l.* a Tun, and makes 9 amount to 216 *l.* for *A*, and 288 *l.* for *B*.

C 504. D 3. F 288. G 216. Suppose *A* sold a; then *B* sells a+3.

And as a . G :: a+D. $\frac{Ga+DG}{a}$. And as a+D . F :: a . $\frac{Fa}{a+D}$.

Then both Quotients added are $\frac{Faq+Gaq+2DGa+DqG}{aq+Da}$=C.

And Faq+Gaq+2DGa+DqG=Caq+CDa. And because by the *Data* it appears C was equal to F and G; therefore their Squares are also equal, and so omitted. And then 2DGa+DqG=CDa; and CDa—2DGa=DqG.

And a=$\frac{DqG}{CD-2DG}$. Or $\frac{1944}{1512-1296}$=$\frac{1944}{216}$=9.

In *Cossicks*, supposing 1 ℨ for *A*; Then must *B* sell 1 ℨ+3.

And 1 ℨ . 216 :: 1 ℨ+3 . $\frac{216ℨ+648}{1ℨ}$. And as 1 ℨ+3 . 288 :: 1 ℨ . $\frac{288ℨ}{1ℨ+3}$.

Total $\frac{504ʒ+1296ℨ+1944}{1ʒ+3ℨ}$=504. And by Reduction

504ʒ+1296 ℨ+1944=504ʒ+1512 ℨ. And 216 ℨ=1944. And 1 ℨ=9.

Q. Of Cloth shrunk.

31. Eight Ells of Cloth being $2\frac{1}{4}$ broad, after shrinking $3\frac{3}{4}$ Ells thereof make but $3\frac{2}{5}$ in length, and $2\frac{1}{2}$ Ells in breadth, make but $2\frac{1}{3}$. A second sort of Cloth being $1\frac{3}{4}$ Ell broad; when it is wet is no broader than $1\frac{2}{5}$, and the length of 6 Ells but $5\frac{1}{4}$: how much of this second sort of Cloth will serve to line the first 8 Ells after the shrinking.

Resolution.

Answ. $12\frac{76}{175}$ Ells; which will appear, if the length and breadth of each dry, or the length and breadth of each wet and shrunk, be multiplied one by the other, because the Product of the one will be equal to the Product of the other.

As $\frac{8}{1}$ × $\frac{9}{4}$. Or $10\frac{2}{7}$ × $1\frac{3}{4}$ = $\frac{72}{4}$ = 18.

Omitting the *Species*, for brevity-sake, the Work by *Cossicks* is thus:

As $3\frac{3}{4}$ dry . $3\frac{2}{5}$ wet :: 8 dry . $7\frac{19}{75}$ wet and shrunk in length.

As $2\frac{1}{2}$ dry . $2\frac{1}{3}$ wet :: $2\frac{1}{4}$ dry . $2\frac{1}{10}$ wet and shrunk in breadth.

$7\frac{19}{75}$ and $2\frac{1}{10}$, or $\frac{544}{75}$ × $\frac{21}{10}$ = $\frac{11424}{750}$ Square Content.

Then

Then for the length of the second Cloth, suppose $1\mathfrak{z}$.
As 6 dry . $5\frac{1}{4}$ wet :: $1\mathfrak{z}$ dry . $\frac{7}{8}\mathfrak{z}$, wet and shrunk in length.
The breadth given $1\frac{1}{2}$ multiplied by $\frac{7}{8}\mathfrak{z} = \frac{21}{16}\mathfrak{z}$, Square Content.
Then $\frac{21}{16}\mathfrak{z} = \frac{4032}{250}$. And $5250\mathfrak{z} = 64512$.
And $1\mathfrak{z} = 12\frac{756}{2625}$.

32. If 6 Ells of Black, 5 Ells of Red and 7 *l.* be worth as much as 9 Ells of Black, and $3\frac{1}{3}$ Ells of Red: And if at the same Price 8 Ells of Black, and 7 of Red lacking 5 *l.* cost as much as 6 Ells of Black, 5 Ells of Red and 9 *l.* how much shall 1 Ell of Black and 1 Ell of Red cost severally? *Q. Of the Price of Black and Red.*

Answ. One Red 3 *l.* and 1 Black 4 *l.* Resolution.

Here being two *Equations* given, there needs no Supposition.

Equations given.	*Reduced.*
$6B + 5R + 7 = 9B + 3\frac{1}{3}R$	$1\frac{2}{3}R + 7 = 3B.$
$8B + 7R - 5 = 6B + 5R + 9$	$1B + 1R = 7.$
Both reduced *Equations* added	$1B + 2\frac{2}{3}R + 7 = 3B + 7.$
This last *Equation* reduced is	$1\frac{1}{3}R = 1B.$
The second reduced *Equation* added	$1B + 1R = 7.$
Total	$1B + 2\frac{1}{3}R = 1B + 7.$

Then $2\frac{1}{3}R = 7$. And $7R = 21$. And $1R = 3$.
And if 1R be 3, then 1B must be 4, because by the second reduced *Equation* both were equal to 7.

33. A King lying with a great Army before an Enemy, the Adversary endeavoureth to corrupt one of the Heralds to declare the Strength of the Army: who willing to receive the Reward, dealeth subtilly, and deviseth this Answer, (nevertheless true) Look, saith he, how many Dukes there are, and for each of them there are twice so many Earls, and under every Earl there are 4 times so many Souldiers as there be Dukes in the Field: And when the Muster of the Souldiers was taken, the 200 Part of them was 9 times so many as the Number of the Dukes: how many then were there of each Sort? *Q. Of an Army, how many therein.*

Answ. 15 Dukes, 450 Earls, (that is 15 times 15 twice); and the Number of Souldiers (being 4 times 15 multiplied by 450) is 27000, the 200 Part whereof (being 135) is 9 times 15 the Number of the Dukes. Resolution.

B 200. Supposition for the Dukes A.
Then the Earls 2Aq; that is $A \times A + A \times A$.
And the Souldiers 8Ac, that is $2Aq \times 4A$.

$$\text{Then } \frac{8Ac}{B} = 9A. \text{ And } 8Ac = 9BA; \text{ and } \frac{Ac}{A} = \frac{9B}{8}.$$

$$\text{And } Aq = \frac{9B}{8}. \text{ Or } \sqrt{q}\,\frac{1800}{8} = \sqrt{q}\,225(15.$$

Dukes suppose $1\mathfrak{z}$. Then Earls $2\mathfrak{Z}$. And Souldiers 8ϕ.

$$\text{Then } \frac{8\phi}{200} = 9\mathfrak{z}. \text{ And } 8\phi = 1800\mathfrak{z}. \text{ And } 8\mathfrak{Z} = 1800N.$$

And $1\mathfrak{Z} = 225N$; whose Square Root is 15.

34. Among 4 Walls, the 2 longest are in proportion to the shortest, as 5 to 3; and to the Height they be double Sesquialter: now multiplying the longest by the shortest, and the Total by the Height, there will arise 39930 Feet: what then is the Length and Height of each Wall? *Q. Of the length and height of four Walls.*

Answ. The Height of the shortest is 22 Feet, and the longest being double *Sesquialter* (that is $2\frac{1}{2}$ as much) must be 55, and the Length of the shortest 33, that is as 5 to 3. Resolution.

B 39930. Supposition for the Height of the shortest A.
Then the longest must be $2\frac{1}{2}$ A. And because the longest to the shortest are as 5 to 3, the shortest must be $1\frac{1}{2}$ A. For $5 . 3 :: 2\frac{1}{2} . 1\frac{1}{2}$.
And $A \times 2\frac{1}{2}A \times 1\frac{1}{2}A = \frac{15}{4}Ac$. And $\frac{15}{4}Ac = B$.

$$\text{Wherefore } Ac = \frac{4B}{15}. \text{ or } \frac{159720}{15} = 10648. \text{ And } \sqrt{c}\,10648 = 22.$$

In

In *Cossicks* 1 ℨ. Then 1 ℨ × $2\frac{1}{2}$ ℨ × $1\frac{1}{2}$ ℨ = $\frac{15}{4}$ Φ.

And $\frac{15}{4}$ Φ = 39930. And 15 Φ = 159720. And 1 Φ = 10648.

And √Φ 10648 = 22.

Q. Of Stock traded with. 35. Traffique is made at *Danske*, and Gain thereupon in 100 *l.* as many Pounds as there were in Stock at first; after which Traffique with the Gain only is made at *Hamburgh*, where the Gain of 100 *l.* is as much as the Gain at *Danske*, and the Gain at *Hamburgh* being found to be $16\frac{1}{5}$ *s.* the Question is, what was the Stock at first?

Resolution. *Answ.* 30 *l.* For if 100 *l.* gain 30, then 30 shall gain at *Danske* 9 *l.* And if 100 *l.* gain 9, then 9 shall gain $16\frac{1}{5}$ *s.*

Here to make the Denominations agree, $16\frac{1}{5}$ *s.* is to be turned into a part of a Pound, and is $\frac{81}{100}$ *l.*

B 100 . $\frac{R}{B}$ $\frac{81}{100}$. Suppose the Stock A.

Then B . A :: A . $\frac{Aq}{B}$. Gain at *Danske*.

And B . $\frac{Aq}{B}$ :: $\frac{Aq}{B}$. $\frac{Aqq}{Bc}$. Gain at *Hamburgh*.

Then $\frac{Aqq}{Bc} = \frac{R}{B}$. And $Aqq = \frac{BcR}{B}$. or $\frac{81000000}{100} = 810000$.

And √qq 810000 = 30.

Supposing the Stock in *Cossicks* 1 ℨ.

Then 100 . 1 ℨ :: 1 ℨ . $\frac{1\,ʒ}{100}$. Gain at *Danske*.

And 100 . $\frac{1\,ʒ}{100}$ :: $\frac{1\,ʒ}{100}$. $\frac{1\,ʒʒ}{1000000}$. Gain at *Hamburgh*.

And $\frac{1\,ʒʒ}{1000000} = \frac{81}{100}$. And 1 ʒʒ = 810000; and the Squared square Root of 810000 is 30. 810000 (900 (30.

Q. Of the Number of Merchants and their Gain. 36. Certain Merchants make a Company, and every one putteth in Stock 150 times so many Pounds as there are Merchants, and they gain as much on the Hundred as the Number of Merchants are; and if $9\frac{1}{2}$ be taken out of the Gain, and $9\frac{1}{2}$ be added to the Gain, and the Remain and Total be multiplied together, the Product will be 1550 *l.* how many Merchants were there, and how much did they gain?

Resolution. *Answ.* 3 Merchants; 450 *l.* each Man's Stock (that is 3 times 150) and the Gain $40\frac{1}{2}$ *l.* For $40\frac{1}{2} - 9\frac{1}{2} = 31$; and $40\frac{1}{2} + 9\frac{1}{2} = 50$; and $50 \times 31 = 1550$.

Suppose the Number of Merchants in *Cossicks* 1 ℨ, (omitting the Work in *Species*) so is every Man's Stock 150 ℨ; and the whole Stock 150 ʒ.

And as 100 . 1 ℨ :: 150 ʒ . $\frac{150}{100}$ Φ. the whole Gain.

Then $\frac{3}{2}$ Φ — $\frac{19}{2}$ × $\frac{3}{2}$ Φ + $\frac{19}{2}$ = 1550. that is, $\frac{9\,ʒΦ - 361}{4} = 1550$.

And 9 ʒΦ — 361 = 6200. And 9 ʒΦ = 6561; and 1 ʒΦ = 729, whose Zenzicube Root is 3.

Of Affected Equations.

Touching Mixt or Affected Equations.

It is to be noted, that the *Equation* first found may be reduced to 3 sorts of Numbers or more.

Brought to 3 Numbers. An *Equation* consisting of 3 Numbers or Magnitudes, must have one a Power, another a Root, and the third an Absolute Number; and of these by the Signs + and — there are 3 sorts, in every of which the greatest Denomination is set Solitary as equal to the other two.

As ʒ = ℨ + N. ʒ = N — ℨ. ʒ = ℨ — N.

And accordingly with the Antients, the Extraction, or rather the finding the Value of the Root, is of 3 Varieties; but of late both the Extraction of the Roots of these and others consisting of more than three Magnitudes, comprehended under

der one and the fame Method, is moft ingenioufly taught by the aforefaid Mr. *Oughtred*, as fhall be fhewn hereafter. Notwithftanding becaufe very many Propofitions of ordinary ufe are reduced to *Equations* of 3 Numbers, and the antient Way of finding the Value of the Root thereof is by fome accompted the moft eafy, that fhall be firft examined.

Refolution of fome Affected Equations *the antient Way.* *Refolution the old Way*

When an *Equation* confifts of 3 Numbers or Magnitudes, either they are in their Natural Order, as aforefaid, without omiffion of any Denomination between them, or they are not. *In 3 Numbers.*

1ft. If they are in their Natural Order, and without fuch Omiffion, and the *Equation* be of the firft fort that ʒ = ℨ + N. or ʒ = N + ℨ which is all one: The Rule is, multiply half the middle Quantity fquarely; to the Product add the Abfolute Number, out of the Total take the Square Root, and thereto add the former half of the middle Quantity, and this laft Total fhall be the Value of one Root of that *Equation.* *1. If in a natural Order, and none omitted, the first Sort.*

As fuppofe 1ʒ = 4ℨ + 21N. or 1ʒ = 90N + 9ℨ. Examples.

In the firft, the half of 4 (belonging to ℨ the middle Quantity) is 2, which fquared is 4, to which 21 added makes 25, whofe Square Root is 5, to which 2 the half added, makes the Total 7, for the Value of the Root fought.

For 1ʒ = 4ℨ + 21 N. The Root being 7, fhall make 49 = 28 + 21.

In the other Inftance $4\frac{1}{2}$ℨ or $\frac{9}{2}$ℨ, the half of 9ℨ fquared, produceth $\frac{81}{4}$; this added to 90 or $\frac{360}{4}$, makes $\frac{441}{4}$, the Square Root whereof is $\frac{21}{2}$, to which $\frac{9}{2}$ added makes $\frac{30}{2}$ or 15 for the Value of 1ℨ, and appears true.

Seeing if 15 be 1ℨ, then 9ℨ is 135, whereunto 90 being added makes the Total 225, and fo much is the Square of 15.

But when the *Equation* of this fort omits fome Denomination orderly between the Quantities, then after the Root found in like manner as above, from this fo found Root or Value, extract a Root according to the Number of Quantities omitted, *viz.* if 1 the Square Root, if 2 the Cube Root, *&c.* *If fome Quantity omitted.*

As fuppofe 1ʒʒ = 2ʒ + 8N. Or 1ʒφ = 7φ + 8 N. Examples.

In the firft the Value of the Root gotten as above, will be 4, which becaufe one Denomination is omitted muft be a Square Number, and the Root thereof 2 the Number fought.

Thus $\frac{1}{2}$ of 2 is 1, and 1 fquared is 1, to which 8 added is 9, whofe Root is 3, to which 1 the half added makes 4, and √ʒ 4 = 2.

And 1ʒʒ = 2ʒ + 8N. The Root being 2 fhall make 16 = 8 + 8.

In the other Inftance, the Root gotten as aforefaid will be 8, out of which the Cube Root muft be extracted, becaufe 2 Quantities were orderly omitted in the *Equation*; and this Cube Root 2 fhall be the Value of the Root fought, whereof the *Equation* is framed.

Becaufe $\frac{1}{2}$ of 7φ the Middle Quantity is $\frac{7}{2}$, which fquared is $\frac{49}{4}$, to which 8 added that is $\frac{32}{4}$, makes $\frac{81}{4}$, the Square Root whereof is $\frac{9}{2}$, to which the half $\frac{7}{2}$ added, makes $\frac{16}{2}$ or 8, and √c 8 = 2.

And if 1ℨ be 2, then the Cube will be 8, and 7φ are 56, which with 8 N makes 64, a Zenzicube Number, and hath 2 for the Root.

2ly. Equations affected of the fecond Sort, (that is) confifting of 3 Numbers in their natural Order, without omiffion of any Denominations between them, and have their Middle Quantity joined with the Sign, as ʒ = N − ℨ, have the Rule to find the Value of the Root in all refpects, fave one like the firft Sort of *Affected Equations*; which Difference is only inftead of adding half the Middle Quantity to the Root: here it is to be fubftracted, and the Remainder fhall be the Value of the Root defired. *2. If in a natural Order, and none omitted, the fecond Sort.*

As fuppofe 1ʒ = 60N − 7ℨ. Or 1ʒ = 153N − 8ℨ. Examples.

In the firft, the half of 7, which is $3\frac{1}{2}$ or $\frac{7}{2}$ fquared, is $\frac{49}{4}$; whereto 60, or $\frac{240}{4}$ added, makes $\frac{289}{4}$ the Square Root, whereof is $\frac{17}{2}$; from whence $\frac{7}{2}$ taken, leaves $\frac{10}{2}$ or 5 for the Value of the Root defired.

For 1ʒ = 60 N − 7ℨ: The Root being 5, makes 25 = 60 − 35.

In the other Instance, half 8 is 4, the Square 16; to which 153 added, is 169; the Square Root whereof is 13; from whence 4 taken, leaves 9 for the Value of 1 ℨ.

So shall the Square be 81, and so much is 153 lacking 72 that is 8ℨ.

If some Quantity omitted.

But when in *Equations* of this sort some Quantities are orderly omitted between the Quantities propounded; then after the Root or Value rather found as before, out thereof let there be a Root extracted, according to the Number of Quantities omitted, as before noted in the former sort of *Equations*.

Examples.

As suppose 1ʒʒ = 117N — 4ʒ. Or 1ʒφ = 810N — 3φ.

In the first of these, squaring 2 the Half of 4, there ariseth 4, which added to 117 maketh 121, the Square Root of which is 11; whence if 2 the first Half be substracted, there resteth 9; which because one Denomination is omitted, the Square Root thereof 3 is the Number desired.

In the other $\frac{3}{2}$ squared is $\frac{9}{4}$, which added to 810, makes $\frac{3249}{4}$, the Square Root of which is $\frac{57}{2}$; if then $\frac{3}{2}$ be abated, there remaineth $\frac{54}{2}$, or 27, which is a Cube Number, and the Cube Root thereof 3 is the Value desired.

So the Root being 3: 1ʒʒ = 117N — 4ʒ is as 81 = 117 — 36.
1ʒφ = 810N — 3φ is as 729 = 810 — 81.

3. If in a natural Order, and none omitted, the third Sort.

3ly. *Affected Equations* of the third Sort, to wit, consisting of 3 Quantities in their natural Order, without omission of any Denominations, and having their least Quantity joined with —, as ʒ = ℨ — N, have 2 Roots or Values, and therefore are called *Ambiguous Equations*: For finding both which, the Rule is thus; Square half the middle Quantity as before, from thence substract the smallest Quantity, extract the Square Root of the Residue, and add this Root to the Half first taken for the Value of one, and take it away from the Half for the Value of the other Root: And either of these Numbers shall be as the Root of the *Equation*, and one of them shall serve to resolve the *Equation*, as occasion shall require; and which of the two is necessary, by the Frame of the Question is directed.

For though no Number can have 2 Square Roots, or 2 Cube Roots, *&c.* yet one Number may have both a Square Root and a Cube Root. Yea, as before hath been seen, one Number may have several Roots of different Denominations, as 64 hath a Square Root 8, a Cube Root 4, and a Zenzicube Root 2. And so in *Equations* of this kind, 2 Roots or a double Value may be had, unless the Half of the Middle Quantity be equal to the Absolute Number, for then is the Moiety or half of the Middle Quantity the Number sought. And these 2 Roots so called, must be such, as being added together, will make the Number of the Middle Quantity, and multiplied together, will make the Number of the least Quantity, and so may be found without farther Operation.

Examples.

As suppose 1ʒ = 12ℨ — 32N.

The Parts of 32 examined, are found to have no more to his Composition likely to this purpose, than 2, 4, 8, 16; but if 2 and 16 be taken, their addition 18 will be greater than 12: wherefore 4 and 8 making 12 by *Addition*, and 32 by *Multiplication*, shall be the 2 desired Numbers.

And this is agreeable to the Rule: For the half of 12 is 6, whose Square is 36, from which is to be taken 32 of the Residue; 4 the Square Root is 2, which either taken from 6 leaves 4 for the one, or added to 6 makes 8 for the other Root or Value required.

So 1ʒ = 12ℨ — 32N, let the Root be 4 and 16 = 48 — 32.
8 64 = 96 — 32.

If some Quantity omitted.

But when the *Equation* of this sort omits some Quantities orderly, there happeneth oftentimes but one Root in *Integers*, and not two: From which Root or Value, as in the others before, must be extracted a Root according to the Denominations omitted.

Examples.

As suppose 1ʒʒ = 24ʒ — 135N. Or 1ʒφ = 12φ — 32N.

In the first 12 squared is 144; from which if 135 be taken, the Square Root of the remaining 9 is 3; to which if 12 be added it makes 15, which is no Square Number;

Number; but if 3 be abated from 12, the Remainder 9 is a Square Number, and hath 3 for his Root.

In the other 6 times 6 is 36, whence 32 abated leaves 4, whose Square Root is 2; to which if 6 be added ariseth 8, the Cube Root whereof is 2 for the Number desired; but if 2 be abated from 6, there remaineth 4, which is no Cube Number, nor hath a Cube Root in whole Numbers.

Thus the Root being 3 . 1ʒʒ = 24ʒ — 135N is as 81 = 216 — 135.
2 . 1ʒφ = 12φ — 32N 64 = 96 — 32.

Because in every Operation where an *Equation* ariseth, it is to be noted, That according to the Supposition for Resolution of the Proposal, so will the *Equation* be produced of the one sort or other; and when *Equations* of this sort happen, wherein is a double Valuation of the Root, the form of the Question sheweth which of the 2 Roots is to be taken, unless the Proposition may be truly resolved by both, as sometimes it cometh to pass.

Questions wherein the first sort of Affected Equations *come to be resolved.*

Ex. *Where the first Sort are resolved.*

1. A Merchant buyeth 45 *C.* and 88 lb. of Pepper, the Tare is 12 lb. *per Cent.* the Hundred cost $6\frac{3}{5}$ *l.* and the whole 274 *l.* how many Pound is the Hundred accompted?

Q. *Of the Pounds in an Hundred of Pepper.*

Answ. 132 lb.

Resolution.

Here supposing the Hundred to be 1ℨ, the Tare abated makes it 1ℨ — 12.

Then as 1ℨ . 1ℨ — 12 :: 45ℨ + 88 . $\dfrac{45ʒ - 452ℨ - 1056}{1ℨ}$.

And then as 1ℨ . $6\frac{3}{5}$ :: $\dfrac{45ʒ - 452ℨ - 1056}{1ℨ}$. $\dfrac{297ʒ - 2983\frac{1}{5}ℨ - 6969\frac{3}{5}}{1ʒ}$.

Wherefore $\dfrac{297ʒ - 2983\frac{1}{5}ℨ - 6969\frac{3}{5}}{1ʒ} = 274$.

And 274ʒ = 297ʒ — $2983\frac{1}{5}$ℨ — $6969\frac{3}{5}$. And 23ʒ = $2983\frac{1}{5}$ℨ — $6969\frac{3}{5}$.
And 1ʒ = $129\frac{81}{115}$ℨ + $303\frac{3}{115}$. And 1ℨ = 132 lb.
For $\frac{1}{2}$ of $129\frac{81}{115}$ is $64\frac{98}{115}$ or $\frac{7458}{115}$ squared is $\frac{55621764}{13225}$. and $303\frac{3}{115}$ or $\frac{4007520}{13225}$ added is $\frac{59629284}{13225}$ the √ʒ whereof is $\frac{7722}{115}$ whereto $\frac{7458}{115}$ added makes the Total $\frac{15180}{115}$ or 132.

Proof.

If 132 lb. abate 12 lb. for Tare, then 1 *C.* shall be but 120 lb.

Proof.

And as 1 *C* . 120 lb :: 45 *C* + 88 lb . 5480 lb.
And as 132 lb . $6\frac{3}{5}$ *l* :: 5480 lb . 274 *l.*

2. Two Men have Silks to sell, *viz.* *A* hath 40 Ells, and *B* 90; and the Silk of *A* being not so fine as the Silk of *B*, he selleth in every Angel more by $\frac{1}{3}$ of an Ell than *B* doth, and both their Monies made 42 Angels: now how much did each of them sell for an Angel?

Q. *Of Silks, how much sold for an Angel.*

Answ. *A* sold $3\frac{1}{3}$ Ells, *B* sold 3 Ells for an Angel.

Resolution.

Here supposing the least Quantity which is *B*, to be 1ℨ;
Then must *A* sell 1ℨ + $\frac{1}{3}$ Ell: and each whole Parcel divided thereby,

is for *B* $\dfrac{90}{1ℨ}$ for *A* $\dfrac{40}{1ℨ + \frac{1}{3}N}$ or reduced $\dfrac{120}{3ℨ + 1N}$

And these two Numbers declaring the Angels each Man received;

It follows that $\dfrac{120}{3ℨ + 1N} + \dfrac{90}{1ℨ} = 42$. And reduced $\dfrac{390ℨ + 90}{3ʒ + 1ℨ} = 42$.

And 390ℨ + 90N = 126ʒ + 42ℨ. And 126ʒ = 348ℨ + 90N.
And 21ʒ = 58ℨ + 15N. And 1ʒ = $\frac{58}{21}$ℨ + $\frac{15}{21}$N.
And 1ℨ = 3. For $\frac{1}{2}$ of $\frac{58}{21}$ is $\frac{29}{21}$, squared is $\frac{841}{441}$; whereto $\frac{15}{21}$ or $\frac{315}{441}$ added, is $\frac{1156}{441}$, the √ʒ whereof is $\frac{34}{21}$, to which $\frac{29}{21}$ added makes $\frac{63}{21}$ or 3.

A

Proof.

Proof.

Ells A $3\frac{1}{3}$ or $\frac{10}{3}$) 40 ($\frac{120}{10}$ (12 Angels. *Ells* B 3) 90 (30 Angels.

And $12 + 30 = 42$.

Ex. *Where the second Sort are resolved.*

Questions wherein the second sort of Affected Equations *come to be resolved.*

Q. *Of the Stock and Gain of 3 Merchants.*

1. Three traffique together, *A* putteth in Stock 14 *l.* less than *B*, and *B* and *C* together put in 148 *l.* they gain 42 *l.* more than their Stock, of which *A* taketh $60\frac{20}{33}$ *l.* what was each Man's Stock and Part of the Gain?

Resolution.

Answ. The Stock of *A* 50 *l.* *B* 64 *l.* *C* 84 *l.* Gains in all 240 *l.* of which to *A* $60\frac{20}{33}$ *l.* *B* $77\frac{19}{33}$ *l.* *C* $100\frac{9}{11}$ *l.*

Here supposing the Stock of *A* to be $1\mathfrak{z}$; then must the Stock of *B* be $1\mathfrak{z} + 14$; and *B* and *C* making up 148, the Stock of *C* must be $134 - 1\mathfrak{z}$; and all these three added, make $1\mathfrak{z} + 148$ the whole Stock, which with 42 more, that is $1\mathfrak{z} + 190$, is all the Gains.

Then as $1\mathfrak{z} + 148 \,.\, 1\mathfrak{z} + 190 :: 1\mathfrak{z} \,.\, \frac{1\mathfrak{Z} + 190\mathfrak{z}}{1\mathfrak{z} + 148}$. Gain of *A*

And then $\frac{1\mathfrak{Z} + 190\mathfrak{z}}{1\mathfrak{z} + 148} = 60\frac{20}{33}$. And $33\mathfrak{Z} + 6270\mathfrak{z} = 2000\mathfrak{z} + 296000$.

And $33\mathfrak{Z} = 296000 - 4270\mathfrak{z}$. And $1\mathfrak{Z} = \frac{296000}{33} - \frac{4270}{33}\mathfrak{z}$.

And $1\mathfrak{z} = 50$

For $\frac{1}{2}$ of $\frac{4270}{33}$ is $\frac{2135}{33}$, squared is $\frac{4558225}{1089}$, to which $\frac{296000}{33}$, that is $\frac{9768000}{1089}$, added is $\frac{14326225}{1089}$. the $\sqrt{\mathfrak{Z}}$ whereof is $\frac{3785}{33}$, from which $\frac{2135}{33}$ taken, the Remainder is $\frac{1650}{33}$ or 50.

Proof.

Proof.

The Stock of *A* less by 14 than *B*; if therefore it be 50, that of *B* must be 64, that is $50 + 14$. And because *B* and *C* made 148, if *B* be 64, then must *C* be 84, for $84 + 64 = 148$: and the Gains being 42 more than the Stock, must be 240, for $50 + 64 + 84 + 42 = 240$; and then by the *Rules of Fellowship*,

$$\text{If } 198 \text{ gain } 240, \text{ the Gain of } \begin{cases} 50 & 60\frac{20}{33}\ A. \\ 64 \text{ is} & 77\frac{19}{33}\ B. \\ 84 & 101\frac{2}{11}\ C. \end{cases}$$

Q. *Of the Days in which a Journey is performed.*

2. A Traveller hath a Journey to go of 2955 Miles, and the first Day he goeth $1\frac{1}{2}$ Mile, and every Day afterwards increaseth his Journey by $\frac{1}{6}$ of a Mile as in an *Arithmetical Progression*: in how many Days shall he finish his Journey?

Resolution.

Answ. In 180 Days.

Here are given the first Term of the *Progression* $1\frac{1}{2}$, the Excess $\frac{1}{6}$ and the Sum 2955, to find the Number of Terms; for which supposing $1\mathfrak{z}$, then shall all the Excesses or Number of Spaces be $1\mathfrak{z} - 1N$, by which the Excess multiplied, the Product is $\frac{1\mathfrak{z} - 1N}{6}$ the Sum of all the Excesses; to which the first Term added which is $1\frac{1}{2}$, or reduced to like Denomination $\frac{9}{6}$, so will the Total be $\frac{1\mathfrak{z} + 8N}{6}$ the last Term of that *Progression*; which gotten, because $\alpha + \omega$ in $\frac{1}{2}T = Z$, the first $\frac{9}{6}$ is added to the last Term $\frac{1\mathfrak{z} + 8N}{6}$ and the Total $\frac{1\mathfrak{z} + 17N}{6}$ multiplied by $\frac{1}{2}\mathfrak{z}$, (seeing the whole Number of Places is supposed to be $1\mathfrak{z}$) the Product is $\frac{1\mathfrak{Z} + 17\mathfrak{z}}{12}$, the Sum of the *Progression*.

So is $\frac{1\mathfrak{Z} + 17\mathfrak{z}}{12} = 2955$. And $1\mathfrak{Z} + 17\mathfrak{z} = 35460$.

And

And $1\text{ʒ} = 35460 - 17\mathfrak{z}$. And $1\mathfrak{z} = 180$.

For $\frac{1}{2}$ of 17 is $8\frac{1}{2}$, or $\frac{17}{2}$ squared $\frac{289}{4}$, to which $\frac{141840}{4}$ added, makes $\frac{142129}{4}$ whose Square Root is $\frac{377}{2}$, from whence $\frac{17}{2}$ taken, there remaineth $\frac{360}{2}$, or 180.

Proof. Proof.

Seeing $\alpha\ 1\frac{1}{2}$. $X\ \frac{1}{6}$. $Z\ 2955$, are given thereby, the Rules before in *Progression* direct to the finding of T the Number here sought.

For $\sqrt{:}\ \frac{\alpha q - \alpha X + \frac{1}{4}Xq + 2ZX}{Xq} : -\alpha + \frac{1}{2}X = T$. That is in Numbers

$$\sqrt{:}\ \frac{\frac{9}{4} - \frac{1}{4} + \frac{1}{144} + \frac{5910}{6}}{\frac{1}{36}} : -\frac{3}{2} + \frac{1}{12} = 180.$$

$$\text{Or } \sqrt{:}\ \tfrac{1}{36}\Big) \frac{324 - 36 + 1 + 141840}{144} : -\frac{3}{2} + \frac{1}{12} = 180.$$

$$\begin{array}{r} 141840 \\ 1 \\ 324 \\ \hline 142165 \\ -36 \\ \hline \end{array}$$

$$\tfrac{1}{36}\Big) \frac{142129}{144}\Big(\sqrt{q}\ \frac{5116644}{144}\Big(\frac{2262}{12}. \quad \text{And } \frac{2262 - \frac{3}{2} + \frac{1}{12}}{12} = 180.$$

$$\text{And } \frac{2262 - \frac{36}{2} + \frac{12}{12}}{12} = 180. \quad \text{And } \frac{377 - 18 + 1}{2} = \frac{360}{2} = 180.$$

Questions, wherein the third Sort of Affected Equations *come to be resolved.* Ex. *Where the third Sort are resolved.*

1. Two Numbers added together make 8, but added severally, with their respective Squares and Cubes, make 194: what are those Numbers? Q. *Of 2 Numbers, what they are.*

Answ. 3 and 5.

Here supposing one of the two Numbers to be $1\mathfrak{z}$, then must the other be $8 - 1\mathfrak{z}$. These with their Squares and Cubes are thus: Resolution.

$1\mathfrak{z}$	Numbers	$8 - 1\mathfrak{z}$.
1ʒ	Squares	$1\text{ʒ} + 64 - 16\mathfrak{z}$.
1ϕ	Cubes	$24\text{ʒ} + 512 - 1\phi - 192\mathfrak{z}$.

$1\phi + 1\text{ʒ} + 1\mathfrak{z}$.
$-1\phi + 25\text{ʒ} - 209\mathfrak{z} + 584$.

Total of both $26\text{ʒ} - 208\mathfrak{z} + 584$.

And then $26\text{ʒ} - 208\mathfrak{z} + 584 = 194$. And $26\text{ʒ} = 208\mathfrak{z} - 390$.
And $1\text{ʒ} = 8\mathfrak{z} - 15$. And $1\mathfrak{z} = 3$ or 5.

For $\frac{1}{2}$ of $8\mathfrak{z}$ is 4, squared is 16; from whence 15 substracted, there is left 1, whose Square Root is 1, which added to 4 makes 5, or taken from 4 leaves 3, for the Value of either Root; both which are necessary to the Solution of this Question.

Proof. Proof.

$$\overset{\mathfrak{z}}{3} + \overset{\text{ʒ}}{9} + \overset{\phi}{27} = 39. \quad \text{And } \overset{\mathfrak{z}}{5} + \overset{\text{ʒ}}{25} + \overset{\phi}{125} = 155.$$
And $39 + 155 = 194$.

2. A Man owed a Sum of Money, which might be divided into two such Parts, that being multiplied would make 24, and their Cubes added together 280: what was the Debt? Q. *Of a Debt, what it was.*

Answ. 10 *l.* Resolution.

Here supposing one Part of the two to be $1\mathfrak{z}$, then must the other be $\frac{24}{1\mathfrak{z}}$.

The Cubes of both which are,

$$\overset{\surd}{1ℨ}.\ \overset{q}{1ʒ}.\ \overset{c}{1φ}.\quad \frac{\overset{\surd}{24}}{1ℨ}.\ \frac{\overset{q}{576}}{1ʒ}.\ \frac{\overset{c}{13824}}{1φ}.$$ And $\frac{1φ}{1}+\frac{13824}{1φ}$ added are $\frac{1ʒφ+13824}{1φ}$

And this equal to 280. Wherefore $1ʒφ = 280φ - 13824$.

And $1ℨ = 4$ or 6.

For $\frac{1}{2}$ of 280 is 140, ſquared is 19600; out of which 13824 taken, the Remain is 5776, whoſe Square Root is 76; which added to 140, maketh 216, and abated from it, leaveth 64; out of both which the Cube Root extracted, becauſe there are 2 Quantities omitted in the *Equation*, the Roots deſired are 6 and 4.

Proof.

Proof.

Parts.	*Debt.*		*Cubes.*	*Sum.*
$6+4=10.$		$6\times4=24.$	$216+64=280.$	

Ex. *Where the Equation is according to the Suppoſition.*

Queſtions wherein according to the Suppoſition, ſo the Equation *happens to be of one ſort or other.*

Q. *Of a Number, what it was.*

1. A Number thought upon hath two Parts, of which the one is 4, and the other multiplied into it ſelf; and then alſo with 4 the two Products will be 117: what was that Number?

Reſolution.

Anſw. 13.

Here ſuppoſing the Number to be 1ℨ, then 1 Part being 4, the other Part muſt be $1ℨ - 4$: This Part ſquared is $1ʒ - 8ℨ + 16$. And the ſaid unknown Part (that is $1ℨ - 4$) multiplied by 4, is $4ℨ - 16$; which added to the Square, makes $1ʒ - 4ℨ$; which being equal to 117, makes an *Equation* of the firſt Sort thus:

$1ʒ = 4ℨ + 117.$ And $1ℨ = 13.$

For $\frac{1}{2}$ of 4 is 2, ſquared is 4, to which 117 added is 121, the Square Root of which is 11, and thereto 2 added is 13, the Number deſired.

But if Suppoſition be made for the unknown Part, then if that Part be 1ℨ, the whole Number ſhall be $1ℨ + 4$: And then multiplying this Part by it ſelf, and alſo by 4, the known Part there will ariſe 1ʒ and 4ℨ; which being equal to 117, makes an *Equation* of the ſecond Sort thus: $1ʒ = 117 - 4ℨ$. And $1ℨ = 9$, the unknown Part.

For $\frac{1}{2}$ of 4ℨ is 2, which ſquared is 4, this added to 117 is 121, whoſe Square Root is 11; from whence 2 taken, there remaineth 9 for the Part unknown.

Proof.

Proof.

Parts.	*Whole Number.*			
$4+9=13.$		One Part led into the other	$9\times4=$	36
		One Part ſquared	$9\times9=$	81
				117 Total.

Q. *Of 3 Numbers, what the Leaſt and Mean.*

2. There are 3 Numbers in *Geometrical Proportion*, the greateſt Extream is $20\frac{1}{4}$, the leaſt Extream with the double of the middle Term makes 22: what is the leaſt Extream, and the middle Term?

Reſolution.

Anſw. The leaſt Extream is 4, and the middle Term 9.

Here ſuppoſing the leaſt Extream to be 1ℨ: And becauſe the leaſt with the double of the middle Term muſt make 22, the middle Term ſhall be $11 - \frac{1}{2}ℨ$, for his double is $22 - 1ℨ$, which with 1ℨ is 22. Then becauſe the Product of the Mean multiplied into it ſelf is equal to the Product of the Extreams, the two Extreams multiplied, which are 1ℨ and $20\frac{1}{4}$, produce $\frac{81}{4}ℨ$. And the Mean that is $11 - \frac{1}{2}ℨ$, ſquared is $121 + \frac{1}{4}ʒ - 11ℨ$; which being equal to the other, the *Equation* ſtands thus, $121 + \frac{1}{4}ʒ - 11ℨ = \frac{81}{4}ℨ$. And by Reduction $484 + 1ʒ - 44ℨ = 81ℨ$. And again $1ʒ = 125ℨ - 484$, an *Equation* of the third Sort. And $1ℨ = 4$.

For $\frac{1}{2}$ of 125 is $\frac{125}{2}$, ſquared is $\frac{15625}{4}$; from which 484 that is $\frac{1936}{4}$ abated, the Remain is $\frac{13689}{4}$, whoſe Square Root is $\frac{117}{2}$; this abated from $\frac{125}{2}$, leaves $\frac{8}{2}$ or 4 for the Root deſired. And although by Addition of the Square Root to the Half, as in other *Equations* of the third Sort, that is $\frac{117}{2}$ to $\frac{125}{2}$, another Root will be obtained, to wit $\frac{242}{2}$, or 121; yet cannot this be

be the Root intended by the Propoſition, becauſe the leaſt Extream with the double of the middle Term was bounded to make but 22, when this Root 121 is much more of it ſelf. But if the middle Term be ſuppoſed 1 ℨ, then becauſe the double thereof with the leaſt Term muſt make 22, that leaſt Term muſt be 22—2ℨ. And ſeeing the Square of the Mean 1 ℨ, that is 1ʒ, muſt be equal to the Product of the two Extreams, that is 22 — 2ℨ × 20$\frac{1}{4}$, which is 445$\frac{1}{2}$—40$\frac{1}{2}$ℨ, there ariſeth an *Equation* of the ſecond Sort, that is 1ʒ = 445$\frac{1}{2}$ — 40$\frac{1}{2}$ℨ. And 1 ℨ = 9.

For $\frac{1}{2}$ of 40$\frac{1}{2}$ is $\frac{81}{4}$, whoſe Square is $\frac{6561}{16}$, to which adding 445$\frac{1}{2}$, or $\frac{7128}{16}$, the Total is $\frac{13689}{16}$ the Square of $\frac{117}{4}$; from which $\frac{81}{4}$ abated, there remaineth $\frac{36}{4}$, or 9 for the middle Term.

Proof.

Proof.

Proportionals 4 . 9 . 20$\frac{1}{4}$. And 4 + 9 + 9 = 22.
Extreams 4 × 20$\frac{1}{4}$ = 81. Mean 9 × 9 = 81.

Reſolution of Affected Equations, *by Mr.* Oughtred's *Way.*

Reſolution of Affected Equations by Mr. Oughtred.

Mr. *Oughtred* in the 16*th* Chapter of his *Clavis*, hath delivered two Rules for the Reſolution of every *Equation*, wherein are 3 Species or Quantities orderly aſcending, ſuch as thoſe herein already ſpoken to; and directs, that the Abſolute Number (being one of the three) ſhall be reckoned the Rectangle (or Product) of the two Magnitudes ſought, whether Root, Square, or Cube, &c. that is to ſay, ſuch as is the Power of the middle *Species*. And in the middle *Species*, if the higheſt be Negative, the Coefficient ſhall be counted the Sum of the ſought Magnitudes, and be manifeſt of both. But if the higheſt *Species* be Affirmative, the Coefficient ſhall be the Difference of the Magnitudes ſought; and the ſame *Species* ſhall be ſhewn of the Greater denied, or of the Leſſer affirmed.

And ſeeing (by the ſecond Chapter of this 4*th* Part in the *Invention of Equations* it appeareth) that the Sum and Rectangle of two Magnitudes given, the Difference is given; or the Difference and Rectangle given, the Sum is given; and by the Sum and Difference the Magnitudes themſelves are given. So as Z and X given, A and E may be found by the ſaid Rules, thus by him ſymbolized.

Two Rules in Species by Mr. Oughtred *for the former* 3.

$$1.\quad \tfrac{1}{2}Z \pm \sqrt{:}\,\tfrac{1}{4}Zq - Æ:\ (\tfrac{1}{2}X) = \begin{matrix}A.\\E.\end{matrix}$$

$$2.\quad \sqrt{:}\,\tfrac{1}{4}Xq + Æ:\ (\tfrac{1}{2}Z)\ \pm \tfrac{1}{2}X = \begin{matrix}A.\\E.\end{matrix}$$

Theſe two Rules being the ſame in effect with thoſe before handled in the Antient Coſſical way, for Reſolution of orderly *Affected Equations*, will need no Explanation here; and the rather, for that the ſame Author afterward, in a Tract of 28 *Sections*, or *Precepts*, hath taught the Inveſtigation of the Root of all ſorts of *Affected Equations*, as well diſordered as others, in one Method; which to avoid multiplicity of Rules, is chiefly to be choſen and followed.

In this joint way of Work, contrary to that by *Coſſicks*, the Abſolute Number or Magnitude is ſet ſolitary, and not the higheſt; ſo as the three ſorts of *Equations* before

Difference in the Reſolution from Coſſicks.

Set thus	ʒ = ℨ + N.	ʒ = N — ℨ.	ʒ = ℨ — N.
are here	Aq — A = N.	Aq + A = N.	A — Aq = N.
And ſo	Aq — XA = Æ.	Eq + XE = Æ.	ZA — Aq = Æ.

The Reaſon whereof is, becauſe every *Equation* may, by Reduction, be brought to an Abſolute Number or Magnitude, which being known, is certain and without Fiction or Figuration, and ſo may be ſet againſt all the other unknown Parts of the *Equation*. For it is evident in this *Equation*,

The Number known to be ſet alone againſt all the other.

8ʒφ = 10ſs + 20ʒʒ + 400φ + 31250N. Or in *Species* tranſlated,
8Acc — 10Aqc — 20Aqq — 400Ac = 31250, that the eighth Part of
10ſs + 20ʒʒ + 400φ + 31250N muſt be a Zenzicube Number. And that 20ʒʒ + 400φ + 31250N contain a certain Number of Surſolids: And alſo that 400φ + 31250N, contain certain Zenzizenzikes. And ſo conſequently 31250 ſhall contain therein certain Cubes, as may be proved, the Root being 5.

The

28 Precepts of Mr. Oughtred of Affected Equations.

The Substance of those 28 *Sections*, or *Precepts*, (save what concerns the use of *Logarithms*, already learned) follows in order as the Author hath left them; the Translation whereof, with the following Examples, will be sufficient, without Additional Illustration.

1. To constitute an Affected Equation.

1. The manner of constituting an *Affected Equation*; Let there be taken at pleasure, for B, 3; for Cq, 16; for Dc, 125; for Fqq, 1296, *&c.* Neither is it material, whether the Numbers are truly figurate or not. And let there be constituted of these Coefficients, a *Surfolid Equation*, according to the manner of the *Analytical Table*, (in Chap. 11. *Figuration of Rational Species*) *viz.*

Example.

Lqc — 5BLqq + 10CqLc — 10DcLq + FqqL = Gqc; which in Numbers appointing L(the Root)47, shall be 1qc—15qq+160c—1250q+6480l=170304782: or omitting the Distinction of the *Unciæ*; For 15qq, say BLqq; for 160c, say CqLc; for 1250q, say DcLq; and for 6480l, say FqqL. For if L be 47, then shall Lq=2209, and Lc=103823, and Lqq=4879681, and Lqc=229345007.

The Practice of this Constitution.

BLqq 15×4879681	229345007 —73195215	Lqc.
CqLc 160×103823	156149792 +16611680	
DcLq 1250 x 2209	172761472 —2761250	
FqqL 6480 x 47	170000222 + 304560	
	170304782	Gqc.

2. Every one to be counted as the former Example.

2. Count every propounded *Equation* as this now found.
1qc — 15qq + 160c — 1250q + 6480l = 170304782.
Or the Numbers changed into *Species*.
Lqc — BLqq + CqLc — DcLq + FqqL = Gqc.

How to express more Affections.

And if there were more *Species* of the Affections, consequently they might be expressed by Hcc, Kqqc, Mqcc, Nccc, and so further.

3. To search out the two Parts. Example.

3. Of the Root L there shall by these be two Parts searched out, to wit, A the first Side, and E the second Side, or whatsoever is subsequent; wherefore L=A+E, and all the Powers of L equally to the like Powers of A+E. As Lq=Aq+2AE+Eq. And Lc=Ac+3AqE+3AEq+Ec, *&c.*

4. Heterogeneals not to be added or substracted, &c.

4. In the propounded *Equation*, the Power to be resolved 170304782, or Gqc, is a Surfolid, of which kind also are the several Species of the Affections; for *Heterogeneals* cannot be added nor substracted among themselves.

5. Two things to be considered. (1) The Degree of Affection. (2) The Coefficient.

5. Wherefore in the several Affections, 2 things are to be considered, The Degree of Affection, and the Coefficent; as in 15qq, the Degree of Affection is squared Square, and the Coefficient 15 Root: In 160c, the Degree of Affection is Cube, and the Coefficient 160 Square: In 1250q, the Degree of Affection is Square, and the Coefficient 1250 Cube: Lastly, in 6480l the Degree of Affection is Root, and the Coefficient 6480 squared Square: and hence arise 2 Consectaries for Extraction of the singular Sides.

6. Consectary from thence.

6. The first Consectary is, if the Root of the Coefficient according to his own kind multiplied into the Degree of Affection, shall multiply the same Coefficient, the Product shall be of the same kind with the Power to be resolved: As in the precedent *Equation*, if the Side 15 multiplied squared-squaredly, be multiplied into 15; and if √q 160 cubed be multiplied into 160 squared; and if √c1250 squared be multiplied into the Cube of 1250; Lastly if √qq 6480 be multiplied into the squared Square of 6480: of all these several Multiplications shall arise a Surfolid Number. And this Analytical Multiplication is the manner of reducing every Coefficient to the Species of the Power to be resolved, most used in Extraction of every first Side.

Example.

7. Another Consectary from the first.

7. From whence also most clearly appeareth, That if the Number arising of the Coefficients, in this manner reduced and compared, be less than the Power to be resolved, the Side thereof also is less than the side of the Power to be resolved; but if Greater, it is Greater; and if Equal, Equal. Therefore in this

Equation

Equation 1qc — 15qq + 160c — 1250q + 6480l = 170304782; or 170304782 + 15 qq — 160 c + 1250 q — 6480 l = 1 qc. If then the lateral Coefficient 15, and √q160, and √c1250, and √qq 6480 be made Surfolids, they shall produce four Homogeneal Species of Affections, to wit, 7593 . ., 3238 . ., 1450 . ., 0581 . ., which by Logarithms is most easily done, and sufficiently exact for the purpose. Example.

Logarithms.		*Coefficient Numbers.*
1) 2) 3) 4) are the Dimensions in the Coefficient.		
1) 5 × 1,17609,12591		15 qq
5,88045,62955		+7593 . .
2) 2,20411,99827		160 c
5 × 1,10205,99913	12,65	
5,51029,99565		—3238 . .
3) 3,09691,00130		1250q
5 × 1,03230,33376	10,8 —	
5,16151,66880		+1450 . .
4) 3,81157,50059		6480 l
5 × 0,95289,37514	8,97	
4,76446,87570		—0581 . .

The Species being gathered into one Sum, according to the Order of their Signs among those propounded in the *Equation*, it shall be, that

170304700 + 759300 — 323800 + 145000 — 058100 = 1qc = 170827100.

which also in other *Equations* may likewise be done.

8. The second Consectary is, if the Power to be resolved be divided by the Coefficient, the Quotient shall be referred to the same Degree of Affection; that is, the Quotient shall be the Side if the Affection be under the Side, or the Square if under the Square, and so of other Degrees: as in the former *Equation*, if 170304782 be divided by 15, the Quotient shall be squaredly Quadratical; if by 160, the Quotient shall be Cubical; if by 1250, the Quotient shall be Quadratical; and if by 6480, the Quotient shall be Lateral; wherefore not always the Quotient it self, but for the most part the Root thereof, according to the Degree of Affection, shall be the singular side to be extracted. *8. The second Consectary.* Example.

9. In searching out the second Figure of the Root, this ought to be remembred, that according to the Number of Figures in the Quotient, the Degree thereof shall be very nearly reckoned; as if the Quotient consist in one Figure only, it may be a Side; if in 2, a Square; if in 3, a Cube, &c. And if the Quotient exceed 5, or 50, or 500, &c. it may be extended perhaps to the Degree following, especially in the greater Affections: And these are the Laws of *Analytical Division*. *9. What to be noted in finding the second Figure of the Root.*

10. Neither in this sort of *Multiplication* or *Division*, shall there be need to run through the whole Power to be resolved, with the whole Coefficient, but only to the next Point agreeing thereto. *10. Proceed only to the next Point.*

11. For in the Resolution of *Affected Equations*, all the Pointings of the Degrees ought to be made in the Power to be resolved, as in other Figural Numbers; those of the highest Degrees above, and of the Residue beneath. Also the Coefficients, every one according to his own Kind, are to be pointed. The Points of the former Examples shall be thus: *11. How the Numbers are to be pricked.*

1qc — 15qq + 160c — 1250q + 06480l = 170304782 Example.

12. And regularly (especially if the Coefficient be Negative) the Number of Points in all ought to be equal. Wherefore if the Power to be resolved have more or fewer Points upon it self than the Coefficient, so many Ciphers shall be set before that which is deficient, that the Points to both may be equal. And in getting the several Sides, the Point of the Coefficient proper to that Side, is to be accommodated to the like Point above of the Power to be resolved: Which shall be *12. Points to be equal and void, Places supplied with Ciphers.*

be done, if the Unit's Place in the Coefficient be removed in order to the lower Points of the Power agreeable to his Degree.

13. If the Coefficient be a Fraction or Surd. 13. If any Coefficient be a Fraction, or Surd Side, let it be reduced to Integers with Decimal Parts.

14. Root to be extracted with Decimals. 14. And if need be, to pursue the Extraction of the Root in the Decimal Parts, adjoin as many Ciphers as shall be meet after the Separatrix, and mark them above and below with Points in like sort.

15. The Table for the Divisors and Gnomons. 15. The following Table shews as well the Divisors as the Gnomons, for finding the several Sides in *Affected Equations*; collected and continued out of the *Analytical Table*, before mentioned. And it is to be noted, That all the *Species* of *Affirmative and Negative.* every Coefficient are Affirmative, if the same be Affirmative; but Negative, if Negative.

For the first Side.	*For the several following Sides to compleat the Gnomon.*					
Aq BA	2AE . BE .	Eq } = Cq				
Ac BAq CqA	3AqE. B2AE. CqE.	3AEq . BEq .	Ec } = Dc			
Aqq BAc CqAq DcA	4AcE. B3AqE. Cq2AE. DcE.	6AqEq . B3AEq . CqEq .	4AEc . BEc .	Eqq } = Fqq		
Aqc BAqq CqAc DcAq FqqA	5AqqE. B4AcE. Cq3AqE. Dc2AE. FqqE.	10AcEq . B6AqEq . Cq3AEq . DcEq .	10AqEc. B4AEc. CqEc.	5AEqq. BEqq.	Eqc &c. } = Gqc	

16. Divisors to be orderly collected and added. 16. The Divisors every where are taken of those, which are had in the Measure given, disposed and gathered together in due order, according to their Signs.

17. Ambiguous Equations. 17. If the highest Power of any *Equation* be Negative, that *Equation* is ambiguous.

18. The first of the Side whence. 18. The first singular Side is drawn out of these Rules, taken from the 2 Consectaries in *Sect.* 6. and 8.

(1) When the Coefficient may be neglected. (1.) If the Coefficient so far depart to the latter Part, that it scarce reach to the first Point of the Power to be resolved, neither (also Analytically reduced) make any great change in it; it may altogether be neglected, in Extraction of the first singular Side.

(2) When devolved into the Consequent Points. (2.) If the Coefficient break forth in the fore part, and be Affirmative, it is to be devolved into the Consequent Points, until a Place be made for Division: By which Division the Quotient found shall be referred to the Degree of Affection; which also in extracting the lesser Root of an Ambiguous *Equation* ought to be understood.

(3) When Negative and of many Points. (3.) But if it be Negative, and consist of more Points than the Power to be resolved, the deficient Places may be supplied with Cyphers prefixed; and for the first singular Side, the Root of the Coefficient it self may be taken according to this kind.

(4) When the Points to both equal. (4.) If the Points to both are equal, and the Numbers differ not much in the first Point, both of the Coefficient and of the Power to be resolved; the Coefficient by his Root extracted according to the Species with which he is pointed, under the Point agreeable thereto, reduced to the Species of the Power (by Analytical Multiplication) may be added to the Power to be resolved, if it be Negative, or taken away if it be Affirmative. For if Ac + CqA = Dc, then shall Ac = Dc — CqA; but if the greater Side of an Ambiguous *Equation* be sought, the Power to be resolved may be taken away from the Coefficient reduced: for if CqA — Ac = Dc, then shall Ac = CqA — Dc; then the Root of the Sum or Difference shall be the first Side to be extracted. And note that the greater Side of

of an Ambiguous *Equation* may be found sometime by Division, sometime by Extraction of the Root of the Coefficient; but for the most part by Reduction of the Coefficient.

19. And by these Precepts diligently weighed at last, the first true singular Side shall be that, which first of all sheweth such a Diagonal; which together with the Coefficients (as the Condition of the *Equation* requireth) multiplied according to the precedent Table, and all gathered together into one Sum, (diligent respect every where had as well to the Signs as Places) bringeth forth a Number not greater than the Power to be resolved, from whence it is to be substracted. And it is to be noted, that every Negative Number is less than any Affirmative, and than any lesser Negative, as — 4 is less than 1, and than — 1; also that Substraction changeth the Sign of the Subtrahend; as from 4 take away 6, there resteth 4 — 6, that is — 2; and from — 4 take away — 6, there resteth — 4 + 6, that is 2; again from 4 take away — 6, there resteth 4 + 6, that is 10; wherefore in Extraction of the first singular Side, it is so often to be tried until the true Side be found, which by the next greater shall certainly be known. *19. To find the first of the Side.*

20. In constituting the Divisor for finding the second Side, the Place of the Coefficient multiplied into every Degree, ought to be ordered according to the pointing of his own Degree, that is, the Place of the Coefficient under the Side shall be distant towards the left Hand, one Place from the Point or Place of the same Coefficient; the Place of the Coefficient under the Square, two Places; under the Cube, three, &c. And to avoid Confusion, it will be profitable, in the Residue of the Power to be resolved, to distinguish those Points alone which serve to the present Root to be extracted. *20. To get the Divisor for the Second of the Side.*

21. Then the second singular Side shall be thus gotten; Let the Divisors of every Kind, out of the precedent Table, be gathered together into one Sum, and disposed in due order, and the Residue of the Power to be resolved divided by all that Divisor. For the Quotient, according to the Laws of Analytical Division (if need require it) weighed, shall give the second singular Side to be gotten. But in this search oftentimes great Difficulty happeneth, especially if the Aggregate of the Negative Magnitudes dividing, be almost equal to the Aggregate of the Affirmatives, (so that the Divisor may be less than the Residue of the Power to be resolved); which difficulty notwithstanding the sagacious Analyst will easily avoid. *21. To find the Second of the Side.*

22. Let this Rule therefore be perpetual; That the true singular second Side is that which first of all sheweth such a Gnomon, consisting of the Complements of every Kind, and multiplied Coefficients, as the Condition of the *Equation* requireth, according to the precedent Table, and all gathered together into one Sum, diligent respect had every where, as well to the Signs as Places; which Gnomon may not be greater than the Power to be resolved, from whence it is to be substracted: Wherefore it is often to be tried, until the true Side be found; which also by the next greater will most certainly be known. *22. To know the Second of the Side.*

23. All the singular Sides after the Second, by Simple Division, are most easily obtained. *23. To get the other Figures after the Second.*

24. If the Affections are compounded of Affirmatives and Negatives, the Antecedent Precepts are to be mixt with Discretion and Judgment: And in the Sides to be valued, always the greater Affection shall be considered before the Lesser. *24. If the Affections are mixt.*

25. But because oftentimes above it is said, it will be needful to try, which in many-fold Affections, and where the Degrees are lofty, will be exceeding laborious, I will add here for a Close, two Manners of easing these Trials: One by Depression, another by the Canon of Logarithms. But in both, if the *Equation* shall be ambiguous, all the Signs thereof shall be changed. Here also is to be noted, that every Negative Number is less than any Affirmative, and than any Lesser Negative. *25. Two ways. By Depression. By Logarithms.*

26. The Invention of the Singular Sides by Depression: If the first Side be sought, all the Points after the First, in the several *Species* of the given *Equation*, may be cut off by the Separatrix: Afterwards all the *Species* may be applied to the Side, that is, depressed by one Degree. *26. Invention by Depression.*

Example

Example 1. *Example* 1. 1qq — 72c + 2386001 = 8725815. This by Depression shall be made 1c + 238,6 — 7,2q = L) 872,5.

Let A be 4, then shall 4) 872,5 (218,1 the just Number.

And + 64 + 238,6 — 115,2 = 187,4 less than the Just.

Let A be 5, then shall 5) 872,5 (174,5 the just Number.

And + 125 + 238,6 — 180,0 = 183,6 greater than the Just.

The true Side therefore A = 5 — 1, that is 4.

Example 2. *Example* 2. Of the *Ambiguous Equation*, 1c — 3257l = — 45744

This by Depression shall be made 1q — 32,5 = L) — 45,7.

Let A be 4, then shall 4) — 45,7 (— 11,4 the just Number.

And + 16 — 32,5 = — 16,5 less than the Just.

Let A be 5, then shall 5) — 45,7 (— 9,1 the just Number.

And + 25 — 32,5 = — 7,5 greater than the Just.

The true Side therefore A = 5 — 1, that is 4.

For the second of the Side. If the second Side be sought, all the Points after the second may be cut off in the several *Species*; Afterwards all the *Species* may be applied to the Square, that is depressed by two Degrees. As in the first Example.

1qq — 72c + 2386001 = 8725815. This by Depression shall be made

1q + L) 238600 — 72l = Q) 8725815.

Let A be 47, then shall 2209) 8725815 (3949 the just Number.

And 2209 + 5077 — 3384 = 3896 less than the Just.

Let A be 48, then shall 2304) 8725815 (3787 the just Number.

And 2304 + 4971 — 3456 = 3819 greater than the Just.

The true Side therefore is 48 — 1, that is 47.

27. Omitted. 27. This being wholly about the Use of Logarithms, is omitted here.

28. Examples of Logarithms. 28. This contains nothing but Examples, wherein trial is made by Logarithms. In which Examples all the Points after the two first are cut off by the Seperatrix.

Example 1. 1qq — 72c + 2386001 = 8725815 the just Number.

Let the two first singular Sides be sought.

47 · 1,67209,78579	|—72|	|+238600|
C · 5,01629,35737	1,85733,24964	5,37767,04393
QQ · 6,68839,14316	5,01629,35737	1,67209,78579
	6,87362,60701	7,04976,82972
+ 4879 ...	— 7475 ...	+ 11214 ...

And + 4870 ... + 11214 ... — 7475 ... = + 8618 ... less than the Just.

48 · 1,68124,12374	1,85733,24964	5,37767,04393
C · 5,04372,37122	5,04372,37122	1,68124,12374
QQ · 6,72496,49496	6,90105,62086	7,05891,16767
+ 5308 ...	— 7962 ...	+ 11453 ...

And + 5308 ... + 11453 ... — 7962 ... = + 8799 ... greater than the Just.

The true Root therefore shall be 48 — 1, that is 47.

Example 2. $1c - 3257l = -45744$ the just Number.

Let the two first Singular Sides be sought.

48 .	1,68124,12374	-3257 .	3,51281,77586
C .	5,04372,37122		1,68124,12374
	$+1106$	-1563 .	5,19405,89960

And $+1106 - 1563 = -457$. Less than the Just (at least not Greater).

49 .	1,69019,60800	-3257 .	3,51281,77586
C .	5,07058,82400		1,69019,60800
	$+1176$	-1596 .	5,20301,38386

And $+1176 - 1596 = -420$. Greater than the Just.

The true Root therefore shall be $49 - 1$, that is 48.

The second Side may also be found by Logarithms, Depression preceding.

As in Example, $1qq - 1246,00q = 08972,6256$.

This depressed squaredly, shall be made $1q - 1246 = Q)$ 8972,6.

The two first Singular Sides may be supposed.

34 .	1,53147,89170	8972,6	3,95291,83073	
Q .	3,06295,78340		3,06295,78340	
	$+1156$	Value 7,76	0,88996,04733	The Just.

And $+1156 - 1246 = -90$. Less than the Just.

36 .	1,55630,25008	8972,6	3,95291,83073	
Q .	3,11260,50016		3,11260,50016	
	$+1296$	Value 6,92	0,84031,33057	The Just.

And $+1296 - 1246 = +50$. Greater than the Just.

Upon these Examples, in the *26th* and *28th* Sections, the same Author afterwards hath added some Notes of Explanation; the Substance whereof is thus: *Notes of the Author.*

That is called *The just Number*, which ariseth of the Application of the Power to be resolved to the Degree of the supposed Side, by which Depression is made: For this is the Measure to which all the other *Species* duly gathered together, ought to be equal. As in the first Example of the *26th* Section, $1c + 238,6 - 7,2q = L)$ 872,5. If for the first Side be supposed 5, it must be that $C:5: + 238,6 - 7,2Q:5: = 872,5$ divided by 5; that is $125 + 238,6 - (7,2 \times 25)$ 180, to wit 183,6 to be equal to 174,5 the Just. But it is greater, and therefore the true Side is less than 5; therefore let 4 be again supposed, and make trial whether $C:4: + 238,6 - 7,2$ $Q:4:$ be equal to 872,5 divided by 4. *What called the just Number.*

But lest in these Examples, as also in the following, these Trials be taken up by chance, it must be admonished, Monitions.

1. If the Homogeneal Power of the Extracted Root, exceed the Power to be resolved; or if the Magnitudes increasing the Power to be resolved, exceed them which they lessen; The true Side A (for the most part) shall be less than the Side extracted, but otherwise greater: As in this *Equation*. 1. Monition.

$$1c + 26,00col = 180931713$$

180,9 (4 The Side A.

26,0 Cq.

$\sqrt{q}26$ is 5, in 26 is made 130, taken from 180, there resteth $50, C:3+:$ which is less than 180; wherefore the true Side A is greater than 3.

2. Monition. 2*ly*. If the Divisors under the same Sign with the Residue of the Power to be resolved, exceed them which are under a diverse Sign, the true Side E (for the most part) shall be less than the Quotient; but otherwise greater: As in this *Equation*.

$$15681 - 1c = 21952.$$

The same also happeneth in *Ambiguous Equations*, when the Residue of the Power to be resolved is Affirmative: As in this *Equation*.

$$67681 - 1c = 214273.$$

The Work of both.

21	952	(28 The two first Sides.
15	68	Cq
—8		Ac
+31	36	CqA
+23	36	*Subtrahend.*
R—1	408	
1	2	—3Aq
	6	—3A
—1	26	
+1	568	Cq
+	308	*Divisor.*

The Sign R is —; But — 1,26 is less than + 1,568. Wherefore the true Side E is greater than the Quotient 4.

214	273	(47 The 2 first Sides.
67	68	Cq
—64		—Ac
+270	72	CqA
+206	72	*Subtrahend.*
R+ 7	553	
4	8	—3Aq
	12	—3A
— 4	92	
+ 6	768	Cq
+ 1	848	*Divisor.*

The Sign is +; But the Divisor out of the Degrees of the Side A Negative, is less than the Coefficient Affirmative Divisor; that is — 4,92, is less than + 6,768. Wherefore the true Side E shall be greater than the Quotient 4.

3. Monition. 3*ly*. If after these Monitions some Doubt remain, trial by 5 shall be most fit to be begun; and from thence Inquiry to be continued by odd Numbers: Or the same may be done by Depression, or by Logarithms.

16 Examples of the same Author, and his Notes thereupon.

The other Examples of the same Author, with his Notes thereupon.

Ex. 1. $1qc - 15qq + 160c - 1250q + 06480l = 170304782$

That is $Lqc - BLqq + CqLc - DcLq + FqqL = Gqc$.

1703	04782	(47
1	5	—B
1	250	—Dc
1	60	Cq
	6480	Fqq
1024		Aqc
102	40	CqAc
2	5920	FqqA
+1128	9920	
384	0	—BAqq
20	000	—DcAq
—404	000	
724	9920	*Subtrahend.*
R 978	05582	
128	0	5Aqq
6	40	10Ac
	160	10Aq
	20	5A
7	680	Cq3Aq
	1920	Cq3A
	160	Cq
	6480	Fqq
+ 142	50040	
38	40	—B4Ac
1	440	—B6Aq
	240	—B4A
	15	—B
1	0000	—Dc2A
	1250	—Dc
—40	87665	
+101	62375	*Divisor.*
896	0	5AqqE
313	60	10AcEq
54	880	10AqEc
4	8020	5AEqq
	16807	Eqc
53	760	Cq3AqE
9	4080	Cq3AEq
	54880	CqEc
	45360	FqqE
+1333	62047	
268	80	—B4AcE
70	560	—B6AqEq
8	2320	—B4AEc
	36015	—BEqq
7	0000	—DC2AE
	61250	—DcEq
—355	56465	
978	05582	*Subtrahend.*

In this Example √qc1703 is 4+, by *Sect.* 18. *Rule* 1. For as it appeareth out of *Sect* 7. there is not made any notable Change in the first Point, by the Coefficients analytically reduced; wherefore the true Side A shall be 4.

The true Side E is less than the Quotient 9; because the Divisors under the Sign +, (which is the Sign of the Residue) exceed them which are under the Sign —. *Exam-*

Example 2.

1c + 420000l = 247651713

That is Lc + CqL = Dc.

247	651	713	(417
42	000	0	Cq
64			Ac
168	000	0	CqA
232	000	0	*Subtrahend.*
R 15	651	713	
4	8		3Aq
	12		3A
4	200	00	Cq
9	120	00	*Divisor.*
4	8		3AqE
	12		3AEq
	1		Ec
4	200	00	CqE
9	121	00	*Subtrahend.*
R 6	530	713	
	504	3	3Aq
	1	23	3A
	420	000	Cq
	925	530	*Divisor.*
3	530	1	3AqE
	60	27	3AEq
		343	Ec
2	940	000	CqE
6	530	713	*Subtrahend.*

In this Example

42) 247 (6—, by *Sect.* 18. *Rule* 2. For 42 reduced Analytically by *Sect.* 6 and 8, it is made 252, greater than 247: And the true Side A is less than 6, because C:6—: exceeds 247,6.

Example 3.

1c + 1007q = 247617936

That is Lc + BLq = Dc.

247	617	936	(417
10	07		B
64			Ac
161	12		BAq
225	12		*Subtrahend.*
R 22	497	936	
4	8		3Aq
	12		3A
8	056		B2A
	100	7	B
13	076	7	*Divisor.*
4	8		3AqE
	12		3AEq
	1		Ec
8	056		B2AE
	100	7	BEq
13	077	7	*Subtrahend.*
R 9	420	236	
	504	3	3Aq
	1	23	3A
	825	74	B2A
	1	007	B
1	332	277	*Divisor.*
3	530	1	3AqE
	60	27	3AEq
		343	Ec
5	780	18	B2AE
	49	343	BEq
9	420	236	*Subtrahend.*

In this Example

10) 247 (24 + = Q:5—: by *Sect.* 18. *Rule* 2. But 10 Q:5:=250 ⊏ 247,6 by *Monition* 1.

Example

Example 4.

1qq — 442990051 = 022252086

That is Lqq — DcL = Fqq.

0	2225	2086	(354
—44	2990	05	—Dc
+81			Aqq
—132	8970	15	—DcA
—51	8970	15	*Subtrahend.*
R 52	1195	3586	
10	8		4Ac
	54		6Aq
	12		4A
+11	352		
— 4	4299	005	—Dc
+ 6	9220	995	*Divisor.*
54	0		4AcE
13	50		6AqEq
1	500		4AEc
	625		Eqq
+69	0625		
—22	1495	025	—DcE
+46	9129	975	*Subtrahend.*
R 5	2065	3836	
1	7150	0	4Ac
	73	50	6Aq
		140	4A
+ 1	7223	640	
—	4429	9005	—Dc
1	2793	7395	*Divisor.*
6	8600	0	4AcE
	1176	00	6AqEq
	8	960	4AEc
		256	Eqq
+ 6	9784	9856	
— 1	7719	6020	—DcE
+ 5	2065	3836	*Subtrahend.*

In this Example, √c44,3 is 3+, by *Sect.* 18. *Rule* 3. wherefore the true Side A is 3.

The true Side E is less than the Quotient 8— by *Monition* 2.

Example 5.

1qq — 124600q = 089726256

That is Lqq — CqLq = Fqq.

0	8972	6256	(354
— 12	4600		—Cq
+ 81			Aqq
— 112	1400		—CqAq
— 31	1400		*Subtrahend.*
R 32	0372	6256	
10	8		4Ac
	54		6Aq
	12		4A
+ 11	352		
7	4760	0	—Cq2A
	1246	00	—Cq
— 7	6006	00	
+ 3	7514	00	*Divisor.*
54	0		4AcE
13	50		6AqEq
1	500		4AEc
	625		Eqq
+ 69	0625		
37	3800	0	—Cq2AE
3	1150	00	—CqEq
— 40	4950	00	
+ 28	5675	00	*Subtrahend.*
R 3	4697	6256	
1	7150	0	4Ac
	73	50	6Aq
		140	4A
+ 1	7223	640	
	8722	000	—Cq2A
	12	4600	—Cq
—	8734	4600	
+	8489	1800	*Divisor.*
6	8600	0	4AcE
	1176	00	6AqEq
	8	960	4AEc
		256	Eqq
+ 6	9784	9856	
3	4888	000	—Cq2AE
	199	3600	—CqEq
— 3	5087	3600	
+ 3	4697	6256	*Subtrahend.*

In this Example, √q12,4 is 3+, by *Sect.* 18. *Rule* 3. Wherefore the true Side A is 3.

The true Side E, is less than the Quotient 9— by *Monition* 2.

Example 6.

1qq — 340c = 621066096

That is Lqq — BLc = Fqq.

	6	2106	6096	(354
—	3	40		—B
+	81			Aqq
—	91	80		BAc
—	10	80		*Subtrahend.*
R	17	0106	6096	
	10	8		4Ac
		54		6Aq
		12		4A
+	11	352		
	9	180		—B3Aq
		3060		—B3A
		34	0	—B
—	9	4894	0	
+	1	8626	0	*Divisor.*
	54	0		4AcE
	13	50		6AqEq
	1	500		4AEc
		625		Eqq
+	69	0625		
	45	900		—B3AqE
	7	6500		—B3AEq
		4250	0	—BEc
—	53	9750	0	
+	15	0875	0	*Subtrahend.*
R	1	9231	6096	
	1	7150	0	4Ac
		73	50	6Aq
			140	4A
+	1	7223	640	
	1	2495	00	—B3Aq
		35	700	—B3A
			340	—B
—	1	2530	7340	
+		4692	9060	*Divisor.*
	6	8600	0	4AcE
		1176	00	6AqEq
		8	960	4AEc
			256	Eqq
+	6	9784	9856	
	4	9980	00	—B3AqE
		571	200	—B3AEq
		2	1760	—BEc
—	5	0553	3760	
+	1	9231	6096	*Subtrahend.*

In this Example, The Lateral Coefficient 3, 4, squared-squaredly, multiplied and encreased 6, 2, is made 140, QQ: 3 +: by *Sect.* 18. *Rule* 4. Wherefore the true Side A is 3. The true Side E is less than the Quotient 9—, by *Monition* 2.

Example 7.

1qq — 77108000l = 085530576

That is Lqq — DcL = Fqq

	0	8553	0576	(426
—	77	1080	00	—Dc
+	256			Aqq
—	308	4320	00	—DcA
—	52	4320	00	*Subtrahend.*
R	53	2873	0576	
	25	6		4Ac
		96		6Aq
		16		4A
+	26	576		
—	7	7108	000	—Dc
+	18	8652	000	*Divisor.*
	51	2		4AcE
	3	84		6AqEq
		128		4AEc
		16		Eqq
+	55	1696		
—	15	4216	000	—DcE
+	39	7480	000	*Subtrahend.*
R	13	5393	0576	
	2	9635	2	4Ac
		105	84	6Aq
			168	4A
+	2	9741	208	
—		7710	8000	—Dc
+	2	2030	4080	*Divisor.*
	17	7811	2	4AcE
		3810	24	6AqEq
		36	288	4AEc
			1296	Eqq
+	18	1657	8576	
—	4	6264	8000	—DcE
+	13	5393	0576	*Subtrahend.*

In this Example.
√c77 is 4, by *Sect.* 18. *Rule* 3. Wherefore the true Side A is 4.

Example

Example 8.

32001 — 1c = 46577

That is CqL — Lc = Dc

An Ambiguous *Equation.*

46	577	(47 The greater Root.
32	00	Cq
— 64		—Ac
+128	00	CqA
+ 64	00	*Subtrahend.*
R— 17	423	
4	8	—3Aq
	12	—3A
— 4	92	
+ 3	200	Cq
— 1	720	*Divisor.*
33	6	—3AqE
5	88	—3AEq
	343	— Ec
— 39	823	
+ 22	400	CqE
— 17	423	*Subtrahend.*

In this Example,

√q32, is 5,65 in 32 is made 180,8. lacking 46,5. there remaineth 144 . C: 5 : by *Sect.* 18. *Rule* 4. But 144 exceedeth 46,5. Wherefore the true Side A is less than 5 by *Monition* 1.

The true Side E is less than the Quotient 10, by *Monition* 2.

Example 9.

32001 — 1c = 46577

That is CqL — Lc = Dc

The same Ambiguous *Equation.*

46	577	(15,7 The lesser Root.	
32	00	Cq	
— 1		—Ac	
+ 32	00	CqA	
+ 31	00	*Subtrahend.*	
R 15	577		
	3	—3Aq	
	3	—3A	
—	33		
+ 3	200	Cq	
+ 2	870	*Divisor.*	
1	5	—3AqE	
	75	—3AEq	
	125	— Ec	
— 2	375		
+ 16	000	CqE	
+ 13	625	*Subtrahend.*	
R 1	952	000	
	67	5	—3Aq
		45	—3A
—	67	95	
+	320	0	Cq
+	252	05	*Divisor.*
	472	5	—3AqE
	22	05	—3AEq
		343	— Ec
—	494	893	
+ 2	240	0	CqE
+ 1	745	107	*Subtrahend.*
R	206	893	000, &c.

In this Example,

The Solution is most easy by *Division*, according to *Sect.* 18. *Rule* 3.

Example

Example 10.

$53q - 1c = 13254$

That is BLq — Lc = Dc

An Ambiguous *Equation.*

13	254	(47 The greater Root.
5	3	B
—64		—Ac
+84	8	BAq
+20	8	*Subtrahend.*
R— 7	546	
4	8	—3Aq
	12	—3A
— 4	92	
4	24	B2A
	53	B
+ 4	293	
—	627	*Divisor.*
33	6	—3AqE
5	88	—3AEq
	343	— Ec
—39	823	
29	68	B2AE
2	597	BEq
+32	277	
— 7	546	*Subtrahend.*

In this Example,

C: 5 : is 125, lacking 13, there remaineth 112, C : 5 —: by *Section.* 18. *Rule* 4. But 112 exceedeth 13. Wherefore the true Side A is less than 5 by *Monition* 1.

The true Side E is less than the Quotient 12, by *Monition* 2.

Example 11.

$53q - 1c = 13254$

That is BLq — Lc = Dc

The same Ambiguous *Equation.*

13	254	(20,05, *&c.* The lesser Root.		
5	3	B		
— 8		—Ac		
+21	2	BAq		
+13	2	*Subtrahend.*		
R	54	000	000	
	12	000	0	—3Aq
		6	00	—3A
—	12	006	00	
	21	200		B2A
		5	3	B
+	21	205	3	
	9	199	30	*Divisor.*
	60	000	0	—3AqE
		150	00	—3AEq
			125	— Ec
—	60	150	125	
	106	000		B2AE
		132	5	BEq
+	106	132	5	
	45	982	375	*Subtrahend.*
R	8	017	625	000, &c.

In this Example,

The Solution is most easy by *Division*, according to *Section* 18. *Rule* 3.

Example 12.

600341 — 1c = 1023768

That is CqL — Lc = Dc.

An Ambiguous *Equation.*

1	023	768	(236. The greater Root.
6	003	4	Cq
— 8			—Ac
+12	006	8	CqA
+ 4	006	8	*Subtrahend.*
R— 2	983	032	
1	2		—3Aq
	6		—3A
— 1	26		
+	600	34	Cq
—	659	66	*Divisor.*
3	6		—3AqE
	54		—3AEq
	27		— Ec
— 4	167		
+ 1	801	02	CqE
— 2	365	98	*Subtrahend.*
R—	617	052	
	158	7	—3Aq
		69	—3A
—	159	39	
+	60	034	Cq
—	99	356	*Divisor.*
	952	2	—3AqE
	24	84	—3AEq
		216	— Ec
—	977	256	
+	360	204	CqE
—	617	052	*Subtrahend.*

In this Example.

√q 6 is 2+, in 6 is made 12 lacking 1, there remaineth 11, C : 2,5 by *Sect.* 18. *Rule* 4. But 11 exceedeth 1; wherefore the true Side A a little less than 2+ by *Monition* 1.

The true Side E is less than the Quotient 5— by *Monition* 2.

Example 13.

600341 — 1c = 1023768

That is CqL — Lc = Dc.

The same Ambiguous *Equation.*

1	023	768			(17,13, &c. The lesser Root.
	600	34			Cq
	—1				—Ac
+	600	34			CqA
+	599	34			*Subtrahend.*
R	424	428			
		3			—3Aq
		3			—3A
—		33			
+	60	034			Cq
—	59	704			*Divisor.*
	2	1			—3AqE
	1	47			—3AEq
		343			— Ec
—	3	913			
+	420	238			CqE
+	416	325			*Subtrahend.*
R	8	103	000		
		86	7		—3Aq
			51		—3A
—		87	21		
+	6	003	4		Cq
+	5	916	19		*Divisor.*
		86	7		—3AqE
			51		—3AEq
			1		— Ec
—		87	211		
+	6	003	4		CqE
+	5	916	189		*Subtrahend.*
R	2	186	811	000	
		591	562	57	*Divisor.*
	1	774	656	903	*Subtrahend.*
R		412	154	097	000, &c.

In this Example.

The Solution is most easy by *Division*, according to *Section* 18. *Rule* 3.

Example 14.

1qq—72c+2386001=8725815,7056.

That is Lqq—BLc+DcL=Fqq.

872	5815,	7056	(47,6
— 7	2		—B
+ 238	600		Dc
256			Aqq
954	400		DcA
+1210	400		
— 460	8		—BAc
+ 749	600		*Subtrahend.*
R 122	9815,	7056	
25	6		4Ac
	96		6Aq
	16		4A
23	8600		Dc
+ 50	4360		
34	56		—B3Aq
	864		—B3A
	72		—B
— 35	4312		
+ 15	0048		*Divisor.*
179	2		4AcE
47	04		6AqEq
5	488		4AEc
	2401		Eqq
167	0200		DcE
+ 398	9881		
241	92		—B3AqE
42	336		—B3AEq
2	4696		—BEc
— 286	7256		
+ 112	2625		*Subtrahend.*
R 10	7190,	7056	
1	7698	808	*Divisor.*
10	7190,	7056	*Subtrahend.*

In this Example,
QQ:7,2: is —2687; and √c238,6 is 6,2, whose QQ is +1480. Then —2687 +1480 =—1207. This added to 872, giveth 2079, QQ:6+: by *Sect.*18.*Rule* 4. And because —2687 to be added, is greater than +1480 to be substracted, the true Side A shall be less than 6, by *Monition* 1. The true Side E is less than the Quotient 9, by *Monition* 2.

Example 15.

3l. — 1c = 1,258640782100.

That is CqL—Lc=Dc.

1,	258	640	782	100	(0,4499,&c.
3	3				Cq
—	64				—Ac
+1	2				CqA
+1	136				*Subtrahend.*
R	122	640	782	100	
	4	8			—3Aq
		12			—3A
—	4	92			
+	3				Cq
	25	08			*Divisor.*
	19	2			—3AqE
	1	92			—3AEq
		64			— Ec
—	21	184			
+	12				CqE
+	98	816			*Subtrahend.*
R	23	824	782	100	
		580	8		—3Aq
		1	32		—3A
—		582	12		
+	3				Cq
	2	417	88		*Divisor.*
	5	227	2		—3AqE
		106	92		—3AEq
			729		—Ec
—	5	334	849		
+	27				CqE
+	21	665	151		*Subtrahend.*
R	2	159	631	100	
		60	480	3	—3Aq
			13	47	—3A
—		60	493	77	
+		3			Cq
		239	506	23	*Divisor.*
		544	322	7	—3AqE
		1	091	07	—3AEq
				729	—Ec
—		545	414	499	
+	2	7			CqE
+	2	154	585	501	*Subtrahend.*
R		5	045	599	00, &c.

In this Example,
Because the lesser Root of the Ambiguous *Equation* is sought, the Coefficients, altho reduced, hinder not. The *Analysis* shall be made by *Division*, according to *Sect.*18.*Rule* 1.

Ex-

Example 16.

$$1qc - 5c + 5l = 1,147152872702092.$$

That is $Lqc - CqLc + FqqL = Gqc$.

1	14715	28727	02092	(0,2437 The Subtense of 14 Degrees.
+	5			Fqq
—	5			—Cq
	32			Aqc
1	0			FqqA
+1	00032			
—	40			—CqAc
	96032			*Subtrahend.*
R	18683	28727	02092	
	8	0		5Aqq
		80		10Ac
		40		10Aq
		10		5A
	5			Fqq
+	5008	8410		
	60			—Cq3Aq
	30			—Cq3A
		5		—Cq
—	630	5		
+	4378	3410		*Divisor.*
	32	0		5AqqE
	12	80		10AcEq
	2	560		10AqEc
		2560		5AEqq
		1024		Eqc
	20			FqqE
+	20047	62624		
	240			—Cq3AqE
	480			—Cq3AEq
	32	0		—CqEc
—	2912	0		
+	17135	62624		*Subtrahend.*
R	1547	66103	02092	&c.

The farther *Divisors* and Sums to be substracted, may in like sort be gotten for the other Figures of the Root after 24.

In this Example,

The Author's Note being the same with that on the last, needs not be repeated here; nor yet his Rules for the *Genesis* and *Analysis* of the *Six Binomials*, Chap. 10. of his *Clavis*, as being more proper for *Trigonometry* than *Arithmetick*. Wherefore having now waded thus far into the Deeps of that Curious, but Mysterious Mathematician, as to untie the Knots of *Affected Equations*, it is high time to desist: for whatsoever may seem to be omitted, the Ingenious may supply, by diligent Observation, and often Practice.

The mention of Side for Root in the *Sections*, and consequently L or l, for *Latus* the Side or Root, instead of A the Suppostitious Root in the *Examples*, being frequent in *Species*, needs no Memorandum; So as a Question or two being added, wherein *Side used for Root.*

wherein *Affected Equations* will arise; as well all this general Survey of *Equations*, as the whole Review of *Arithmetick*, may be shut up together.

Q. Of a Number, what it is. 1. There is a Number, whose Square abated by 16, and the first Number augmented by 8; and the Total of one multiplied by the Remainder of the other, will produce 2560: what is that Number?

Resolution. *Answ.* Twelve; For the Square of 12 being 144, lessened by 16, leaves 128; this multiplied by 12 and 8, that is 20, produceth 2560.

The Work by Cossicks.

Suppose 1 ℨ: Then the Square is 1 ʒ, abating 16, the Remain is 1ʒ—16. And the Number increased by 8, is 1ℨ + 8. And 1ʒ — 16 × 1ℨ + 8, makes 1φ + 8ʒ — 16ℨ — 128 = 2560. And by Reduction, 1φ + 8ʒ — 16ℨ = 2688.

The Work by Species.

— B 16. C 8. D 2560. Suppose the Number A, then the Square is Aq: And from thence abating 16, leaves Aq—B; and to the Number adding 8, makes the Total A + C. Then multiplying Aq — B into A + C, there is produced Ac + AqC — BA — BC, which are equal to D.

And by Reduction, Ac + AqC — BA = D + BC.

Or set after the other Mode, Lc + BLq — CqL = Dc.

The Resolution.

1c + 08q — 0161 ==	2	688	(12 Root.
	—	8	B
	—	16	—Cq
	1		Ac
		8	BAq
	1	8	
	—	16	—CqA
	+1	64	*Subtrahend.*
	R 1	048	
		3	3Aq
		3	3A
		16	B2A
		8	B
	+	498	
	—	16	—Cq
	+	482	*Divisor.*
		6	3AqE
		12	3AEq
		8	Ec
		32	B2AE
		32	BEq
	+1	080	
	—	32	—CqE
	1	048	*Subtrahend.*

Q. Of a Fat of Wine mixed with Water, what drawn out at a time. 2. Suppose out of a Fat of Wine of 360 Gallons, be drawn out a certain Number of Gallons; and as many of Water as were drawn out be put into the Fat; and the like be done the second and third Times, and at last there be found to remain in the Fat of Wine (besides the Water mixed therewith) 208⅓ Gallons: how much Wine was drawn out at each time?

Answ.

Answ. Sixty Gallons: As may be tried by *Alligation*, and *Tripled Proportions*, in the Second Part of this 4*th* Book; and by the *Mean Proportionals* in the third Part, where a like Question to this is resolved. Resolution.

To the Resolution, 2 Proportionals gotten (that is 1 less than the Draughts) between the whole Quantity 360, and the Remainder given $208\frac{1}{3}$, and let B be 360 and D $208\frac{1}{3}$; the Proportionals in *Species* stand thus:

D. √cBDq. √cBqD. B.

Then supposing the Draught sought to be A, it shall be that

A = B — √cBqD.

And by exalting (as was shewed in *Reduction*) the plain *Species* to an equal Power with the other in this *Equation*, B—A shall be cubed.

And so Bc — 3BqA + 3BAq — Ac = BqD.

Translated Ac — 3BAq + 3BqA = Bc — BqD.

Reduced into Numbers 1c—1080q+388800l=19656000, and resolved.

19	656	000	(60
—	108	0	—3B
+ 3	888	00	3Bq
	216		AC
23	328	00	3BqA
+23	544	00	
— 3	888	0	—3BAq
19	656	00	*Subtrahend.*
R		000	

Several of the Questions in this Chapter resolved by *Equations*, fall under some or other of the *Rules of Proportions disjunct* or *continued*, before handled, whereby the Truth of the Operations here may be tried. But if not, the *Resolution of the Equations*, both by *Cossicks* and *Species*, where both are used, evidence the Truth of both Conclusions by their Agreement. And where *Affected Equations* are resolved by this latter Way of Mr. *Oughtred* only, the Resolution agreeing in all things with the Tenor of the Question, is as in all other Works, a Proof sufficient of the Truth thereof. *Proof of the Resolution of Equations.*

Partis quartæ & Libri quarti

FINIS.

AN APPENDIX
OF THE
Properties of ſome Numbers.

Abſtract Numbers of 3 ſorts. ALL Abſtract whole Numbers being expreſſed by 9, ſignifying Figures and the Cipher, (as in the beginning of this Treatiſe was noted) the Numbers made up thereby were divided into three Sorts, *Digits*, *Articles*, and *Mixt Numbers*, of all which ſome peculiar Properties may be obſerved; but leſt it prove tedious, Content ſhall be taken with the firſt 12.

Digits. *Whence ſo called.* A *Digit* is always wrote with one Figure; of which there being 9, with the Cipher 0, makes up the firſt Article 10, the Number of the Fingers in Latin known by the Name of *Digiti*, from whence the 9 Figures came to be called *Digits*.

Articles. An *Article* hath always a Cipher in the firſt Place.

Mixt. And a *Mixt Number* hath a Digit there.

Properties of the Unit. *One*, Though ſome differ about its being a Number, yet all agree it is the entire Foundation, and the Root and Meaſure of every Number, every Number meaſuring another ſo many times as there are Units therein: For every Number, whom beſides the Unit, no other Number meaſureth, is meaſurable by no other than the Unit. And as 1 is the Foundation of Number, ſo abides he firm and unalterable, he cannot break, nor will be broken by others, but remains whole. He neither multiplieth nor divideth, nor will be multiplied nor divided by himſelf; if he be ſquared, cubed, &c. he is ſtill no more than 1.

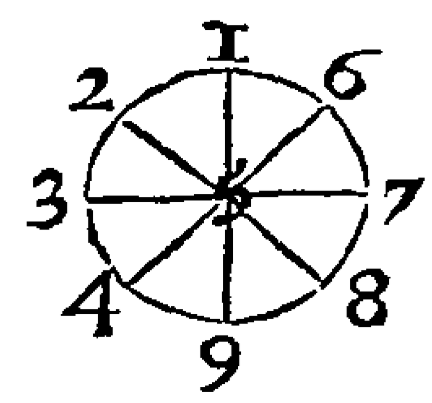
Of 2. *Two*, The only even prime Number (all other even Numbers being compound) meaſureth every even Number; and is the only Integer that multiplied by or added to himſelf, makes the Total equal to the Product; for 2 and 2 is 4, and no more is twice 2. He is ſometime called the *firſt Lineary Number*, becauſe a Line is bounded with two Points thus •——•

Of 3. *Three*, Is the firſt Simple odd Number made by *Addition* of Units, and not by *Multiplication*; and the firſt that multiplies an even Number to make the Product even, and an odd Number odd: As twice 3 is 6, and 3 times 5 is 15. It compoundeth and meaſureth every Number, whoſe ſeveral Notes taken by themſelves, and added to themſelves, are numbred from the ſame Ternary; as 39 . 54. &c. It is ſometime called the *Muſical Number*, becauſe the third Concord is the Chief in Muſick: Sometimes it is called a *Syſtatical*, or *Subſtantial Number*, becauſe all Sublunary Bodies conſiſt of the three principal Subſtances, *Sal*, *Sulphur*, and *Mercury*. Alſo it is the firſt that diſpoſed in Units, hath Beginning, Middle, and End; and in *Geometry* may repreſent the three Angles of a Triangle, thus; ▽ and ſo came to be called the *firſt Figural Number*, becauſe a Triangle is the firſt Figure in *Geometry*.

Of 4. *Four*, Is the firſt even Compound Number, begotten by the Multiplication of 2 by 2, and the firſt proper Square Number, and repreſenteth the ſame, if the Units thereof be placed in oppoſition one to the other, as . It meaſureth compounded, and numbereth every Number, whoſe Figure comprehended under the two firſt Sides, it can number: As becauſe it can meaſure 16, it ſhall meaſure 69816. It is called ſometime a *Worldly*, or *Mundane Number*, becauſe the Sublunary World conſiſts of 4 Elements.

Of 5. *Five* meaſureth and numbereth every Number, in whoſe firſt Place is 5 or 0; It is the ſecond Simple odd Number, and the firſt Circular Number; becauſe as a Circle turns to the Point whence it begun, ſo 5 multiplied by it ſelf, ends in 5: Wherefore whatever Equilateral figural Number hath 5 for his Root, in the firſt Place of his Quantity, will 5 be ſtill retained. Five is alſo called the *firſt Central Number*,

Number, becauſe 5 placed in the Center of a Circle, all the reſt of the Digits may be ſo diſpoſed about the Circle, that every two Oppoſites ſhall make 10, the firſt Article, and taking in the Central Number, make every way 15, thus:



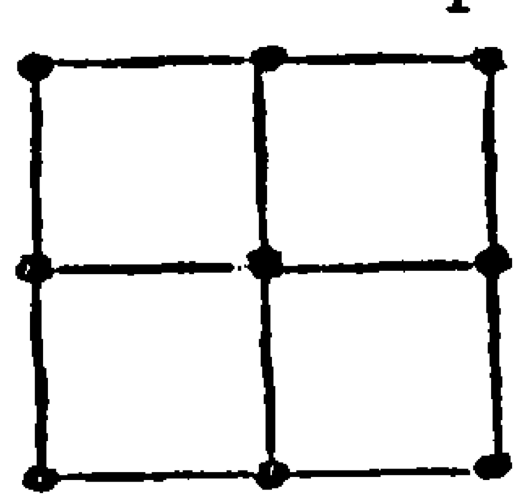
Six, Compoundeth and meaſureth every even Number, which 3 can meaſure. *Of* 6.
It is the ſecond even Compound and Circular Number, and the firſt perfect Number, whoſe even Parts are equal to himſelf, as $1 + 2 + 3 = 6$. Alſo Six is called the firſt *Pyramidal Number*; for the Units therein may be ſo placed, as to repreſent a Pyramis, thus:

.
. .
. . .

Seven, The old *Magi* called a *Virgin Number*, ſuppoſing the Force thereof great, as a Virgin in her full ſtrength: But this Force is diſcerned in things concrete, and dependeth not on the Quantity of 7; and ſo every 7*th* Year bringing ſome change in Nature, is Climacterical. It is ſometime called the *Sacred and Quiet*, or *Sabbatory Number*, becauſe in Sacred Writ, the Seventh Day and Year were appointed to be reſted in. *Of* 7.

Eight, Is the firſt proper Cubick Number, every Cube having 8 Corners. It compoundeth and numbereth every Number, whoſe Figure comprehended under the three firſt Figures it can number. It is a chief Note in Muſick, and taketh turns with 6 in the Termination of perfect Numbers, for they alternately end in 6 and 8. as 6. 28.96.8128. 130816. 2096128. 33550336. 536854528. &c. *Of* 8.

Nine, Is the ſecond Square Number, the firſt compound odd Number, and the laſt Digit. What Number ſoever it is applied to, rejecting the Nines of that whole Number taken in Groſs, or the Nines of the ſeveral Parts taken Simple, will leave the Remains alike. As 27 makes 3 times 9: Wherefore whether the Nines of 27 be rejected, or of 2 and 7, the Remain will be all one (to wit 0.) The Units in 9 regularly diſpoſed, repreſent 4 the Square of 2. And by an orderly placing in every Point and Corner one of the Digits, the whole 9 will not only be taken up, but counted up any way as they ſtand, will make 15. *Of* 9.



8	3	4
1	5	9
6	7	2

Scaliger called 9, the *chief and moſt perfect Number*; and *Exerc.* 365. *S.* 1. ſaith, It can be increaſed of none but the *Unit* to be made 10; and containeth in it ſelf all Species and Proportions of Quantity, as well Primary as Conſequent and Reſultant: For in it are *Length*, *Breadth*, *Depth*, *Perfect*, *Imperfect*, *Diviſible*, *Indiviſible*, *Triangle*, *Cube*, *Oblong*, *Plurilateral*, *Equality*, *Inequality*, *Abſolute*, *Comparate*, *Simple*, *Manifold*; and in *Specie* Double, *Seſquialter*, *Triple*, *Seſquitertia*, *Quadruple*, *Superpartiens*, &c.

Ten, Is the firſt Article, and by adjoining Ciphers to the right Hand, or increaſing the Unit to the left Hand, other Articles will be produced, as 10. 100. 1000, *&c.* or 20, 30, 40, *&c.* It meaſureth and compoundeth every Number, in whoſe Right-hand Place is a Cipher. It comprehendeth all the Digits. The firſt round Number, becauſe now they begin with the Digits again as in a Circle; and hence by the *Pythagoreans* called *Circular*, and counted a perfect Number. *Of* 10.

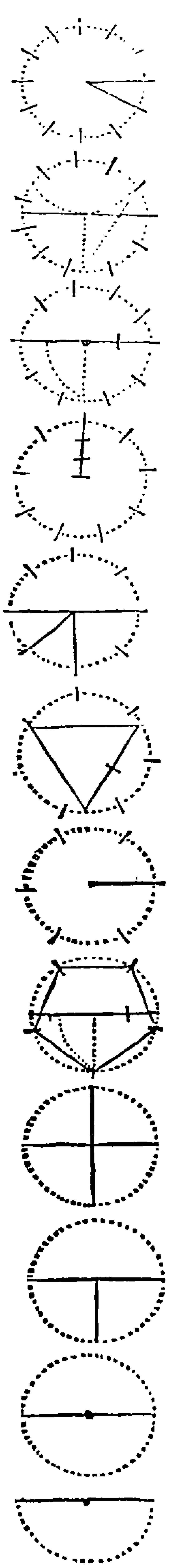
Eleven, multiplied by any Digit, beginneth and endeth alike; as 11 by 2, is 22; by 3, is 33, *&c.* *Of* 11.

Twelve, Is the Perimeter of a Triangle, whoſe *Area* is 6. For if 3, 4, 5, which make 12, be the Sides of a Triangle, half the Product of 3×4, which is 12, ſhall be the *Area*, as in Figural Numbers before was demonſtrated. *Of* 12.

In

Sir Balt. Gerbier's *Notes of the Circle.*

In imitation of these 12 Notes, because a Circle may be divided into such Portions, as the *Polygones* of several regular Works may be noted thereby. The Divisions of a Circumference in concordancy to 12, taken out of Sir *Baltasar Gerbier*, with a little Alteration, shall serve for a final Conclusion of all this Work.



1. The Circle being a round Figure, representeth a Cipher or 0. The Center of which doth denote the *Unit*, or 1. From whence infinite Deductions are into Multitude; as from the Center infinite Lines may be drawn to the Circumference: And by cutting the whole Circle into Parts, shall be infinite Sections beneath the Whole; as from 1 broken into Pieces, ariseth infinite Fractions. And thus the Diameter terminating, the Circumference represents one half the Semicircle.

2. The Diameter drawn through the Center to the Circumference, divideth the Circle into two equal Parts.

3. A Perpendicular falling from the Circumference on the Diameter in the Center, parteth the Circumference into 3 Parts, of which the lesser 2 are equal to the Third.

4. Two Diameters crossing each other at Right Angles in the Center, divide the Circle into four equal Parts, called *Quadrants*.

5. From the half of the Semidiameter, to the half of an Arch drawn from the Circumference to the touch of the other Semidiameter; the distance of the Touch to the first half, gives a Fifth of the Circumference, or very near it.

6. The Semidiameter applied to the Circumference, is little less than the sixth Part thereof.

7. Half one of the Sides of the greatest Equilateral Triangle inscribed in a Circle, shall equal the seventh Part of the Circumference pretty exactly.

8. The Quadrant equally bissected, shall exactly part the Circumference into eight Parts.

9. Two thirds of the Semidiameter, parteth the Perimeter into 9 equal Divisions.

10. From the half of the Semidiameter, to the half of an Arch drawn from the Circumference to the touch of the other Semidiameter, the Distance of the Touch to the Center will give nigh the Tenth.

11. The Length of a Line from the Point where two Circles cut each other to the Semidiameter, may be taken from the eleventh Part of the Perimeter.

12. The third Part of the Quadrant is the twelfth Part of the Circle, and of the Semicircle the Sixth.

Totius Operis Finis.

Soli Deo Gloria.

A N

Alphabetical Table,

Directing to ſome ſpecial Points noted in the *precedent Treatiſe.*

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To the READER.

I Know no Reasons that can be assigned for suppressing the Errata: unless to impose on a Credulous Buyer, that the Impression needs no Correction; or that the Author (if found in an Error) may take Sanctuary under the Mistakes of the Press. But neither of these Motives, have any Prevalence with me; as being fully assured, that the Success would prove as contemptible as the Design: When by the Discovery of some Errata; the Reader would (as the natural Consequence of such an unjust Concealment) be tempted, to judge it unsafe to rely in any thing, either upon the Printer or the Author.

I do therefore give this Advertisement; That though the distance of my habitation from London, permitted me not to correct the Sheets as they were printed off. Yet I have (before the Publication) carefully examined the whole Impression by the Original Manuscript left by my Father at his death. And have exactly noted all the deviations of the Press, that can possibly mislead the most unexperienced Tyro. So that I may perhaps incurr the censure of being unnecessarily scrupulous; in that, together with the Errors which are more material; I have also inserted those that the meanest Genius will scarce think worth the trouble of noting with his Pen; as being only literal.

London March 26th.
1696.

S. Jeake.

ERRATA sic corrigas.

PAge 1. line ult. read p. 66. p. 2. l. 24. r. Hypsometria, l. 35. r. Hypogeiodia, p. 3. l. 23. r. D. Sixty, l. 24. r. ID. Fifteen, l. 55. r. XVIII for XXII, p. 4. l. 14. r. Affections, l. 39. r. Lesse, p. 5. l. 35. dele the latter [into], p. 7. l. 1. r. Solid, & Marg. l. 1. r. 3. Solid, p. 8. l. 31. r. mixed either, p. 10. l. 17. Lat. r. Grana, l. 51. r. Æs, p. 11. l. 16. r. Imum, p. 12. l. penult. r. ▭ next Greater, p. 13. l. 1. r. .. l. 54. r. Second Book, p. 14. l. 53. r. Quintillion, p. 16. l. 16. r. —Prime Part of Composition, which is Addition, p. 19. l. 8. r. Cyphers, l. 17. r. leaves, p. 21. l. 2. r. 40, l. 14. r. preceding, p. 22. l. 15. r. If of two, p. 23. l. penult. r. The, p. 25. l. 28. r. substracted, p. 27. l. 29. r. Rectangles, p. 29. l. 14. Marg. for Geometry, r. Grounded on, p. 30. l. 46. r. Multiplee, p. 31. l. 54. r. figures, p. 32. l. 14. r. comprehended, p. 36. set the figures over the Dividends a little more to the Right hand, p. 44. l. 17. r. Fractions of Fractions, p. 46. l. 42. for $\frac{1}{1}\frac{2}{2}$. r. $\frac{13}{16}$, and transpose the Comma that is there set after [16 and 12] setting it after [Denominator], l. antepenult. for $\frac{16}{13}$. r. $\frac{16}{12}$, p. 48. l. 31 In the latter Dividend, r. $\frac{18}{4}$ p. 49. l. 1. for, in, r. is, and for, is, r. in, p. 50. l. 25. Marg. r. pag. 59, p. 51. l. 12. r. Case, l. 15. r. $\frac{3}{4}$ & $\frac{3}{4}$ & $\frac{3}{2}$, And r. $\frac{8\ \&\ 9\ \&\ 18}{12}$. l. 19. at C. for $\frac{3}{4}$ r. $\frac{3}{4}$, p. 53. l. 21. r. Subtrahends, p. 59. l. ult. Marg. r. p. 50, p. 62. l. 8. r. considered, l. 16. r. Vide pag. 106. to 152, p. 63. l. 31. r. Verstegan, p. 66. l. 2. r. Tail, l. 22. r. if sold, p. 68. l. 40. r. Washers, l. penult. r. the Billet, p. 70. l. 13. r. Bushels, p. 73. l. 33. r. pag. 95, p. 74. l. 14. r. Wastel, l. 19. r. White-Bread, l. 35. r. such Horse loaves, l. 37. r. assized, p. 75. l. 43. in the last Column, Tit. 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Exam. 2. *l. ult.* of Caſe 2. *r.* 85 : 3 : 4. *p.* 435. *l.* 6. *for* 35 l. *r.* 35 lb. *p.* 446. *l.* 24. *for* $\frac{207}{48}$. *r.* $\frac{207}{84}$. *p.* 447. *l.* 21. *for* $2\frac{1}{3}$. *r.* $2\frac{2}{3}$. *p.* 449. *l.* 33, 34. *r.* Suppoſititious, *p.* 453. *l.* 28. in the multipl. by 2, *for* 8. *r.* 84. *p.* 455. *l.* 36. in the Quot. *r.* 20 Crowns, *l.* 38. Divid. *for* 024. *r.* 240. *p.* 459. *l.* 18. *r.* abbreviate, *p.* 469. *l.* 2. *for* $\frac{11}{1}$. *r.* $\frac{11}{17}$. *l.* 10. *dele* the line between 1680. and 840. *l.* 21. *for* $\frac{}{2}$. *r.* $\frac{1}{2}$. *l.* 27. *r.* orderly by, *p.* 472. *l.* 37. *for* three Sexes, *r.* ſeveral Sexes, *p.* 477. Col. *ult. l.* 6. *for* 33. *r.* 13. *l.* 7. *for* 23. *r.* 33. *p.* 478. *l.* 8. *for* 38. *r.* 14. *p.* 482. *l.* 44. *r.* added, *p.* 486. *l.* 11. *r.* 6566(40. *l.* 16. *dele* the line between 29184. and 3648. *p.* 503. *l.* 40. *for* $2\frac{1}{1}$. *r.* $2\frac{1}{2}$. *p.* 525. *l.* 3. & Marg. *r.* Iſoperimeters, *l.* 8. *r.* Iſoperimetral, *l.* 45. *r.* *Spheroide*, *p.* 531. *l.* 20. *for* of *r.* or, *p.* 534. *l.* 32. *r.* := T. *p.* 541. *l.* 30. *r.* allotted, *p.* 551. *l.* 2. *r.* he hath, *p.* 553. *l.* 31. *r.* Numbers, *p.* 560. *l.* 21. place a Semicolon after [Integers], *l.* 28. place the Comma before [is], *p.* 566. *l.* 23. *r.* in the Third, *p.* 567. *l.* 40. *r.* till 3. *l.* 42. *dele* and, *p.* 568. *l.* 37. place a Comma after [ſought] and *dele* the latter [ſelf], *l.* 43. *r.* Quotients, *p.* 572. *l.* 16. *for* ⊏ *r.* ⊏ *p.* 573. *l.* 16. place I. over $\frac{3}{4}$. R. over $1\frac{1}{2}$ &c. *for* P. 577, 578. *r.* 578, 579. *p.* 578. *for* 579. *l.* 22. *r.* Pence in a, *p.* 585. *l.* 43. *r.* thereto, *p.* 591. *l.* 34. *r.* take, *p.* 595. *l.* 8. *dele* the negative Sign —. *p.* 613, &c. *for* Equations, *r.* Æquations, *p.* 613. *l.* 23. *r.* Andreas, *p.* 622. *l.* 11. *for* + 3. *r.* + 2 3. *p.* 630. *l.* 8. *r.* Ports, *p.* 631. *l.* 44. *r.* C to be, *p.* 637. *l.* 11. *r.* that is, *l.* 48. *for* Sign, *r.* Sign —, *p.* 643. *l.* 19. *for* Number *r.* Magnitude given. *for* the Rectangle *r.* a Rectangle, *l.* 20. *for* of *r.* under, *l.* 23. 25. manifeſt, ſhewn, *i. e.* expounded, *l.* 25. *for* denied *r.* negatively, *or* being negative. *for* affirmed *r.* affirmatively, *p.* 644. *l.* 10. *r.* 5 F. *l.* 25. *r.* 2. Let any Æquation be propounded : As this, &c. *l.* 31. *r.* Of the Root L. to be found out of theſe, there are 2. parts, *l.* 33. *r.* equal, *l.* 39. *r.* Coëfficient, *l.* 49. *for* ſquared *r.* the Square, *l.* 50. *dele* of, *l.* 51. *dele* the firſt [of], *p.* 645. *l.* 2. *dele* then, *l.* 21. *r.* in the propounded Æquation, *l.* 33. *dele* Figure of the, *p.* 646. *l.* 36. *for* alſo *r.* although, *p.* 647. *l.* 19. *for* his own *r.* that ſame, *l.* 32. *for* may be *r.* be much, *l.* 46. *r.* uſed mixtly, *l. penult.* and *p.* 648. *l.* 15, 16. *for* may *r.* are to, *l.* 26: Marg. *for* of *r.* by, *l.* 30. *dele* ſought, *l.* 36. *r.* 4879. *p.* 649. *l.* 2. *dele* ſought. After *l.* 23. Add, Therefore the true Root falls between 34 and 36. *l.* 31. *for* to be *r.* be, *l.* 39. *for* they leſſen *r.* leſſen it, *p.* 650. *l.* 21. *r.* Sign of R. *l.* 29. *for* Or *r.* And, *p.* 652. Ex. 3. *l.* 2. place a tittle over 7 q. *l.* 35. *dele* it, *p.* 654. *l. ult.* ſave 3, place the Comma after [multiplied], *p.* 658. *l. ult.* ſave 3, Col. 2. *r.* and the Coëff : *l. antepen. dele* the Comma, and ſet it in the Period's place, *p.* 663. *l.* 13. *r.* Sabbatary, *p.* 664. The Diagrams are ſet the wrong ſide upwards, *l.* 11. *dele* the Comma, and inſert it *l.* 12. after [half], *l.* 38. *for* from *r.* for.

THE Pointings of the Remains in Mr. Oughtred's 16. Examples of Affected Æquations (*in the Lines marked* R) *are omitted here. If any be deſirous to point them, they may have recourſe to his* Clavis, Edit. 3. Oxon. 1652. *which I eſteem the beſt.*

The Prefaces and Contents at the Beginning, and Table at the End of the Book; as alſo the preceding accompt of the Errata, *have all been examined and corrected by me at the Preſs: And I do not know of any miſtake in the Printing them.*

S. J.